International Global Navigation Satellite Systems Society
IGNSS Symposium 2011

University of New South Wales, Sydney, NSW, Australia
15 – 17 November 2011

GNSS ambiguity resolution: which subset to fix?

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ABSTRACT

A key issue with GNSS carrier phase ambiguity resolution is that often the full set of ambiguities cannot be fixed fast and reliably. A possible strategy is then to resolve only a subset of ambiguities, one for which the probability of correct fixing, the so-called success rate, is sufficiently close to 1. However, a proper subset selection criterion is still lacking. This criterion should on the one hand guarantee an acceptably high success rate, and at the same time result in a significant performance improvement with respect to the remaining parameters, like the baseline parameters. The second requirement is important and has not yet been addressed in literature. As an extreme example consider the case where the float ambiguities are not correlated with the other float parameters. Ambiguity resolution would then be useless, since it will then not allow for an improvement of these other parameters. This indicates that resolving a subset of ambiguities (in the extreme example, an empty subset) may lead to the same, or almost same, performance improvement of the other parameters, notably the baseline solution.

This contribution presents two approaches to subset selection, where both the requirements on success rate and performance improvement are taken into account. With this approach the user can set a threshold for the success rate to be obtained. It will be shown how much better the baseline solution will be after reliable fixing of the subset of ambiguities.

KEYWORDS: Partial ambiguity resolution, success rate, precision and reliability.

1. INTRODUCTION

Precise and reliable GNSS positioning is of crucial importance in many applications. As such, carrier-phase observations, known to be much more precise than the code observations, are commonly used, both in relative positioning but nowadays also precise point positioning (PPP) set-ups. As a consequence, the unknown integer number of carrier cycles, called integer ambiguities, must be estimated, since only after successful ambiguity fixing the carrier-phase observations start to act as very precise pseudoranges. The problem of integer estimation has long been solved. However, in many situations reliable ambiguity resolution is not feasible due to high noise levels and/or biases in the data. In those cases, partial ambiguity resolution (PAR) may be useful. It means that a subset of the ambiguities is selected which can be fixed reliably. Ideally, this should result in an improved precision of the position solution compared to the case where no ambiguities are fixed. The main questions are then, how to select the subset to be fixed and what is the effect on the position estimates. Different approaches have been proposed in literature, and in this contribution two more approaches will be presented.

This paper starts with a review of the carrier-phase ambiguity resolution problem in Section 2. Then a review of existing partial ambiguity resolution methods will be given in Section 3, followed by a description of the proposed new methods. A performance analysis is presented in Section 4.

2. CARRIER-PHASE AMBIGUITY RESOLUTION

2.1 General

As point of departure the following system of linear(ized) observation equations will be taken:

$$y = Aa + Bb + e \qquad (1)$$

where y is the given GNSS data vector of order m, a and b are the unknown parameter vectors of order n and p, respectively, and where e is the noise vector. In principle all the GNSS models can be cast in this frame of observation equations.

The data vector y will usually consist of the 'observed minus computed' single- or double-difference (DD) phase and/or pseudorange (code) observations accumulated over all observation epochs. The entries of vector a are then the DD carrier phase ambiguities, expressed in units of cycles rather than range. They are known to be integers, $a \in Z^n$.

The entries of the vector b will consist of the remaining unknown parameters, such as for instance baseline components (coordinates) and possibly atmospheric delay parameters (troposphere, ionosphere). They are known to be real-valued, $b \in R^p$.

The procedure which is usually followed for solving the GNSS model (1), can be divided into three or four steps. In the first step, one simply disregards the integer constraints $a \in Z^n$ on the ambiguities and applies a standard least-squares adjustment, resulting in real-valued estimates of a and b, together with their variance-covariance (vc-) matrix

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}, \begin{pmatrix} Q_{\hat{a}\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}\hat{b}} \end{pmatrix} \tag{2}$$

This solution is referred to as the 'float' solution. In the second step, the 'float' ambiguity estimate $\hat{a}$ is used to compute the corresponding integer ambiguity estimate $\breve{a}$. This implies that a mapping, $S: R^n \mapsto Z^n$, from the n-dimensional space of reals to the n-dimensional space of integers is introduced such that

$$\breve{a} = S(\hat{a}) \tag{3}$$

The three best known integer estimators are integer rounding, integer bootstrapping and integer least-squares, see Section 2.2.

Once the integer ambiguities are computed, they can be used in the third step as input to decide whether or not to accept the integer solution, although sometimes this step is disregarded. In literature several such 'validation' tests have been proposed in order to decide whether or not to fix the ambiguities. A review and evaluation of the tests can be found in Verhagen (2004). Well-known examples are the ratio test, distance test and projector test. Among the most popular tests is the ratio test, where the decision is made as follows:

1. Apply ILS to obtain $\breve{a}$ and $\breve{a}_2$
2. Evaluate ratio $\dfrac{\|\hat{a}-\breve{a}\|^2_{Q_{\hat{a}\hat{a}}}}{\|\hat{a}-\breve{a}_2\|^2_{Q_{\hat{a}\hat{a}}}}$
3. Decision: If $\dfrac{\|\hat{a}-\breve{a}\|^2_{Q_{\hat{a}\hat{a}}}}{\|\hat{a}-\breve{a}_2\|^2_{Q_{\hat{a}\hat{a}}}} \begin{cases} \leq \mu & \text{use } \breve{a} \\ > \mu & \text{use } \hat{a} \end{cases}$

where $\breve{a}_2$ is the second-best integer solution in the integer least-squares sense, and with the squared norm $\|\cdot\|^2_Q = (\cdot)^T Q^{-1}(\cdot)$.. Note that in practice the reciprocal of the test statistic is mostly used.

The third step is often referred to in the literature as the 'validation' step. It should be noted, however, that this step is not designed to validate the underlying model. Furthermore, the choice of the threshold value μ is often ad hoc or based on false theoretical grounds, see (Teunissen and Verhagen, 2004). Often a fixed value of $\mu = 1/2$ or $1/3$ is used. In Teunissen and Verhagen (2009) the fixed failure rate approach was introduced, which implies that the value of μ is chosen such that it is guaranteed that the probability of wrong fixing will be below a user-defined value. In the remainder, this approach will be called the FF-RT approach.

Once the integer solution is accepted, the fourth step consists of correcting the 'float' estimate of b. As a result one obtains the 'fixed' solution

$$\breve{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}\hat{a}}^{-1} (\hat{a} - \breve{a}) \tag{4}$$

2.2 Integer estimation

The most simple approach for integer estimation would be to simply round the entries of the float ambiguity vector to their nearest integers. However, the correlations between the ambiguities are neglected and this will generally result in high probabilities of wrong fixing. In the following, two integer estimators will be presented which are known to result in higher probabilities of correct fixing, i.e. success rates.

Bootstrapping estimator

A generalization of the integer rounding method is the *sequential* integer rounding method, also referred to as the integer bootstrapping method, see e.g. (Blewitt, 1989; Dong and Bock, 1989). In contrast to integer rounding, the integer bootstrapping estimator takes the correlation between the ambiguities into account. It follows from a sequential conditional least-squares adjustment with a conditioning on the integer ambiguity values from the previous steps (Teunissen, 1993; Teunissen, 1998a). The components of the bootstrapped estimator are given as:

$$
\begin{aligned}
\breve{a}_{n,\mathrm{B}} &= [\hat{a}_n] \\
\breve{a}_{n-1,\mathrm{B}} &= [\hat{a}_{n-1|n}] = [\hat{a}_{n-1} - \sigma_{\hat{a}_{n-1}\hat{a}_n}\sigma_{\hat{a}_n}^{-2}(\hat{a}_n - \breve{a}_{n,\mathrm{B}})] \\
&\vdots \\
\breve{a}_{1,\mathrm{B}} &= [\hat{a}_{1|N}] = \left[\hat{a}_1 - \sum_{i=2}^{n}\sigma_{\hat{a}_1\hat{a}_{i|I}}\sigma_{\hat{a}_{i|I}}^{-2}(\hat{a}_{i|I} - \breve{a}_{i,\mathrm{B}})\right]
\end{aligned}
\tag{5}
$$

where $\hat{a}_{i|I}$ stands for the i-th ambiguity obtained through a conditioning on the $I = \{i+1,\ldots,n\}$ sequentially rounded ambiguities. One should start with the most precise float ambiguity, which in this case is assumed to be $\hat{a}_n$. The real-valued sequential least-squares solution can be obtained by means of the triangular decomposition of the vc-matrix of the ambiguities: $Q_{\hat{a}\hat{a}} = L^T DL$, where L denotes a unit lower triangular matrix with entries

$$
l_{i,j} = \sigma_{\hat{a}_j\hat{a}_{i|I}}\sigma_{\hat{a}_{i|I}}^{-2} \tag{6}
$$

and D a diagonal matrix with the conditional variances $\sigma_{\hat{a}_{i|I}}^2$ as its entries.

The success rate of integer bootstrapping can be evaluated exactly (Teunissen, 1998b):

$$
P_{s,\mathrm{B}} = P(\breve{a}_{\mathrm{B}} = a) = \prod_{i=1}^{n}\left(2\Phi(\frac{1}{2\sigma_{\hat{a}_{i|I}}}) - 1\right) \tag{7}
$$

with a the true ambiguity vector and $\Phi(x)$ the cumulative normal distribution.

It was already mentioned that the bootstrapping procedure should start with rounding the most precise float ambiguity. Moreover, from Eq.(5) it is clear that bootstrapping will generally result in different outcomes if applied to re-parameterized ambiguities. It is known that bootstrapping performs close to optimal if applied to the decorrelated ambiguities $\hat{z} = Z^T\hat{a}$, with $Q_{\hat{z}\hat{z}} = Z^T Q_{\hat{a}\hat{a}} Z$. This is because the sequential conditional variances are largely reduced by the decorrelation.

The decorrelating Z-transformation is implemented in the LAMBDA method such that the n-th transformed ambiguity is the most precise one. Moreover, the decorrelation algorithm is implemented such that after each decorrelation step a reordering is performed, which guarantees that

$$\sigma_{\hat{z}_{i|I}} \leq \sigma_{\hat{z}_{j|I}} \quad \text{for } j < i \tag{8}$$

Eq.(5) in terms of the decorrelated ambiguities $\hat{z}$ can be expressed as:

$$\breve{z}_{j,\mathrm{B}} = [\hat{z}_{j|N}] = \left[\hat{z}_j - \sum_{i=j+1}^{n} \sigma_{\hat{z}_j \hat{z}_{i|I}} \sigma_{\hat{z}_{i|I}}^{-2} (\hat{z}_{i|I} - \breve{z}_{i,\mathrm{B}}) \right] \tag{9}$$

The term $\sigma_{\hat{z}_j \hat{z}_{i|I}} \sigma_{\hat{z}_{i|I}}^{-2}$ can be readily obtained from the $L^T DL$-decomposition of $Q_{\hat{z}\hat{z}}$, see Eq.(6).

Integer least-squares estimator
When solving the GNSS model of Eq.(1) in a least-squares sense, but now with the additional constraint that the ambiguity parameters should be integer-valued, the integer estimator of the second step in the procedure becomes:

$$\breve{a}_{\mathrm{LS}} = \min_{z \in Z^n} \left\| \hat{a} - z \right\|_{Q_{\hat{a}\hat{a}}}^2 \tag{10}$$

This estimator is known to be optimal, cf. (Teunissen, 1999), which means that the probability of correct integer estimation is maximized. In contrast to integer rounding and integer bootstrapping, an integer search is needed to compute $\breve{a}_{\mathrm{LS}}$.

This ILS procedure is efficiently mechanized in the LAMBDA method, see e.g. (Teunissen, 1998a). Note that the success rate of integer least-squares estimation is independent of the parameterization of the float ambiguities. The decorrelating Z-transformation is only required in order to largely reduce the search times, such that the LAMBDA method is highly efficient.

3. PARTIAL AMBIGUITY RESOLUTION

Many options would be possible to select a subset of ambiguities to be fixed in case fixing the full set is not possible. Several approaches have been proposed in literature in which it is tried to fix only the wide-lane ambiguities in case two or more frequencies are being used, cf. (Mowlam, 2004; Cao et al., 2007).

Dai et al. (2007) selects the subset as those integer ambiguities which are identical in the best and second-best solution obtained with LAMBDA. However, it is doubtful whether this is an appropriate selection criterion, since those ambiguities might also be identical but still be wrong in the presence of high noise or large biases.

Henkel and Günther (2010) propose a partial ambiguity resolution scheme in the presence of biases. For that purpose a worst-case accumulation of biases is assumed, which may make the procedure very conservative and some knowledge about the biases to be expected is needed.

Parkins (2011) presents an algorithm where first all possible subsets are formed, which are then ordered according to either the corresponding ADOP (ambiguity dilution of precision, a theoretical performance measure) or the mean SNR (observed signal strength). Starting with the subset with the 'best' value of either the ADOP or mean SNR, the corresponding ambiguities are fixed and validated. If the subset is not accepted, the next best subset is chosen, until a subset is found that can be fixed, if there is any. In a later step, it is tried to extend the subset with ambiguities that were not fixed yet. Success rates obtained with this method are higher than with full-set fixing, as expected. However, the effect on baseline precision was only presented for one static scenario, and it turned out that the precision might be improved, but in some cases also degraded.

Here, two new approaches will be presented. The first is purely model-driven and is based on (Teunissen, 2001). The second approach is both model- and data-driven.

3.1 Partial ambiguity resolution with minimum required success rate

If the success rate obtained with fixing the full set of decorrelated ambiguities is deemed too low, one could opt for resolving only a subset of the ambiguities for which the success rate is sufficiently high. If the minimum success rate required is given by the user as P_0, the goal is then to select the largest possible subset such that:

$$\prod_{i=k}^{n}\left(2\Phi\left(\frac{1}{2\sigma_{\hat{z}_{i|I}}}\right)-1\right)\geq P_0 \tag{11}$$

where only the last $n-k+1$ ambiguities are sequentially rounded. Because of the property in Eq.(8) one can simply start with the n-th decorrelated ambiguity, and check if the success rate of rounding this ambiguity is at least equal to P_0. If that is the case, one can continue with rounding the next conditional ambiguity $\hat{z}_{n-1|n}$, since this is now the most precise one. This procedure continues until the success rate becomes too small. The remaining $k-1$ ambiguities are then conditioned on the $K=k,\dots,n$ conditionally rounded ambiguities, but are not sequentially rounded themselves. The complete 'fixed' ambiguity vector becomes thus:

$$\breve{z}_{\mathrm{B}}=\begin{pmatrix}\hat{z}_{1|K}\\ \vdots\\ \hat{z}_{k-1|K}\\ [\hat{z}_{k|K}]\\ \vdots\\ [\hat{z}_{n-1|n}]\\ [\hat{z}_n]\end{pmatrix} \tag{12}$$

Note that the 'fixed' solution in terms of the original ambiguities can be obtained after applying the back-transformation:

$$\breve{a}_{\mathrm{B}} = Z^{-T} \breve{z}_{\mathrm{B}} \tag{13}$$

If not the complete set of ambiguities is fixed, $\breve{a}_{\mathrm{B}}$ will generally not contain integer elements, since all elements are a linear function of all decorrelated ambiguities $\breve{z}_{\mathrm{B}}$, which are not all integer-valued.

The approach presented in Khanefseh and Pervan (2010) is based on this procedure, although it is extended with a validation step in the position domain.

Of course it is possible to evaluate (11) before applying the sequential conditional rounding, so that it is known beforehand which $n-k+1$ ambiguities can be fixed to integers. This also implies that rather than applying bootstrapping, one can apply LAMBDA to those $n-k+1$ decorrelated ambiguities, and then calculate the corresponding conditional estimates for the remaining $k-1$ ambiguities. This will result in a success rate which is equal to or higher than the bootstrapped success rate as calculated with Eq.(11), since it is known that ILS is optimal.

3.2 Partial ambiguity resolution based on the ratio test with fixed failure rate

The PAR algorithm described above is purely model-driven and does not include a validation step (the third step described in Section 2.1). Of course, it is possible to simply apply e.g. the FF-RT after fixing the subset to decide on acceptance. However, the data does not affect the subset selection. In this subsection, an alternative approach will be presented which is both model- and data-driven. It was developed in the frame of the MSP3 project, funded by ESA, aiming at improving Precise Point Positioning (PPP) solutions with Galileo and GPS. The procedure is as follows.

Assume to be in a certain epoch, in which the total number of ambiguities is equal to n, and an initial set to be fixed is chosen with n_0 ($n_0 \leq n$) ambiguities. This initial set can be chosen for example as the same set as fixed in a previous epoch, or simply equal to all n ambiguities.

If the initial set cannot be fixed according to the FF-RT outcome, it is tried to remove the ambiguities of one satellite at a time. In each step, the subset with the smallest square norm of residuals is selected. If this subset is rejected, it is tried whether it helps to remove the ambiguities of another satellite until a subset is accepted or none of the ambiguities can be fixed. If at a certain stage a subset is accepted, it is tried in a similar fashion to add the ambiguities of satellites that have not been considered before, as they were not in the initial subset (e.g. the ambiguities of newly risen satellites).

The decision from the FF-RT and the subset selection is based on the squared norm of ambiguity residuals, and therefore this approach is data-driven. It is also model-driven, since it depends on both the model strength and the data whether or not the FF-RT will be accepted.

Note that this approach may be computationally heavy, since it is not known on beforehand how many times LAMBDA has to be applied. Therefore it can be useful to apply a constraint on the minimum number of ambiguities to be fixed. This iterative procedure is expected to work well, since often large residuals will be caused by only one or a few satellites. At the same time, the procedure includes an automatic decision criterion on when to start using the ambiguities of newly risen satellites.

4. PARTIAL AMBIGUITY RESOLUTION: IS IT USEFUL?

4.1 Simulated data

With the partial ambiguity resolution procedure based on the minimum required success rate, only a subset of decorrelated ambiguities is fixed, and it is not yet known how much this contributes to an improved baseline solution. In order to assess this, a small analysis was performed. Three different variance matrices Q_{yy} were constructed. For each variance matrix 10,000 samples of float solutions were simulated, and partial ambiguity resolution was applied. Figure 1 shows on the left the success rate as function of number of fixed decorrelated ambiguities $n_f = n - p + 1$ and on the right the empirical probability

$$P(\|\breve{b} - b\| \leq c \cdot \|\hat{b} - b\|)$$

as function of c for a minimum required success rate of 0.99 and 0.999 in black and green, respectively. In the left figures the cross with the same colour indicates the corresponding n_f. The baseline probabilities are also shown for the case that the full set ($n_f = n$) is fixed. Hence, these figures show the probability that the fixed solution will be better ($c < 1$) or worse ($c > 1$) than the float solution.

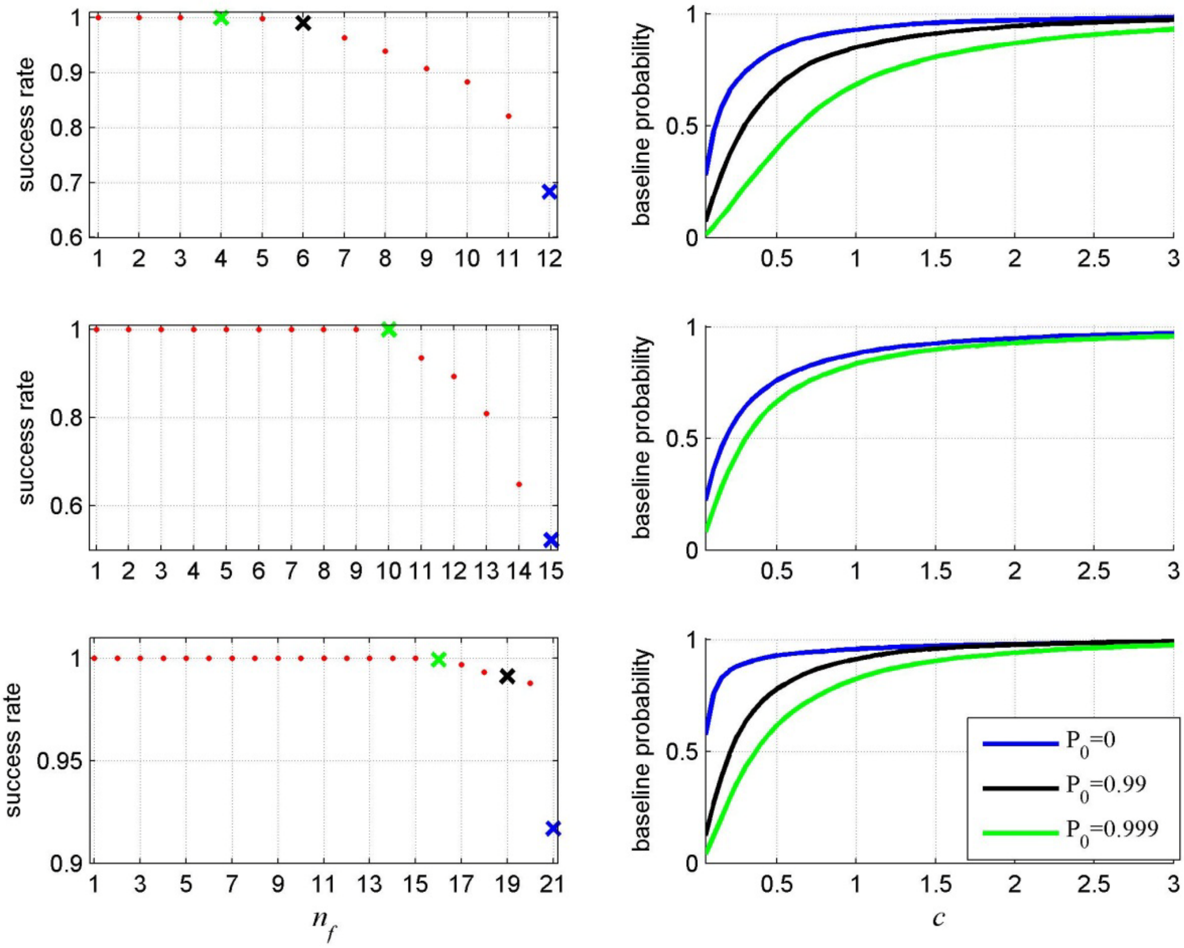

Figure 1. Performance of partial ambiguity resolution with minimum required success rate for three different model. Left: success rate as function of n_f. Right: Probabilities $P(\|\breve{b} - b\| \leq c \cdot \|\hat{b} - b\|)$ as function of c.

It can be observed that even if it is guaranteed that the success rate with partial fixing is above 0.99, the probability that $\breve{b}$ is better than $\hat{b}$ may still be smaller than 0.99. Apparently, fixing a subset of decorrelated ambiguities to integer values does not per se result in an improved baseline solution; it may even become worse while the subset is fixed correctly. Moreover, the baseline probabilities are all smaller than in the case that all ambiguities would be fixed. Therefore it does not seem to make sense to apply PAR based on a minimum required success rate to these weak models.

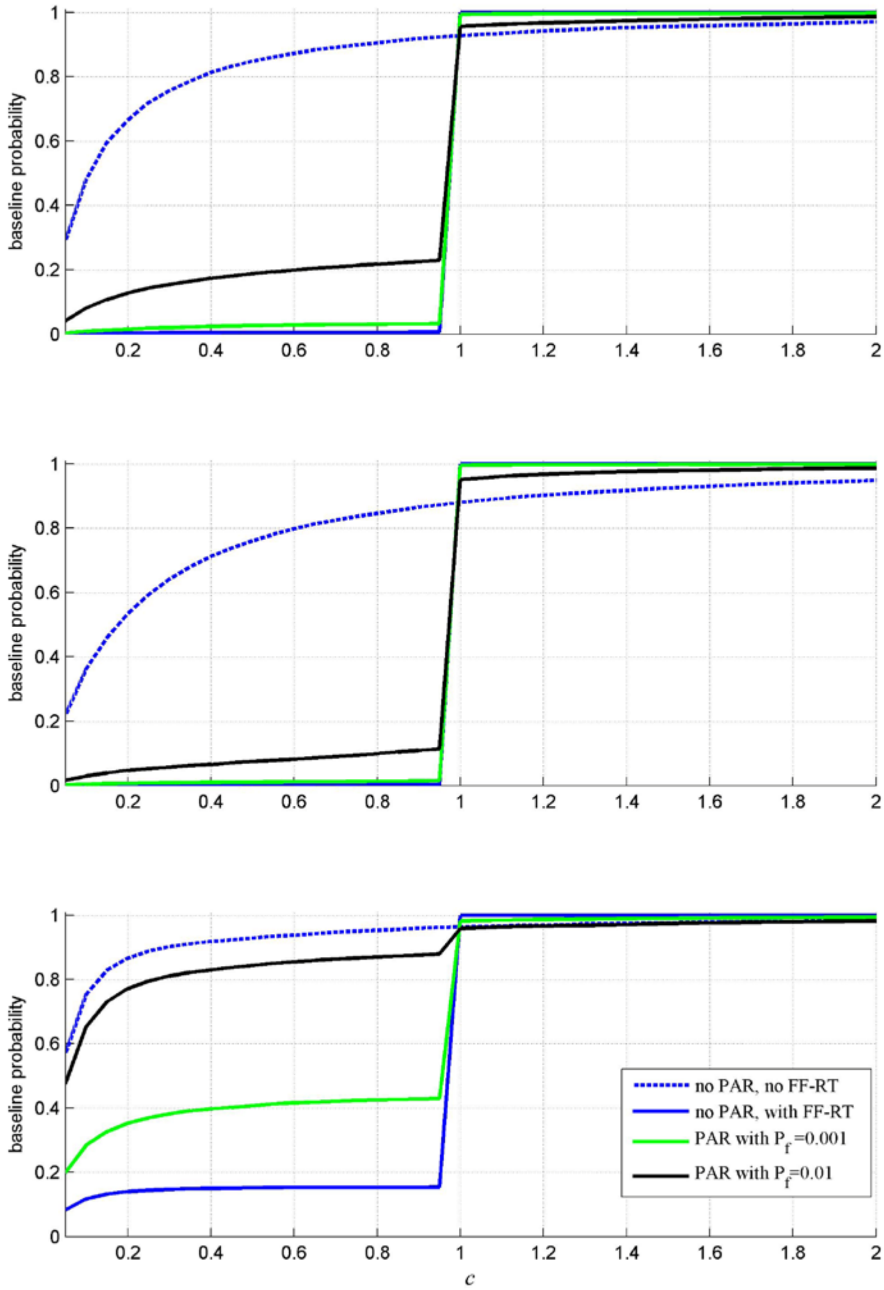

Figure 2. Performance of partial ambiguity resolution based on the ratio test for three different models. Probabilities $P(\|\breve{b}-b\| \leq c \cdot \|\hat{b}-b\|)$ as function of c.

With the success rate-based approach the same subset of *decorrelated* ambiguities will be fixed for all simulated samples (model is the same). With the ratio test-based approach, on the contrary, it depends on the sample which subset of *original* ambiguities is fixed. Similarly as above, this approach was applied to 10,000 simulated samples of float solutions for the different models considered. In this case, for many samples no ambiguities will be fixed at all, such that $\breve{b} = \hat{b}$, and consequently there is a large probability that $\|\breve{b} - b\| = \|\hat{b} - b\|$. Hence the large jumps for $c = 1$ in Figure 2.

The impact of applying the FF-RT can be seen by comparing the blue dotted and blue solid graphs[1]. The first correspond to the case where all ambiguities are fixed (no PAR and/or ratio test applied), the second to the case where the ratio test is applied but no PAR. The models here are all very weak, which results in a very high probability that the ambiguities will not be fixed. Only for the third model (bottom panel), there is a fix probability of 15%, and in that case this always results in a better baseline solution.

The expected advantage of PAR is that there is a higher probability that some ambiguities can be fixed resulting in a better position solution than if the full set of ambiguities would be rejected. This is indeed the case. On the other hand, there is also a somewhat higher probability that $\|\breve{b} - b\| > \|\hat{b} - b\|$ with PAR, but this probability is smaller than if simply all ambiguities would be fixed without ratio test (blue dotted line).

Applying PAR based on the ratio test with a fixed failure rate of 0.001 seems to be a good trade-off: there might still be a reasonable to high probability that the fixed subset results in a better baseline solution, without the risk of unacceptably large position errors due to incorrect fixing.

4.2 MSP3 data results

The simulation results presented above are only indicative for the corresponding weak models. PAR may, however, be especially beneficial in practical situations where the time to fix is long, since the model will become gradually stronger, so that more and more ambiguities can be fixed. In order to investigate how the PAR algorithms may work for a time series of data, a dataset was selected which was generated in the frame of the MSP3 project.

This particular dataset corresponds to a PPP scenario for a static configuration with nominal atmosphere conditions, and with perfect orbit and clock corrections. Of course this is not representative for real-life, however, it does allow for a good analysis of the ambiguity resolution performance, since the solution is not hampered by the accuracy of the orbit and clock products.

All results below are based on the following scheme. First, only the wide-lane ambiguities are selected and fed in the partial fixing algorithm. If some of these wide-lane ambiguities were fixed, an attempt to fix a subset of the narrow-lane ambiguities is made.

The top panels of the figures show the position errors in the float and fixed solutions. The bottom panels show the number of fixed ambiguities together with the total number of unknown ambiguities for each epoch. The figures on the left and right show the same results,

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[1] Results with failure rates, P_f, of 0.01 and 0.001 were visually the same, therefore only one graph is shown.

but on the left the complete time series is shown, whereas on the right only the first 1000 epochs are selected.

Results obtained with the PAR algorithm based on the minimum required success rate are shown in Figure 3 with a minimum required success rate of 0.99. Results obtained with the ratio test-based PAR algorithm are shown in Figure 4 and Figure 5, corresponding to a fixed failure rate of 0.01 and 0.001, respectively. In both cases, this algorithm results in a shorter time to fix the complete set of ambiguities. Note that the number of ambiguities here corresponds to the original ambiguities, whereas the numbers in Figure 3 correspond to the number of decorrelated ambiguities.

The benefit of the ratio test-based algorithm is that it is not only model-driven, but also data-driven. Even if the model is relatively weak, i.e. success rate is low, the ambiguities may still get fixed if their squared norm of residuals is small enough to guarantee a small enough failure rate. Compare the results in Figure 3 and Figure 4, where both a maximum failure rate of 0.001 is allowed. Note, however, that in the first case this is the failure rate without applying the ratio test, whereas in the second case it is the failure rate with applying the ratio test.

Both PAR approaches show a similar behaviour for new rising satellites: if all ambiguities are fixed and a new satellite rises (corresponding to a jump in the red graphs in bottom panels) it takes some time to fix the complete set including the new ambiguities due to the higher noise levels for those satellites.

Careful inspection of the results seems to indicate that only when at least some of the narrow-lane ambiguities can be fixed (i.e. more than half of the ambiguities), this results in a significant improvement in the positioning accuracy.

5. CONCLUSIONS

The results presented here give an idea of the theoretical performance that can be expected with the two proposed partial ambiguity resolution methods. It follows that both PAR algorithms inherently include an automatic procedure for deciding when to include newly risen satellites. Furthermore, it follows that a dramatic baseline precision improvement is only attained when nearly all ambiguities can be fixed.

Still, the two approaches presented here could both be further tuned and/or extended, for instance in terms of the subsets to consider, and the decision criteria.

What still needs to be investigated is how a PAR algorithm may be useful in case of biases in the data. Of course, only approaches that are both model- and data-driven like the FF-RT approach might be beneficial then.

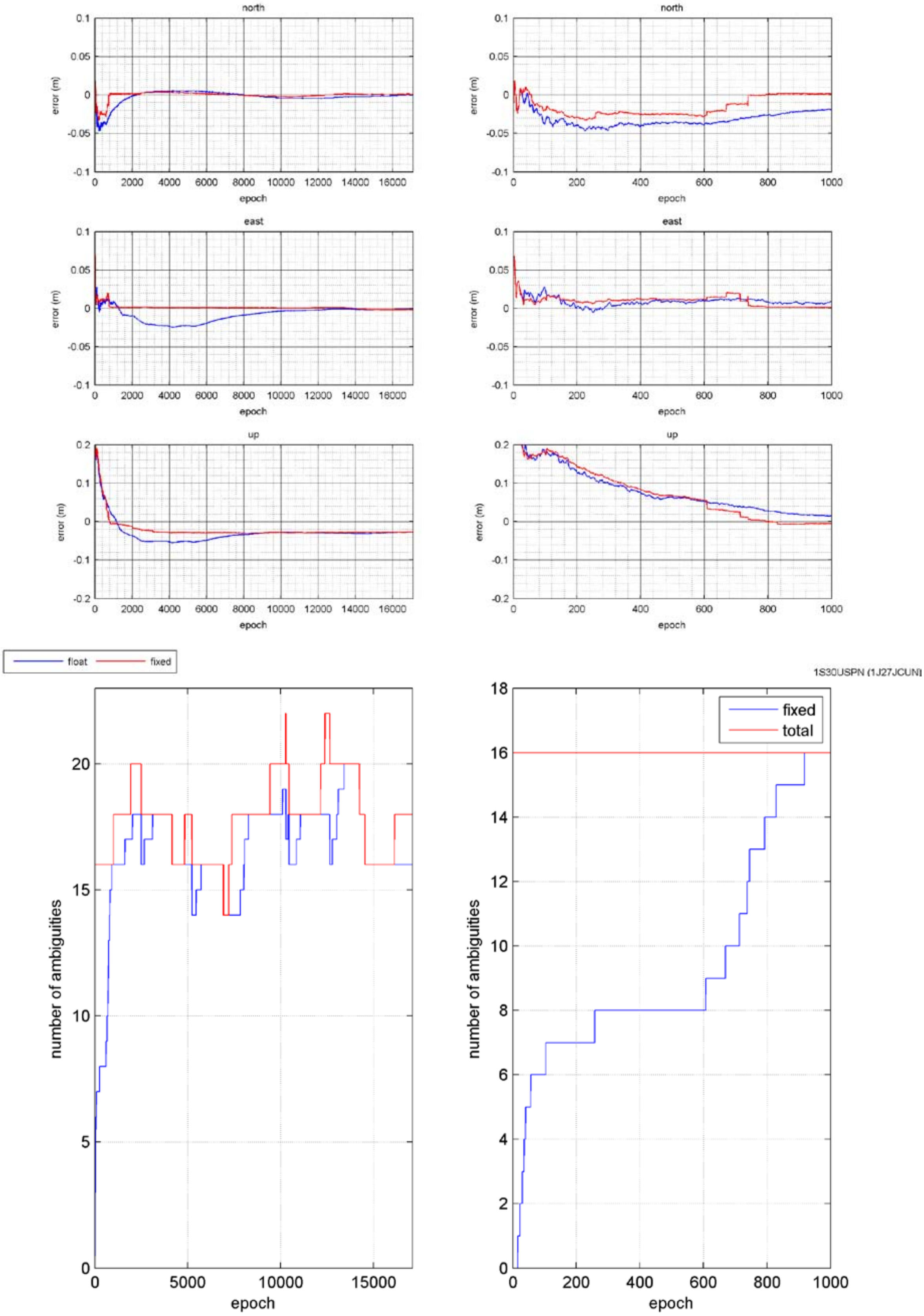

Figure 3. Bootstrapped success rate approach, with minimum required success rate of 0.99.

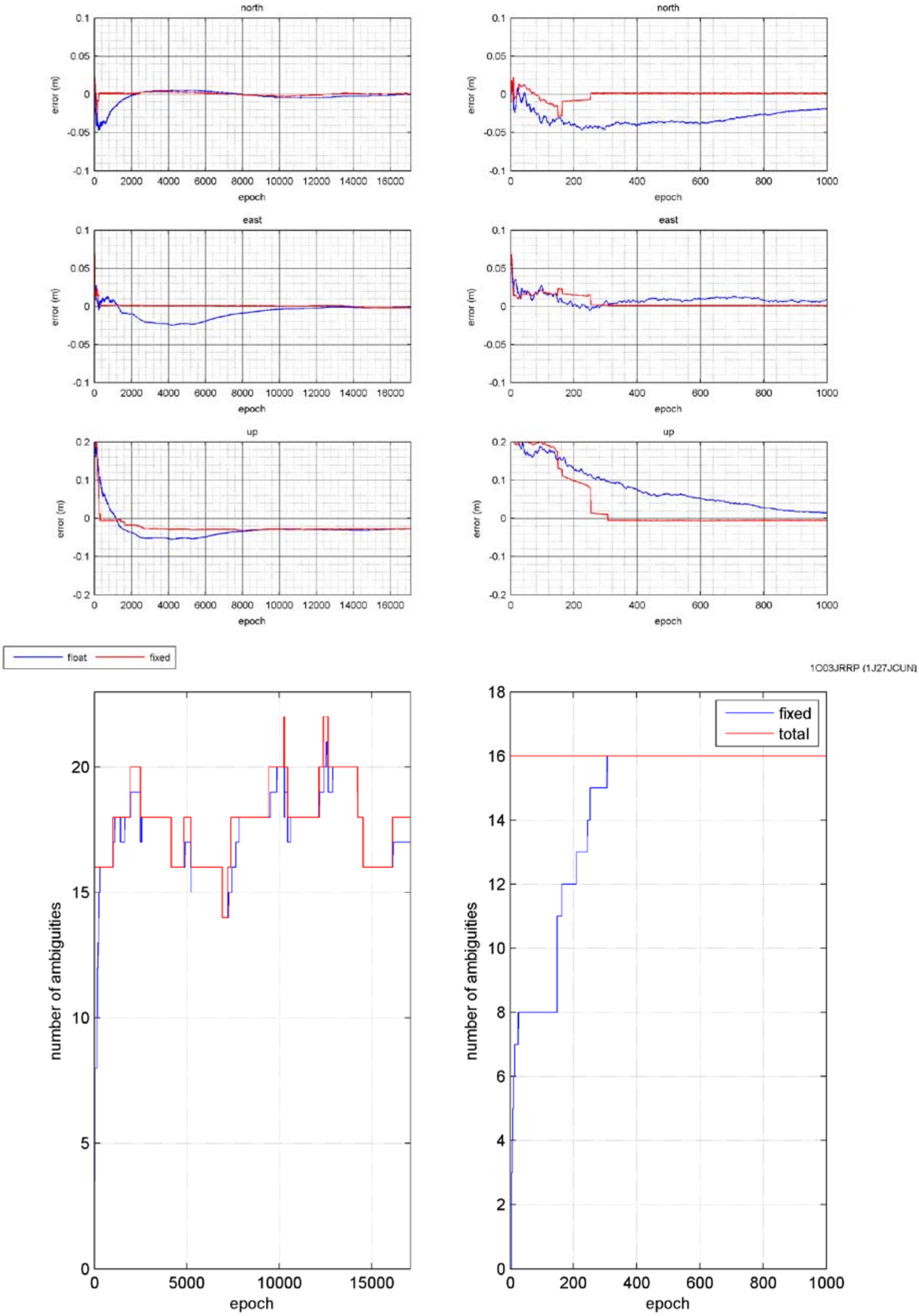

Figure 4. Ratio test-based approach, with fixed failure rate of 0.01.

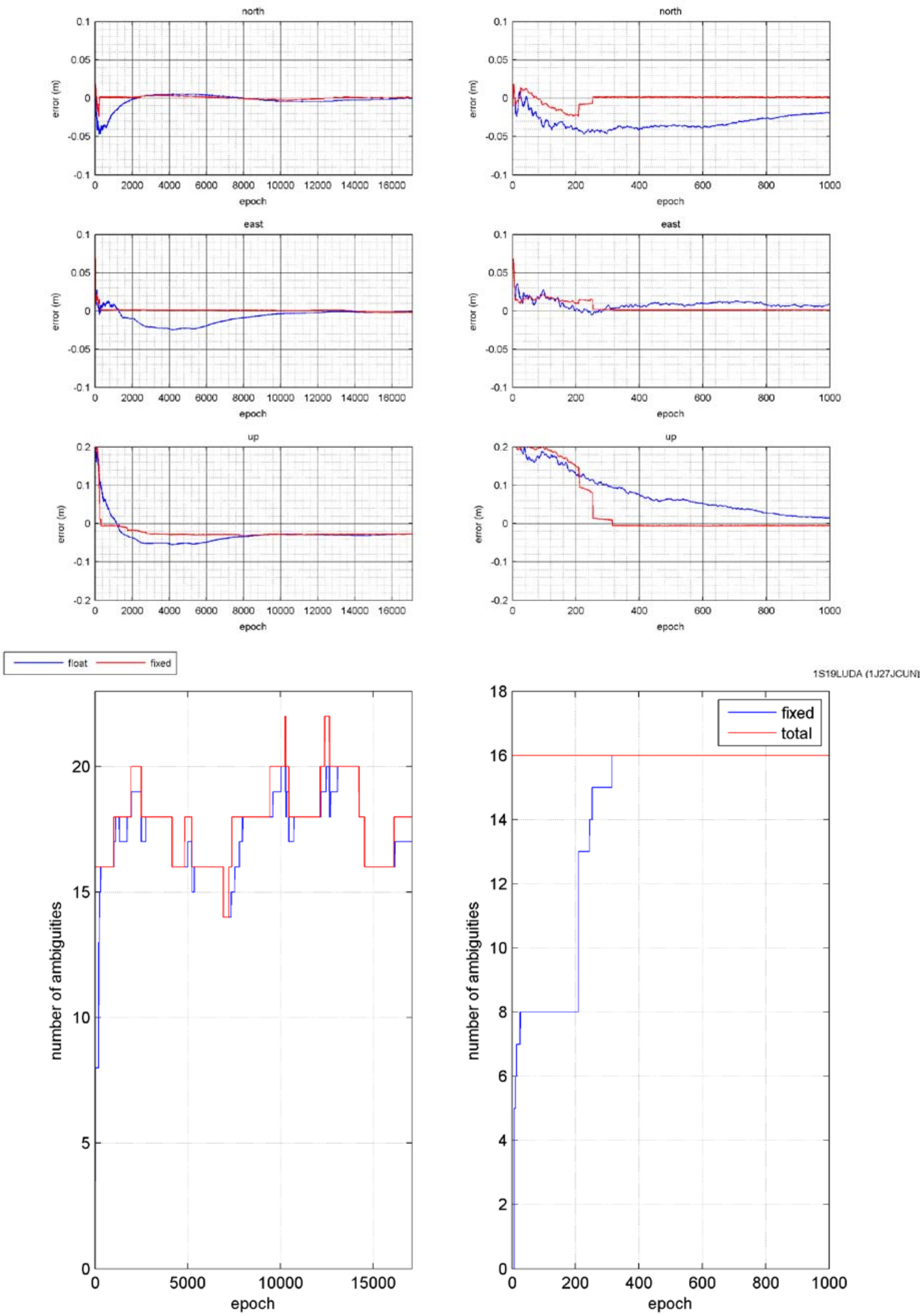

Figure 5. Ratio test-based approach, with fixed failure rate of 0.001.

ACKNOWLEDGEMENTS

The research of Sandra Verhagen is supported by the Dutch Technology Foundation STW, applied science division of NWO, and the Technology Program of the Ministry of Economic Affairs.
Peter Teunissen is Federation Fellow of the Australian Research Council (project FF0883188). This support is gratefully aknowledged.
The MSP3 project was carried out by NSL and Delft University of Technology and funded by ESA.

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