Application-Driven Critical Values for GNSS Ambiguity Acceptance Testing

Sandra Verhagen, Peter J.G. Teunissen, and Jingyu Zhang

Abstract

Integer ambiguity estimation and validation are crucial steps when solving the carrier-phase based GNSS model. For the validation, different ambiguity acceptance tests have been proposed. For those tests often fixed critical values are used, with the important disadvantage that the performance of the tests varies a lot depending on measurement set-up and circumstances. Therefore it is better to use model-driven critical values such that it is guaranteed that the failure rate will not exceed a user-defined threshold.

This contribution will study the model-dependency of the critical values for two well-known acceptance tests, the ratio test and difference test, and then specifically for a given application. This means that mainly the satellite-receiver geometry and number of epochs will be variable. It will be shown that critical values do exhibit a strong dependence on these factors, and it will not be possible to simply use a fixed (i.e., constant) application-driven critical value.

Keywords

Critical value • Integer acceptance test • Model-dependency

1 Introduction

Requirements on both precision and reliability depend on the GNSS application at hand, and drive the choice for receiver and measurement set-up. For (near) real-time applications, very precise positioning is only possible with carrier-phase based GNSS, and consequently relies on the carrier-phase integer ambiguities to be correctly estimated. Therefore both integer ambiguity estimation and validation are crucial steps.

For the validation, different ambiguity acceptance tests have been proposed. For those tests often fixed critical values are used, with the important disadvantage that the performance of the tests in terms of the failure rate and false alarm rate varies a lot depending on measurement set-up and circumstances. Therefore it is better to use model-driven critical values such that it is guaranteed that the failure rate will not exceed a user-defined threshold.

In Verhagen and Teunissen (2013) the model-dependency of the critical value of the popular ratio test was analysed and it was shown how the model-driven values can be determined. The results confirmed that in general it is not advisable to use a fixed critical value for all possible scenarios and/or measurement set-ups.

This contribution aims at analyzing application-driven critical values with the fixed failure rate approach, and to study the dependency on specifically the satellite-receiver geometry and number of observation epochs. The paper starts with a brief description of the procedure to solve the carrier-phase GNSS model, followed by a section on integer

S. Verhagen (✉) • J. Zhang
Department of Geoscience and Remote Sensing, Delft University of Technology, Delft, The Netherlands
e-mail: a.a.verhagen@tudelft.nl

P.J.G. Teunissen
GNSS Research Centre, Curtin University, Bentley, WA, Australia

Department of Geoscience and Remote Sensing, Delft University of Technology, Delft, The Netherlands
e-mail: p.teunissen@curtin.edu.au

International Association of Geodesy Symposia,
DOI 10.1007/1345_2015_36,

acceptance testing. Here, the focus will be on two well-known tests, namely the ratio test and difference test. One specific measurement set-up will be used as an example in Sect. 4 to study the model-dependency of the critical values for both tests.

2 Solving the GNSS Model

The mixed integer GNSS linear(ized) model is defined as

$$\mathsf{E}(y) = Aa + Bb,\ \mathsf{D}(y) = Q_{yy} \tag{1}$$

where E and D denote the expectation and dispersion operators. $a \in \mathbb{Z}^n$ is the integer carrier-phase ambiguity vector and $b \in \mathbb{R}^p$ is the parameter vector with remaining unknown parameters, such as baseline parameters, residual zenith troposphere delays (ZTD) and ionosphere delays. The design matrices are $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{m\times p}$ with $[A\ B]$ of full column rank. The observation vector $y \in \mathbb{R}^m$ contains the double-difference (DD) code and phase observations and is assumed to be contaminated by normally distributed random errors with zero means and variance-covariance matrix Q_{yy}. In general, a four-step procedure is employed to solve model (1).

Step 1: Float Solution The integer property of the ambiguities $a \in \mathbb{Z}^n$ is disregarded and the so-called float solution,

$$\begin{bmatrix}\hat{a}\\ \hat{b}\end{bmatrix} \sim \mathrm{N}\left(\begin{bmatrix}a\\ b\end{bmatrix},\ \begin{bmatrix}Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}}\\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}}\end{bmatrix}\right) \tag{2}$$

is computed with (recursive) weighted least-squares or Kalman filtering. Ideally, this step includes testing for outliers, cycle slips, or other modeling errors.

Step 2: Integer Estimation The float ambiguity estimate $\hat{a}$ is used to compute its integer counterpart, denoted as

$$\check{a} = \mathcal{I}(\hat{a}) \quad \text{with} \mathcal{I} : \mathbb{R}^n \mapsto \mathbb{Z}^n \tag{3}$$

There are different choices of mapping function $\mathcal{I}$ possible, which correspond to different integer estimation methods. Integer rounding, integer bootstrapping and integer least-squares (ILS) are examples of such integer estimators. Of all choices, ILS is proven to be optimal as it achieves the lowest probability of incorrect fixing, referred to as failure rate (Teunissen 1999). ILS is efficiently mechanized in the LAMBDA method (Teunissen 1995; Verhagen and Li 2012).

Step 3: Integer Acceptance Test An integer acceptance test is devised to decide whether or not the integer solution from step 2 is sufficiently more likely than any other integer candidate. Several tests have been proposed in the literature and are currently used in practice. Examples are the ratio test, the difference test and the projector test. Ambiguity acceptance tests are discussed in Sect. 3. If the integer solution is accepted, it is possible to re-evaluate the validation of the GNSS model, since knowing the ambiguities strengthens the model.

Step 4: Fixed Solution The float solution of the baseline parameters is updated using the fixed integer parameters,

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}}Q_{\hat{a}\hat{a}}^{-1}(\hat{a} - \check{a}),\quad Q_{\check{b}\check{b}} = Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{a}}Q_{\hat{a}\hat{a}}^{-1}Q_{\hat{a}\hat{b}} \tag{4}$$

It is pointed out that the VC-matrix $Q_{\check{b}\check{b}}$ is derived based on the error propagation law under the assumption that the integer solution $\check{a}$ is deterministic. This holds true only when the probability of correct integer estimation is sufficiently close to 1. In that case, $Q_{\check{b}\check{b}} \ll Q_{\hat{b}\hat{b}}$, since after successful ambiguity fixing the carrier-phase measurements start to act as very precise pseudorange measurements. However, if the success rate is not sufficiently high, the fixed solution $\check{b}$ is not necessarily better than the float solution $\hat{b}$ (Verhagen et al. 2013).

3 Integer Acceptance Tests

Ambiguity acceptance testing concerns the third step in the procedure described in Sect. 2. It is common practice to use the ILS failure rate and/or a discrimination test to decide on acceptance or rejection of the integer ambiguity solution. Obviously, the ILS failure rate should be sufficiently close to 0, since incorrect fixing may lead to unacceptably large positioning errors. Apart from the ILS failure rate, a discrimination test allows to test whether or not the found integer solution is sufficiently more likely than any other integer candidate. Several tests have been proposed in literature (Abidin 1993; Chen 1997; Euler and Schaffrin 1991; Frei and Beutler 1990; Han 1997; Han and Rizos 1996; Landau and Euler 1992; Tiberius and De Jonge 1995; Wang et al. 1998; Wu et al. 2010). All these tests compare in one way or another the ILS solution $\check{a}$ with a so-called ‘second best’ integer solution $\check{a}_2$. Based on this comparison, the outcome of the discrimination tests is either to accept the integer solution $\check{a}$, or to reject it in favor of the float solution $\hat{a}$.

For all these tests the choice of the corresponding acceptance criterion is one of the challenges to which the recently developed integer aperture estimation theory provides an answer (Teunissen 2003). This theory namely allows to choose the critical value for the tests in such a way that the user gains control over the probability of incorrect fixing, the failure rate. This is referred to as the fixed failure

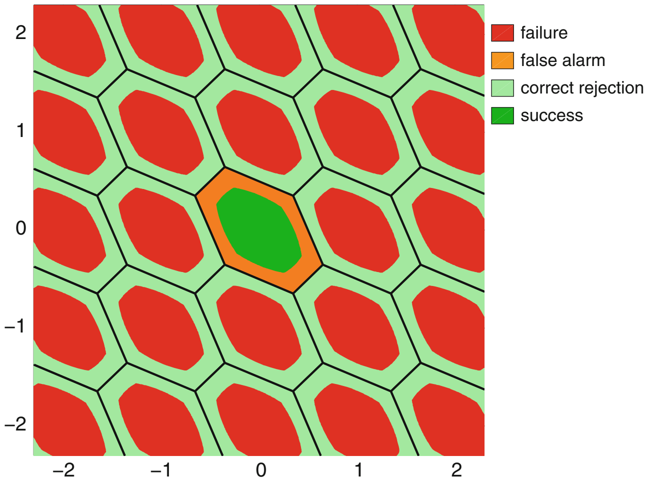

Fig. 1 Two-dimensional ILS pull-in regions (*black*) with acceptance regions. If the float solution resides in a *red* acceptance region, it will be incorrectly fixed (failure), if it resides in the (*dark green*) region it will be correctly fixed (success), otherwise it will be rejected (either a detection or false alarm)

rate approach. It should be stressed, that this is the failure rate *after* the acceptance test, which will be smaller than the ILS failure rate (the failure rate with unconditional acceptance).

The principle is illustrated in Fig. 1 for a two-dimensional example. The ILS pull-in regions are shown; these are regions centered at the integer grid points such that if the float solution resides in a specific pull-in region, the corresponding integer grid point is the ILS solution. The acceptance regions are contained by the ILS pull-in regions, and the size is determined by the critical value of the discrimination test. According to the fixed failure rate approach, the size is thus determined by choosing the maximum failure rate that one finds acceptable. From the figure it will be clear, that a smaller choice for the fixed failure rate will result in a smaller acceptance region.

One question still to be answered then is which discrimination (or acceptance) test to use. This choice will determine the 'shape' of the acceptance region. Interestingly, the integer aperture estimation theory now allows for defining an optimal test (Teunissen 2005). As can be seen from Fig. 1 the size of the acceptance region namely not only affects the failure rate, but the probability of correct fixing as well. The idea is then to define the test such that the probability of correct fixing is maximized for a given failure rate. Disadvantage of the optimal test remains the computational complexity and efficiency. In Verhagen (2005) and Verhagen and Teunissen (2006) the performance of the optimal test was compared with other acceptance tests, from which followed that especially the well-known ratio test and difference test (as defined below) generally exhibit close-to-optimal performance. Therefore these tests will be subject to further analysis in this contribution.

Let the squared norm of ambiguity residuals with respect to integer candidate i be given as

$$R_i = \|\hat{a} - \check{a}_i\|^2_{Q_{\hat{a}\hat{a}}} = (\hat{a} - \check{a}_i)^T Q^{-1}_{\hat{a}\hat{a}} (\hat{a} - \check{a}_i) \tag{5}$$

with

$$\check{a}_i = \arg\min_{z \in \mathbb{Z}^n} \{\|\hat{a} - z\|^2_{Q_{\hat{a}\hat{a}}} \geq R_{i-1}, R_0 = 0\} \tag{6}$$

The ratio test (RT) is then defined as:

$$\text{Accept } \check{a} \text{ iff: } \quad \frac{R_1}{R_2} \leq \mu_{\text{RT}}, \quad 0 \leq \mu_{\text{RT}} \leq 1 \tag{7}$$

The difference test (DT) is defined as:

$$\text{Accept } \check{a} \text{ iff: } \quad R_2 - R_1 \geq \mu_{\text{DT}}, \quad \mu_{\text{DT}} \geq 0 \tag{8}$$

The critical values are denoted μ_{RT} and μ_{DT}.

In Verhagen and Teunissen (2013) the model-dependency of the ratio test was analyzed by considering many different scenarios. Furthermore, it was shown that using fixed critical values - as is common practice - will often lead to unnecessarily high false alarm rates (critical value is too conservative), implying longer times-to-fix, or conversely to high failure rates, which may lead to unacceptably large positioning errors.

4 Application-Driven Critical Values: Example

4.1 Scenario

In order to investigate whether application-dependent critical values for the ratio test and difference test can be determined, one specific scenario is selected here as an example. Table 1 presents an overview of the model parameters for this specific scenario. A medium-length single baseline scenario is considered, implying that the ZTD is estimated, and between-receiver single-difference ionosphere constraints are applied with a standard deviation of 1cm in zenith (see Fig. 2), cf. Odijk et al. (2012).

Table 1 Model parameters

System	All combinations of GPS, Galileo, BeiDou
# frequencies	2
Locations	Netherlands and Australia
Noise	See Fig. 2
# epochs	1, 2, 3, or 4
Atmosphere	Ionosphere-weighted model ZTD estimated
Fixed failure rate	0.1%

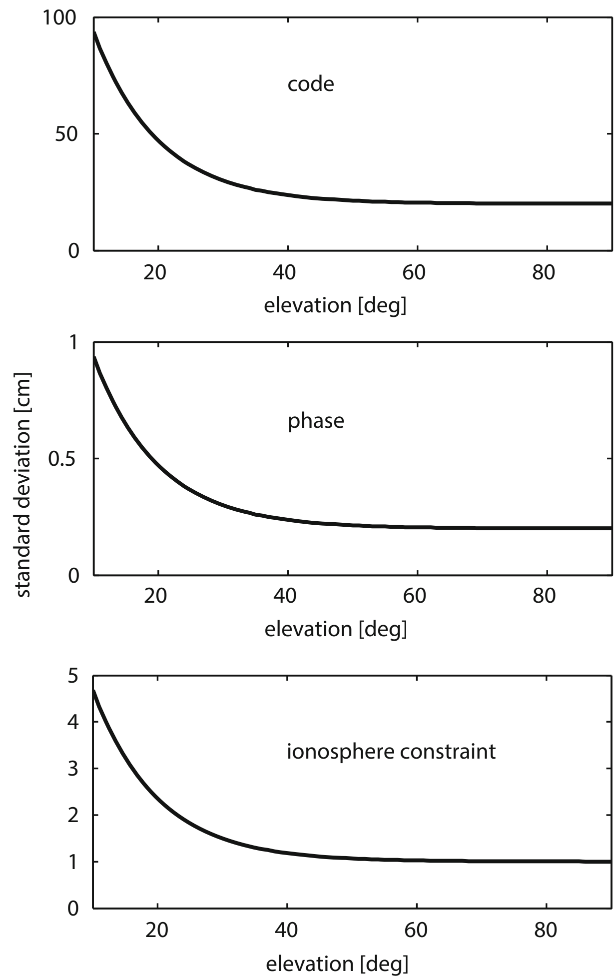

Fig. 2 Noise as function of elevation for code and phase observations, as well as for ionosphere constraints

All results are based on simulations assuming the availability of the GPS, Galileo and BeiDou systems at full operational capability. All combinations of the three systems are considered as well, where double-differencing is employed per system (i.e., one reference satellite per system). For GPS the constellation as of July 2013 is used.

Two different geographic locations are considered, one in Europe at 50°N, and one in Australia at 18°S. The latter was chosen because of the excellent visibility of the BeiDou geostationary and inclined geosynchronous orbiting satellites (which are not visible in Europe). In order to study many different satellite-receiver geometries, more than 1,000 different observation times were selected over a 10-day period. The procedure for determining the critical values of an acceptance test based on simulations with many different models as described in Verhagen (2005) and Verhagen and Teunissen (2013) is used here.

4.2 Critical Values

Figures 3 and 5 shows the critical values of the ratio and difference test as function of number of satellites. Each marker corresponds to a specific satellite-receiver geometry. The corresponding number of systems is indicated by the marker-type. If the relation between critical value and number of satellites as shown in Figs. 3 and 5 is plotted per system and also per geographic location, this does not alter the pattern. Therefore, the results with different systems and locations are combined and not presented separately. In a specific measurement set-up, not only the satellite-receiver geometry will change, but also the number of epochs used to estimate the parameters. Here, the 1- up to 4-epoch models (from top to bottom) are considered.

Obviously, the satellite-receiver geometry is an important factor in most scenarios: there is quite some variability in the critical values for a given number of satellites.

For the 1-epoch scenarios, the model strength is generally poor. Even though the ratio test critical value shows less dependence on the satellite-receiver geometry for a given number of satellites, the corresponding fix rates (see Fig. 4) are generally low.

With the single-epoch model there are instances where the 3-system provides higher ILS failure rates than the 1- or 2-system constellation for the same time of observation; as a result the ratio test critical values can then be chosen larger with the 1- or 2-system constellation than with the 3-system (see Fig. 3: the **maximum** critical values with the 3-system are lower than the **maximum** values with the 1- and 2-systems). This implies that there is a dimensional curse: the much larger number of integer ambiguities to be estimated is not compensated sufficiently by improved float parameter precision due to better geometry and redundancy. This occurs for instance if the ILS failure rate with system A is very low, but with system B is very high; combining the two systems may then result in a higher ILS failure rate than with only system A.

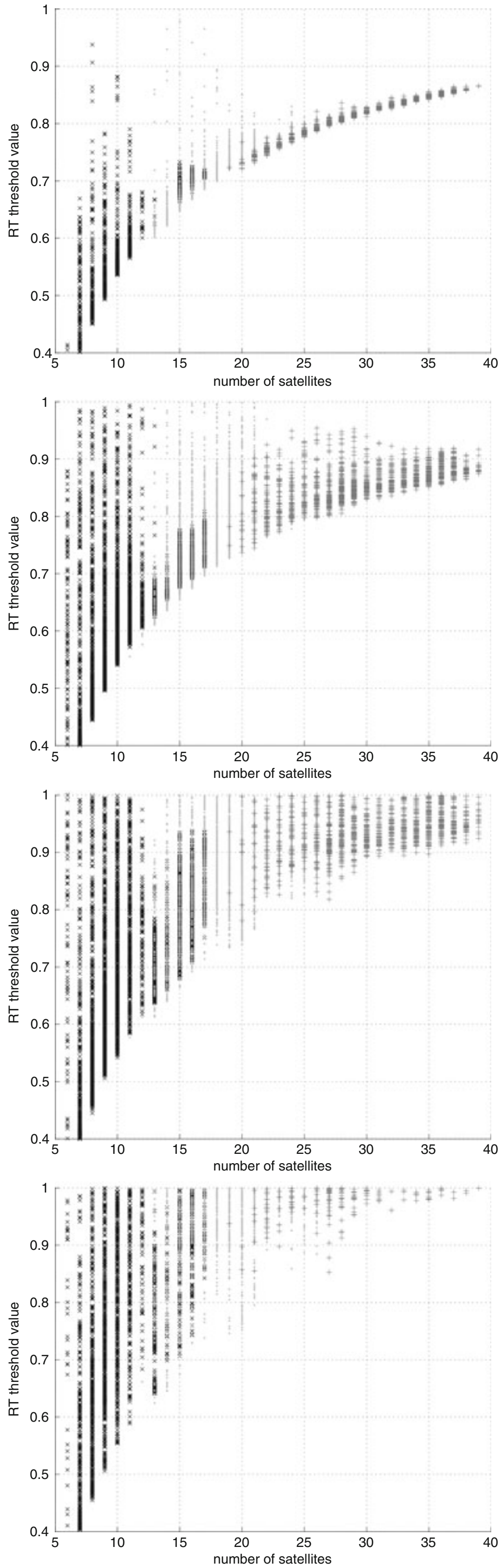

Fig. 3 Ratio test critical value with a fixed failure rate of 0.1% as function of number of satellites for many different times of observation and for all combinations of GPS, Galileo and BeiDou. From *top to bottom*: 1 to 4 epochs (*black crosses*: 1 system, *light grey filled circles*: 2 systems, *dark grey plus symbols*: 3 systems)

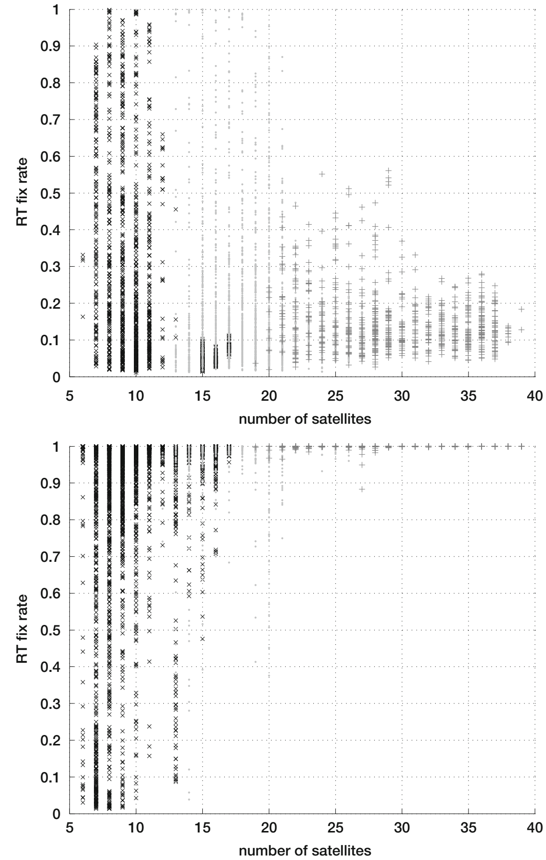

Fig. 4 Ratio test fix rate with a fixed failure rate of 0.1% as function of number of satellites for many different times of observation and for all combinations of GPS, Galileo and BeiDou. From *top to bottom*: 1 and 4 epochs (*black crosses*: 1 system, *light grey filled circles*: 2 systems, *dark grey plus symbols*: 3 systems)

For the ratio test it is clear that with increasing dimension, the minimum critical value over all satellite geometries is increasing as well. This is not per se related to more model strength, but also an effect of the dimension, i.e. number of ambiguities, itself. This is especially obvious for the 1-epoch models, where the fixing rate can be lower while the critical value is generally higher for larger number of satellites.

The relation between number of satellites and difference test critical value is not as obvious, as can be seen from Fig. 5; even for a fixed number of satellites, the variability in the critical value is very large.

The fix rates for the difference test are not separately shown, but show an almost similar pattern as for the ratio test in Fig. 4. With more epochs, the precision of the float solution will obviously improve. This implies that the acceptance regions can be chosen larger, which means a higher critical value for the ratio test and a smaller critical value for the difference test. As a consequence the fix rates will increase. The same applies if we would consider the same scenario but then for the triple-frequency case.

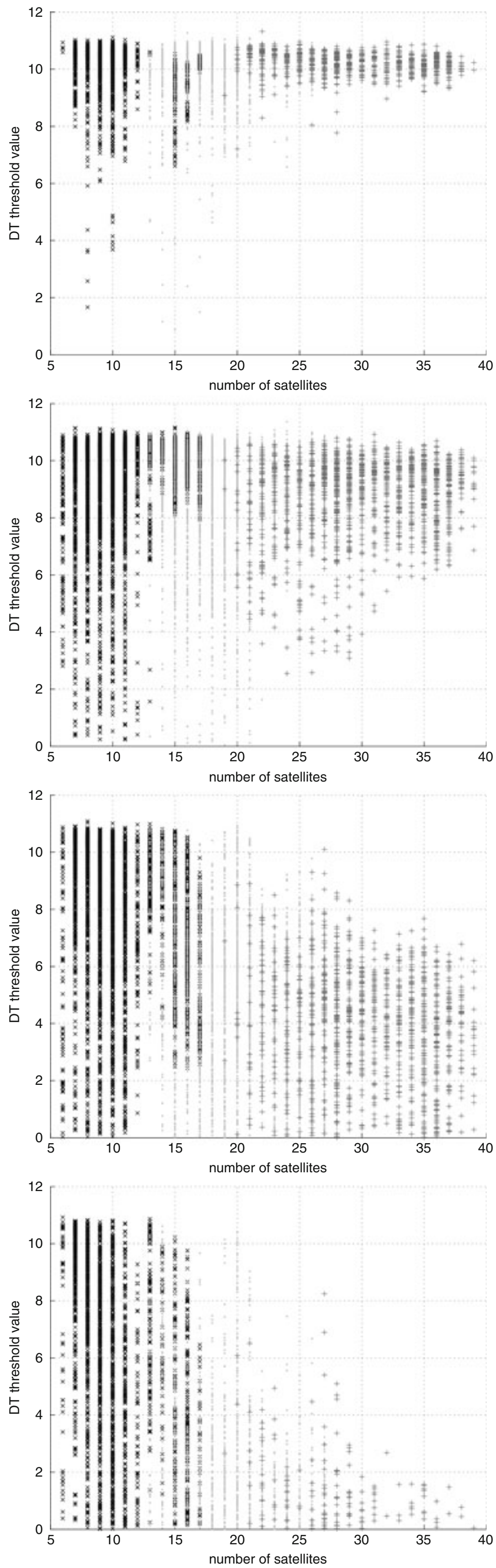

Fig. 5 Difference test critical value with a fixed failure rate of 0.1% as function of number of satellites for many different times of observation and for all combinations of GPS, Galileo and BeiDou. From *top to bottom*: 1 to 4 epochs (*black crosses*: 1 system, *light grey filled circles*: 2 systems, *dark grey plus symbols*: 3 systems)

5 Concluding Remarks

It is not advised to use a fixed critical value for the ratio test or difference test, even for a given application for which many model parameters will be fixed, such as the observation types (depending on system and frequencies), measurement noise, geographic region, and baseline length. The results in this contribution show that for a fixed failure rate, the corresponding critical values can be highly variable depending on satellite-receiver geometry and number of epochs.

For the ratio test the procedure as sketched in Verhagen and Teunissen (2013) can be used to devise look-up tables from which the appropriate critical value can be determined for a given dimension and ILS failure rate (these parameters can be determined prior to the actual IAR). The present contribution shows that it can be interesting to create such a table for the application (or: scenario) at hand and the required maximum allowable failure rate, i.e. the so-called fixed failure rate. In this way, users can determine the 'best' critical values for their needs.

The procedure is as follows. The GNSS model for many different satellite-receiver geometries and different numbers of epochs must be set-up. For each of these models, the critical value can then be empirically determined with a simulation procedure: generate a large number of float ambiguity samples for the model at hand, and tune the critical value such that the failure rate becomes equal to the required value. The current contribution shows an example for single baseline dual-frequency GNSS; future research will also address single- and multi-frequency scenarios, as well as PPP or multi-baseline processing.

For the difference test, a useful relation between critical value, and the dimension and ILS failure rate has not been found. In future work, it will be further investigated why this is different from the ratio test and also how it will be possible to efficiently determine the appropriate critical value for a fixed failure rate.

Acknowledgements This work has been executed in the framework of the Positioning Program Project 1.01 "New carrier phase processing strategies for achieving precise and reliable multi-satellite, multi-frequency GNSS/RNSS positioning in Australia" of the Cooperative Research Centre for Spatial Information.

References

Abidin HA (1993) Computational and geometrical aspects of on-the-fly ambiguity resolution. Ph.D. Thesis, Department of Surveying Engineering, Technical Report no.104, University of New Brunswick, 314 pp.

Chen Y (1997) An approach to validate the resolved ambiguities in GPS rapid positioning. In: Proceedings of the international symposium on kinematic systems in geodesy, geomatics and navigation, Banff

Euler HJ, Schaffrin B (1991) On a measure for the discernibility between different ambiguity solutions in the static-kinematic GPS-mode. In: IAG symposia no. 107, kinematic systems in geodesy, surveying, and remote sensing. Springer, New York, pp 285–295

Frei E, Beutler G (1990) Rapid static positioning based on the fast ambiguity resolution approach FARA: theory and first results. Manuscr Geod 15:325–356

Han S (1997) Quality control issues relating to instantaneous ambiguity resolution for real-time GPS kinematic positioning. J Geod 71:351–361

Han S, Rizos C (1996) Validation and rejection criteria for integer least-squares estimation. Surv Rev 33(260):375–382

Landau H, Euler HJ (1992) On-the-fly ambiguity resolution for precise differential positioning. In: Proceedings of ION GPS 1992, The Institute of Navigation, Fairfax, pp 607–613

Odijk D, Verhagen S, Teunissen PJG (2012) Medium-Distance GPS Ambiguity Resolution with Controlled Failure Rate. In: Kenyon S et al (eds) Geodesy for planet earth. International association of geodesy symposia, vol 136. Springer, Berlin, pp 745–751

Teunissen PJG (1995) The least-squares ambiguity decorrelation adjustment: a method for fast GPS integer ambiguity estimation. J Geod 70:65–82

Teunissen PJG (1999) An optimality property of the integer least-squares estimator. J Geod 73:587–593

Teunissen PJG (2003) Integer aperture GNSS ambiguity resolution. Artif Satell 38:79–88

Teunissen PJG (2005) GNSS ambiguity resolution with optimally controlled failure-rate. Artif Satell 40:219–227

Tiberius CCJM, De Jonge PJ (1995) Fast positioning using the LAMBDA method. In: Proceedings of DSNS'95, Bergen, paper no. 30

Verhagen S (2005) The GNSS integer ambiguities: estimation and validation. PhD thesis, Delft University of Technology, Netherlands Geodetic Commission, No. 58

Verhagen S, Teunissen PJG (2006) New global navigation satellite system ambiguity resolution method compared to existing approaches. J Guid Control Dyn 29:981–991

Verhagen S, Li B (2012) LAMBDA Software Package - Matlab implementation, version 3.0. Delft University of Technology and Curtin University, 39p

Verhagen S, Teunissen PJG (2013) The ratio test for future GNSS ambiguity resolution. GPS Solutions 17:535–548

Verhagen S, Li B, Teunissen PJG (2013) Ps-LAMBDA: Ambiguity success rate evaluation software for interferometric applications. Comput Geosci 54:361–376

Wang J, Stewart MP, Tsakiri M (1998) A discrimination test procedure for ambiguity resolution on-the-fly. J Geod 72:644–653

Wu Y, Jin SG, Wang Z, Liu J (2010) Cycle slip detection using multi-frequency GPS carrier phase observations: a simulation study. Adv Space Res 46(2):144–149