# A CARRIER PHASE AMBIGUITY ESTIMATOR WITH EASY-TO-EVALUATE FAIL-RATE

P.J.G. Teunissen
Department of Mathematical Geodesy and Positioning
Delft University of Technology
Kluyverweg 1
2629 HS Delft, The Netherlands
Fax: ++ 31 15 278 3711

**Abstract**
In (*Teunissen*, 2003) we introduced the class of integer aperture ($IA$) estimators. This class is larger than the class of integer ($I$) estimators, but smaller than the class of integer equivariant ($IE$) estimators, $I \subset IA \subset IE$. The IA-estimator is of a hybrid nature since its outcome may be integer-valued or real-valued. For its probabilistic evaluation one needs to take both the success-rate and fail-rate into account, since these two probabilities do not sum up to one as it is the case with integer estimators. The IA-estimators also take care of the so-called discernibility problem of GNSS ambiguity resolution.

In the present contribution we will introduce one particular integer aperture estimator, the ellipsoidal IA-estimator. This estimator has the advantage that a rigorous and easy-to-evaluate probabilistic description of its performance can be given. It will also be shown that some well-known discernibility tests which are used in practice are in fact examples of IA-estimators.

## 1 Introduction

Global Navigation Satellite System (GNSS) ambiguity resolution is the process of resolving the unknown cycle ambiguities of double difference (DD) carrier phase data. Its practical importance becomes clear when one realizes the great variety of current and future GNSS models to which it applies. An overview of GNSS models, together with their applications in surveying, navigation, geodesy and geophysics, can be found in textbooks such as (*Hofmann-Wellenhof et al.*, 2001), (*Leick*, 1995), (*Misra and Enge*, 2001), (*Parkinson and Spilker*, 1996), (*Strang and Borre*, 1997) and (*Teunissen and Kleusberg*, 1998).

In (*Teunissen*, 2003) we introduced the class of integer aperture ($IA$) estimators for carrier phase ambiguity resolution. This class allows one to design ambiguity estimators such that the ambiguity resolution process will have a user-defined *fixed fail-rate*. In this contribution we will introduce one particular integer aperture estimator, the *ellipsoidal IA-estimator*. This estimator has the advantage that a rigorous and easy-to-evaluate probabilistic description of its performance can be given. It will also be shown that some well-known discernibility tests which are used in practice are in fact examples of IA-estimators. For these IA-estimators, however, no easy-to-evaluate fail-rates exists.

As our point of departure we take the following system of linear observation equations

$$E\{y\} = Aa + Bb \ , \ a \in Z^n \ , \ b \in R^p \tag{1}$$

with $E\{.\}$ the mathematical expectation operator, $y$ the $m$-vector of observables, $a$ the $n$-vector of unknown integer parameters and $b$ the $p$-vector of unknown real-valued parameters. All the linear(ized) GNSS models can in principle be cast in the above frame of observation equations. The data vector $y$ will then usually consist of the 'observed minus computed' single- or dual- frequency double-difference (DD) phase and/or pseudorange (code) observations accumulated over all observation epochs. The entries of vector $a$ are then the DD carrier phase ambiguities, expressed in units of cycles rather than range, while the entries of the vector $b$ will consist of the remaining unknown parameters, such as for instance baseline components (coordinates) and possibly atmospheric delay parameters (troposphere, ionosphere).

The procedure which is usually followed for solving the GNSS model can be divided into three steps. In the *first* step one simply discards the integer constraints $a \in Z^n$ and performs a standard least-squares adjustment. As a result one obtains the least-squares estimators $\hat{a}$ and $\hat{b}$, together with their variance-covariance (vc-) matrices. This solution is referred to as the 'float' solution. In the *second* step the 'float' estimator $\hat{a}$ is used to compute an improved estimator which in some pre-defined sense incorporates the integerness of the ambiguities. This estimator is then used in the final and *third* step to adjust the 'float' estimator $\hat{b}$ so as to obtain the so-called 'fixed' estimator $\check{b}$. In this contribution however we will focuss on the integer aperture mappings of $\hat{a}$ and introduce the ellipsoidal IA-estimator.

## 2 Integer aperture estimation with examples

The three classes of ambiguity estimators discussed in (*Teunissen*, 2003) are the class of integer ($I$) estimators, the class of integer equivariant ($IE$) estimators and the class of integer aperture ($IA$) estimators. These three classes are related as $I \subset IA \subset IE$. In this contribution we will restrict attention to the IA-estimators. This class is defined as follows.

**Definition 1** (*Integer aperture estimators*)
Let $\Omega \subset R^n$ be integer translational invariant, i.e. $\Omega = \Omega + z, \forall z \in Z^n$ and let $\Omega_z = \Omega \cap S_z$ with $S_z$ the pull-in region of an admissible integer estimator. Then integer aperture estimators are defined as

$$\hat{a}_{IA} = \hat{a} + \sum_{z \in Z^n} (z - \hat{a})\omega_z(\hat{a}) \tag{2}$$

with $\omega_z(x)$ the indicator function of $\Omega_z$.

The IA-estimator is a hybrid estimator having as outcome either the real-valued float solution $\hat{a}$ or an integer solution. The IA-estimator returns the float solution if $\hat{a} \notin \Omega$ and it will be equal to $z$ when $\hat{a} \in \Omega_z$. Note, since $\Omega$ is the collection of all $\Omega_z = \Omega_0 + z$, that the IA-estimator is completely determined once $\Omega_0$ is known. Thus $\Omega_0 \subset S_0$ plays the same role for the IA-estimators as $S_0$ does for the I-estimators. By changing the size and shape of $\Omega_0$ one changes the outcome of the IA-estimator. The subset $\Omega_0$ can therefore be seen as an adjustable pull-in region with two limiting cases. The limiting case in which $\Omega_0$ is empty and the limiting case when $\Omega_0$ equals $S_0$. In the first case the IA-estimator becomes identical to the float solution $\hat{a}$, and in the second case the IA-estimator becomes identical to an I-estimator. The subset $\Omega_0$ therefore determines the *aperture* of the pull-in region.

In order to evaluate the performance of an IA-estimator as to whether it produces the correct integer outcome $a \in Z^n$, it is helpful to classify its possible outcomes. An IA-estimator can produce one of the following three outcomes: $a \in Z^n$ (correct integer), $z \in Z^n \setminus \{a\}$ (incorrect integer), or $\hat{a} \in R^n \setminus Z^n$ (no integer). A correct integer outcome may be considered a *success*, an incorrect integer outcome a *failure*, and an outcome where no correction at all is given to the float solution as indeterminate or *undecided*. The probability of success, the *success-rate*, equals the integral of the pdf of $\hat{a}$, $f_{\hat{a}}(x)$, over $\Omega_a$, whereas the probability of failure, the *fail-rate*, equals the integral of $f_{\hat{a}}(x)$ over $\Omega \setminus \Omega_a$. The respective probabilities are therefore given as

$$\begin{cases} P_S &= \int_{\Omega_a} f_{\hat{a}}(x)dx \quad \text{(success)} \\ P_F &= \sum_{z\neq a}\int_{\Omega_z} f_{\hat{a}}(x)dx \quad \text{(failure)} \\ P_U &= 1 - P_S - P_F \quad \text{(undecided)} \end{cases} \tag{3}$$

Note that these three probabilities are completely governed by $f_{\hat{a}}(x)$, the pdf of the float solution, and by $\Omega_0$, the aperture pull-in region which uniquely defines the IA-estimator. Hence one can proceed with the evaluation of IA-estimators once this information is available. We will now give three examples of IA-estimators.

**Example 1:** In the practice of GPS carrier phase ambiguity resolution various tests are in use for discriminating between the 'best' and the so-called 'second-best' solution. These tests are usually referred to as discernibility tests. One such test is the popular *ratio-test.* The ratio-test is defined as follows. Let $\hat{a}$ be the float solution, $\check{a} = \arg\min_{z\in Z^n} \parallel \hat{a} - z \parallel^2_{Q_{\hat{a}}}$ the integer least-squares solution and $\check{a}' = \arg\min_{z\in Z^n\setminus\check{a}} \parallel \hat{a} - z \parallel^2_{Q_{\hat{a}}}$ the so-called 'second-best' solution. Then $\check{a}$ is accepted as the fixed solution if

$$\frac{\parallel \hat{a} - \check{a} \parallel^2_{Q_{\hat{a}}}}{\parallel \hat{a} - \check{a}' \parallel^2_{Q_{\hat{a}}}} \leq \rho \tag{4}$$

This test has been used in e.g. (*Euler and Schaffrin*, 1990), (*Wei and Schwarz*, 1995) and (*Han and Rizos*, 1996). Thus with the ratio-test $\check{a}$ is accepted as the fixed solution if the float solution $\hat{a}$ is sufficiently more closer to $\check{a}$ than to the 'second-best' solution $\check{a}'$. The non-negative scalar $\rho$ is a user-defined tolerance level.

It can be shown that the procedure underlying the above test is actually that of an IA-estimator. The rejection region of the above test is integer translational invariant and thus an example of $R^n \setminus \Omega$. For this region the outcome will be $\hat{a}$. The outcome will be the integer $z \in Z^n$ however, when the test is passed and $\hat{a}$ lies in the least-squares pull-in region of $z$.

In the GPS literature it has been suggested, by refering to the classical theory of hypothesis testing in linear models, that $\rho$ could be computed as critical value from a chosen level of significance using the F-distribution, see e.g. (*Erickson*, 1992). This procedure is however flawed. A ratio of quadratic forms is F-distributed if both quadratic forms are Chi-square distributed and independent. But already the Chi-square distribution fails to hold. It can be shown that the ambiguity residual $\check{\epsilon} = \hat{a} - \check{a}$ is not Gaussian distributed even if the float distribution $\hat{a}$ is. Another way of seeing that the ratio of (**??**) can never be F-distributed goes as follows. Since the domain of an F-distribution is $[0, \infty)$, the value of $\rho$ could be chosen larger than one in case the ratio would be F-distributed. But it will be immediately clear that any value for $\rho$ larger than or equal to 1 will always lead to acceptance, since by definition $\parallel \hat{a} - \check{a} \parallel^2_{Q_{\hat{a}}} \leq \parallel \hat{a} - \check{a}' \parallel^2_{Q_{\hat{a}}}$. The values of $\rho$ are therefore bounded as $0 \leq \rho \leq 1$.

The aperture of the pull-in region of the ratio-test is governed by the choice of the single parameter $\rho$. One has a zero aperture in case $\rho = 0$ and a maximum aperture in case $\rho = 1$. In the first case the procedure of the ratio-test will always output the float solution, while in the second case it will always output the integer least-squares solution $\check{a}$. Changing the value of the aperture parameter $\rho$ will thus change the performance of the ratio-test, in particular its probabilities of success, failure or undecided. With the use of (**??**) one can thus now choose an appropriate value for $\rho$.

It will now also be clear, since the procedure of the ratio-test is that of an IA-estimator, that the ratio-test can not be seen as a classical hypothesis test for testing the validity of $\check{a}$. In fact, the validity of $\check{a}$ is not tested at all with the ratio-test. Any arbitrary integer perturbation of $\hat{a}$ will namely result in the same outcome of the ratio-test. Thus what is actually measured with the ratio-test is the closeness of $\hat{a}$ to an integer.

Another version of the ratio-test which is often used is the one in which the norm of the vector of least-squares residuals $\hat{e}$ is taken into account as well. In this case $\check{a}$ is accepted as the fixed solution if

$$\frac{\| \hat{e} \|^2_{Q_y} + \| \hat{a} - \check{a} \|^2_{Q_{\hat{a}}}}{\| \hat{e} \|^2_{Q_y} + \| \hat{a} - \check{a}' \|^2_{Q_{\hat{a}}}} \leq \rho \tag{5}$$

This version of the ratio-test has been used in e.g. (*Frei and Beutler*, 1990), (*Abibdin*, 1993) and (*Corbett and Cross*, 1995). The above remarks apply to this version of the ratio-test as well. And also this test is insensitive to arbitrary integer pertubations. To see this, consider the underlying system of linear observation equations $E\{y\} = Aa + Bb$ and assume that $y$ is perturbed to $y + Az$, with arbitrary $z \in Z^n$. This pertubation does not get propagated into the least-squares residuals. Thus $\hat{e}$ remains invariant. The float solution $\hat{a}$ and the two integer solutions $\check{a}$ and $\check{a}'$ do change. But since all three of them undergo the same change, the differences $\hat{a} - \check{a}$ and $\hat{a} - \check{a}'$ also remain invariant. Thus also the above version of the ratio-test measures only the closeness of $\hat{a}$ to an integer.

**Example 2:** Although perhaps less popular, tests other than the ratio test have been proposed in the GPS literature as well. One such test is the *difference-test*. This test was introduced in (*Tiberius and de Jonge*, 1995). This test also makes use of the integer least-squares solution and the 'second best' solution. It is defined as follows. The integer least-squares solution $\check{a}$ is accepted as the fixed solution with the difference-test if

$$\| \hat{a} - \check{a}' \|^2_{Q_{\hat{a}}} - \| \hat{a} - \check{a} \|^2_{Q_{\hat{a}}} \geq \delta \tag{6}$$

where the non-negative scalar $\delta$ is a user-defined tolerance level. As with the ratio-test, the difference-test accepts $\check{a}$ as the fixed solution if the float solution is sufficiently more closer to $\check{a}$ than to the 'second best' solution $\check{a}'$. 'Closeness' is however measured differently. The procedure underlying the difference-test can also be shown to be that of an IA-estimator. Note that the difference-test will always lead to acceptance in case $\delta > \| \hat{a} - \check{a}' \|^2_{Q_{\hat{a}}}$.

As it was the case with the ratio-test, one should be careful in finding the motivation for the above acceptance test by refering to the classical theory of hypothesis testing in linear models. It is true that the above difference (**??**) equals the Likelihood Ratio Teststatistic when testing

$$\mathrm{H_o} : E\{\hat{a}\} = \check{a} \ \text{ versus } \ \mathrm{H_a} : E\{\hat{a}\} = \check{a}' \tag{7}$$

This motivation and interpretation of (**??**) is however flawed, despite the apparent similarity with the classical likelihood ratio principle. The erroneous assumption underlying formulation (**??**) is that $\check{a}$ and $\check{a}'$ are both deterministic, while $\hat{a}$ is a random vector. But this is impossible. If $\hat{a}$ is a random vector then so are $\check{a}$ and $\check{a}'$, since both are functions of $\hat{a}$. All three vectors $\hat{a}$, $\check{a}$ and $\check{a}'$ are therefore random. This shows that (**??**) is an impossible formulation, since the deterministic expectation of a random vector can never be set equal to a random vector.

In (*Wang et al.*, 1998) the above difference of quadratic forms is used to formulate a test statistic which they claim to be standard normally distributed. Their reasoning is based on the following. Since $\hat{a}$ is normally distributed and since $\hat{a}$ appears linearly in $\| \hat{a} - \check{a}' \|^2_{Q_{\hat{a}}} - \| \hat{a} - \check{a} \|^2_{Q_{\hat{a}}} = -2(\check{a}' - \check{a})^T Q_{\hat{a}}^{-1} (\hat{a} - \check{a}) + \| \check{a}' - \check{a} \|^2_{Q_{\hat{a}}}$, they conclude that the difference of the two quadratic forms is normally distributed too. But this conclusion is false, since $\check{a}$ and $\check{a}'$, although integer, are also random variables. The conclusion that the difference of the two quadratic forms is normally distributed would only be correct in case $\check{a}$ and $\check{a}'$ would be deterministic instead of random. But this is not true, since both are functions of the random vector $\hat{a}$.

**Example 3:** As a third example of an IA-estimator we have the procedure underlying the *projector-test*. The integer least-squares solution $\check{a}$ is accepted as the fixed solution with the projector-test if

$$\left| \frac{(\check{a} - \check{a}')^T Q_{\hat{a}}^{-1} (\hat{a} - \check{a})}{\| \check{a} - \check{a}' \|_{Q_{\hat{a}}}} \right| \leq \varpi \tag{8}$$

where the non-negative scalar $\varpi$ is a user-defined tolerance level. This test is referred to as the projector-test since the ratio of (??) equals a projector which projects $\hat{a} - \check{a}$ orthogonally onto the direction of $\check{a} - \check{a}'$, where orthogonality is measured with respect to the metric of $Q_{\hat{a}}$. As with the previous two tests the projector-test accepts $\check{a}$ as the fixed solution if the float solution is sufficiently more closer to $\check{a}$ than to the 'second best' solution $\check{a}'$. 'Closeness' is however again measured differently.

For the same reasons as above the projector-test may also not be motivated by referring to the classical theory of hypothesis testing in linear models, despite the fact that the absolute value of the projector equals the Generalized Likelihood Ratio Teststatistic when testing

$$\mathrm{H_o} : E\{\hat{a}\} = \check{a} \;\text{ versus }\; \mathrm{H_a} : E\{\hat{a}\} = \check{a} + c\nabla \;,\; \text{ with } c = (\check{a}' - \check{a}) / \parallel \check{a}' - \check{a} \parallel_{Q_{\hat{a}}} \tag{9}$$

with $\nabla$ unknown. Using the normal distribution for the computation of $\varpi$ as critical value, as suggested in (*Wang*, 199?), is therefore also incorrect. Due to the randomness of both $\check{a}$ and $\check{a}'$, the projector is not normally distributed even if the float solution is. A closer look at (??) also shows that it does not make sense to choose the tolerance parameter too large. The projector-test will always lead to acceptance in case $\varpi > \frac{1}{2} \parallel \check{a} - \check{a}' \parallel^2_{Q_{\hat{a}}}$.

# 3 The ellipsoidal IA-estimator

In the previous section it has been shown that some of the current procedures in place for carrier phase ambiguity resolution can be seen as particular cases of IA-estimation. These procedures are usually motivated by referring to the classical theory of hypothesis testing in linear models. But one can not evaluate the probabilistic performance of these procedures by simply using the distributional results known from the classical theory. In order to evaluate and compare these procedures properly one needs the unified framework of IA- estimation and in particular compute their success-rate and fail-rate.

In this section we will introduce a new IA-estimator. Instead of finding our motivation in the classical theory of hypothesis testing, we directly proceed from the class of I-estimators. The prime difference between I-estimators and IA-estimators is that the latter have pull-in regions which are *subsets* of the space-filling pull- in regions of the I-estimators. Hence, one can design ones own IA- estimator by simply defining the size and shape of these subsets. This gives one therefore also the possibility to choose the integer aperture pull-in region $\Omega_0$ such that their probabilistic evaluation is made easier. For the ratio-test, the difference-test and the projector-test one has to resort to computations of Monte-Carlo-type in order to evaluate their fail-rates and success-rates. This can be avoided however if $\Omega_0$ is chosen such that the probabilistic evaluation can be based on analytical closed form expressions. This will therefore be the approach followed in the current contribution.

The procedures currently in place for GPS ambiguity resolution all make use of comparing, in some pre-defined sense, the 'best' solution with the so-called 'second best' solution. But when one thinks of the concept of the aperture region, there is in principle no need to compute or to make use of the 'second-best' solution. That is, one can do without the 'second-best' solution, as long as one is able to measure and evaluate the closeness of the float solution to an integer. The *ellipsoidal IA*-estimator is one such IA-estimator. It is defined as follows.

**Definition 2** (*Ellipsoidal IA estimation*)
The aperture pull-in regions of the ellipsoidal integer aperture (EIA) estimator are defined as

$$E_z = E_0 + z \;,\; E_0 = S_0 \cap C_{\epsilon,0} \;,\; \forall z \in Z^n \tag{10}$$

with $S_0$ being the least-squares pull-in region and $C_{\epsilon,0} = \{x \in R^n \mid \parallel x \parallel^2_{Q_{\hat{a}}} \leq \epsilon^2\}$, an origin-centred ellisoidal region of which the size is controlled by the aperture parameter $\epsilon$.

Thus the EIA-estimator equals $\hat{a}_{EIA} = z$ if $\hat{a} \in E_z$ and $\hat{a}_{EIA} = \hat{a}$ otherwise. From the definition follows that $E_z = \{x \in S_z \mid \parallel x - z \parallel^2_{Q_{\hat{a}}} \leq \epsilon^2\}$. This shows that the procedure for computing the

EIA-estimator is rather straightforward. Using the float solution $\hat{a}$, its vc- matrix $Q_{\hat{a}}$ and the aperture parameter $\epsilon$ as input, one only needs to compute the integer least-squares solution $\check{a}$ and verify whether or not the inequality

$$\| \hat{a} - \check{a} \|^2_{Q_{\hat{a}}} \leq \epsilon^2 \tag{11}$$

is satisfied. If the inequality is satified then $\hat{a}_{EIA} = \check{a}$, otherwise $\hat{a}_{EIA} = \hat{a}$. A comparison with the ratio-test (**??**), with the difference-test (**??**) and with the projector-test (**??**), shows that (**??**) is indeed the simplest of the four inequalities. Instead of working with a distance-ratio, a distance-difference or a projected distance, the EIA-estimator simply evaluates the distance to the closest integer directly. There is therefore no need to make use of a 'second- best' solution.

The simple choice of the ellipsoidal criterion (**??**) is motivated by the fact that the squared-norm of a normally distributed random vector is known to have a Chi-square distribution. That is, if $\hat{a}$ is distributed as $\hat{a} \sim \mathrm{N}(a, Q_{\hat{a}})$ then $P(\hat{a} \in C_{\epsilon,z}) = P\left(\chi^2\left(n, \mu_z\right) \leq \epsilon^2\right)$, in which $\chi^2(n, \mu_z)$ denotes a random variable having as pdf the noncentral Chi-square distribution with $n$ degrees of freedom and noncentrality parameter $\mu_z = (z-a)^T Q_{\hat{a}}^{-1}(z-a)$. This implies that one can give *exact* solutions to the fail-rate and to the success-rate of the EIA-estimator, provided the ellipsoidal regions $C_{\epsilon,z}$ do not overlap. These probabilistic results of the EIA-estimator are given in the following theorem.

**Theorem** (*The EIA-probabilities of success, failure and undecided*)
Let the float solution be distributed as $\hat{a} \sim \mathrm{N}(a, Q_{\hat{a}})$ and let the aperture parameter satisfy $\epsilon \leq \frac{1}{2}\min_{z \in Z^n \setminus \{0\}} \| z \|_{Q_{\hat{a}}}$. Then the EIA-probabilities of failure, success and undecided are given as

$$\left\{ \begin{array}{lcl} P_F &=& \sum_{z \in Z^n \setminus \{0\}} P\left(\chi^2(n, \lambda_z) \leq \epsilon^2\right) \\ P_S &=& P\left(\chi^2(n, 0) \leq \epsilon^2\right) \\ P_U &=& 1 - P_F - P_S \end{array} \right. \tag{12}$$

in which $\chi^2(n, \lambda_z)$ denotes a random variable having as pdf the noncentral Chi-square distribution with $n$ degrees of freedom and noncentrality parameter $\lambda_z = z^T Q_{\hat{a}}^{-1} z$. The above given probabilities for $P_F$ and $P_S$ become upper bounds in case $\epsilon > \frac{1}{2}\min_{z \in Z^n \setminus \{0\}} \| z \|_{Q_{\hat{a}}}$. To obtain the corresponding lower bounds, one has to replace $\epsilon$ by $\frac{1}{2}\min_{z \in Z^n \setminus \{0\}} \| z \|_{Q_{\hat{a}}}$.

In order to understand the situation of overlap, note that the region for which the EIA-estimator outputs an integer as solution is given as $E = \cup_{z \in Z^n} E_z$. Since this region is independent of $S_z$ it follows that $E = \cup_{z \in Z^n} C_{\epsilon,z}$. Thus $E$ equals the union of all integer translated copies of the origin- centred ellipsoidal region $C_{\epsilon,0}$. Depending on the value chosen for the aperture parameter $\epsilon$, these ellipsoidal regions may or may not overlap. The overlap will occur when $\epsilon$ is larger than half times the smallest distance between two integer vectors. Thus when $\epsilon$ is larger than half times the length of the smallest nonzero integer vector, $\epsilon > \frac{1}{2}\min_{z \in Z^n \setminus \{0\}} \| z \|_{Q_{\hat{a}}}$.

The results of the above theorem are easy to apply and they allow for an exact evaluation of the EIA-estimator in the absence of overlap. Such an exact evaluation is not possible for the ratio-test, the difference-test and the projector-test. The steps for computing and evaluating the EIA- estimator are as follows. Depending on the application the user will usually start by requiring a certain small and fixed value for the fail-rate $P_F$. Hence, the user will start by choosing the aperture parameter $\epsilon$ such that it meets his requirement on $P_F$. If one requires the same value for $P_F$, the value of the aperture parameter $\epsilon$ needs to be chosen smaller when the precision of the ambiguities becomes poorer. This can be understood by looking at the noncentrality parameter $\lambda_z = z^T Q_{\hat{a}}^{-1} z$. Note that the strength of the underlying GNSS model is reflected in the vc-matrix $Q_{\hat{a}}$. A weaker GNSS model will result in ambiguities having a poorer precision. But a poorer precision will result in a smaller value for $\lambda_z$ and thus in a larger value for the probability $P\left(\chi^2(n, \lambda_z) \leq \epsilon^2\right)$. Hence, in order to keep $P_F$ at the same level, while the precision of the ambiguities gets poorer, one will have to choose a smaller value for $\epsilon$. Once $\epsilon$ has been chosen, one verifies whether or not (**??**) holds true. If it does, the outcome equals $\check{a}$, otherwise the outcome equals $\hat{a}$. The probability that the EIA-estimator equals the correct integer ambiguity is then given by $P_S = \left(\chi^2(n,0) \leq \epsilon^2\right)$.

# 4 References


Abidin, H.A. (1993): *Computational and geometrical aspects of on-the-fly ambiguity resolution.* Dept. of Surveying Engineering Tech. Rept. No. 164, Univ. of New Brunswick, Fredericton, New Brunswick, Canada, 314 pp.

Corbett, S.J. and P.A. Cross (1995): GPS single epoch ambiguity resolution. *Survey Review*, Vol. 33, N0. 257, 149-160.

Erickson, C. (1992): *Investigations of C/A code and carrier measurements and techniques for rapid static GPS surveys.* Dept. of Geomatic Engineering Tech. Rept. No. 20044, The University of Calgary, Calgary, Alberta, Canada, 180pp.

Euler, H.-J. and B. Schaffrin (1990): On a measure of the discernibility between different ambiguity solutions in the static-kinematic GPS-mode. *Kinematic Systems in Geodesy, Surveying and Remote Sensing* (Ed. K.P. Schwarz and G. Lachapelle), IAG Symposium No. 107, Banff, Canada, 285-295.

Frei, E. and G. Beutler (1990): Rapid static positioning based on the fast ambiguity resolution approach: theory and first results. *Manuscripta Geodaetica*, 15: 325-356.

Han, S. and C. Rizos (1996): Integrated method for instantaneous ambiguity resolution using new generation GPS receivers. *Proc IEEE PLANS'96*, Atlanta, USA, pp. 254-261.

Hofmann-Wellenhof, B., H. Lichtenegger, J. Collins (2001): *Global Positioning System: Theory and Practice.* 5th edition. Springer Verlag.

Leick, A. (1995): *GPS Satellite Surveying.* 2nd edition, John Wiley, New York.

Misra, P., P. Enge (2001): *Global Positioning System: Signals, Measurements, and Performance.* Ganga-Jamuna Press, 2001.

Parkinson, B., J.J. Spilker (eds) (1996): *GPS: Theory and Applications*, Vols 1 and 2, AIAA, Washington DC.

Strang, G., K. Borre (1997): *Linear Algebra, Geodesy, and GPS*, Wellesley-Cambridge Press.

Teunissen, P.J.G., A. Kleusberg (eds) (1998): *GPS for Geodesy*, 2nd enlarged edition, Springer Verlag.

Teunissen, P.J.G. (2003): Integer aperture GNSS ambiguity resolution. *Artificial Satellites*, Vol. 38. In print.

Tiberius, C.C.J.M. and P. de Jonge (1995): Fast positioning with the LAMBDA method. *Proc DSNS 95*, Bergen, Norway, paper No. 30.

Wang, J, M.P. Stewart, M. Tsakiri (1998): A discrimination procedure for ambiguity resolution on-the-fly. *Journal of Geodesy*, 72: 644-653.

Wei, M. and K.P. Schwarz (1995): Fast ambiguity resolution using an integer nonlinear programming method. *Proc. ION GPS-95*, Palm Spring, USA, pp. 1101-1110.