Probabilistic properties of GNSS integer ambiguity estimation

P. J. G. Teunissen

Department of Mathematical Geodesy and Positioning, Delft University of Technology, Thijsseweg 11, 2629 JA Delft, The Netherlands

(Received November 4, 1999; Revised August 7, 2000; Accepted August 7, 2000)

Successful integer estimation of carrier phase ambiguities of Global Navigation Satellite Systems (GNSS) is the key to many high precision positioning applications. In order to describe the quality of the positioning results rigorously, one needs to know the probabilistic properties of both the integer and noninteger parameters in the GNSS model. In this contribution these probability distributions are presented and discussed. The probability mass function of the integer ambiguities is needed to evaluate the ambiguity success rate and the distribution of the GNSS baseline is needed to evaluate the relevant confidence regions for positioning.

1. Introduction

Global Navigation Satellite System (GNSS) ambiguity resolution is the process of resolving the unknown cycle ambiguities of double difference (DD) carrier phase data as integers. Ambiguity resolution applies to a great variety of current and future GNSS models. Apart from the current Global Positioning System (GPS) models, it also applies to the future modernized GPS and the future European Galileo GNSS. An overview of GNSS models, together with their applications in surveying, navigation, geodesy and geophysics, can be found in textbooks such as Leick (1995), Parkinson and Spilker (1996), Strang and Borre (1997), Hofmann-Wellenhof *et al.* (1997), and Teunissen and Kleusberg (1998).

Despite the differences in application of the various GNSS models, it is important to recognize that their ambiguity resolution problems are intrinsically the same. This implies that it is possible to develop a single theoretical framework that applies to every GNSS model for which ambiguity resolution would make sense. Such a framework is available for the integer estimation part of ambiguity resolution. Rigorous and efficient methods of estimation exist for the determination of the integer carrier phase ambiguities. This is not yet true however when one considers the probabilistic aspects of ambiguity resolution. To fill in this gap, one should first realize that ambiguity resolution is not an end in itself. After all, the sole purpose of ambiguity resolution is to use the integer ambiguity constraints as a means of improving significantly on the precision of the remaining model parameters, such as baseline coordinates and/or atmospheric (tropopshere, ionosphere) delays. Hence, the qualitative aspects of ambiguity resolution should be seen in the context of how well these parameters can be determined. It is therefore the purpose of this contribution to show how well these model parameters can be determined by means of ambiguity resolution. Such a qualitative description then also enables one to formulate and test the requirements which ambiguity resolution has to fulfil in order to be successful.

As our point of departure we will take the following system of linear(ized) observation equations

$$y = Aa + Bb + e \tag{1}$$

where y is the given GNSS data vector of order m, a and b are the unknown parameter vectors respectively of order n and o, and where e is the noise vector. In principle all the GNSS models can be cast in this frame of observation equations. The data vector y will usually consist of the 'observed minus computed' single- or dual-frequency double-difference (DD) phase and/or pseudorange (code) observations accumulated over all observation epochs. The entries of vector a are then the DD carrier phase ambiguities, expressed in units of cycles rather than range. They are known to be *integers*, $a \in Z^n$. The entries of the vector b will consist of the remaining unknown parameters, such as for instance baseline components (coordinates) and possibly atmospheric delay parameters (troposphere, ionosphere). They are known to be real-valued, $b \in R^o$.

The procedure which is usually followed for solving the GNSS model (1), can be divided into three steps. In the *first* step one simply disregards the integer constraints $a \in Z^n$ on the ambiguities and performs a standard least-squares adjustment. As a result one obtains the (real-valued) estimates of a and b, together with their variance-covariance (vc-) matrix

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}, \begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix}. \tag{2}$$

This solution is referred to as the 'float' solution. In the *second* step the 'float' ambiguity estimate $\hat{a}$ is used to compute the corresponding integer ambiguity estimate $\check{a}$. This implies that a mapping $S : R^n \mapsto Z^n$, from the n-dimensional space of reals to the n-dimensional space of integers, is introduced such that

$$\check{a} = S(\hat{a}). \tag{3}$$

Once the integer ambiguities are computed, they are used in the *third* step to finally correct the 'float' estimate of b. As a result one obtains the 'fixed' solution

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a}). \quad (4)$$

In this contribution we will discuss the probabilistic consequences of relation (3). We will refrain however, from discussing the computational intricacies of integer estimation. For a discussion of these aspects, we refer to e.g. Teunissen, (1993), de Jonge and Tiberius (1996a), Hassibi and Boyd (1998) or to the textbooks (Hofmann-Wellenhof *et al.*, 1997; Strang and Borre, 1997; Teunissen and Kleusberg, 1998). Practical results can also be found in e.g. Tiberius and de Jonge (1995), Han (1995), de Jonge *et al.* (1996), de Jonge and Tiberius (1996b), Boon and Ambrosius (1997), Boon *et al.* (1997), Tiberius *et al.* (1997) and Jonkman (1998).

2. The Probability Distribution of Ambiguities and Baseline

There are many ways of computing an integer ambiguity vector $\check{a}$ from its real-valued counterpart $\hat{a}$. To each such method belongs a mapping $S : R^n \mapsto Z^n$ from the n-dimensional space of real numbers to the n-dimensional space of integers. Due to the discrete nature of Z^n, the map S will not be one-to-one, but instead a many-to-one map. This implies that different real-valued ambiguity vectors will be mapped to the same integer vector. One can therefore assign a subset $S_z \subset R^n$ to each integer vector $z \in Z^n$:

$$S_z = \{x \in R^n \mid z = S(x)\}, \quad z \in Z^n. \quad (5)$$

The subset S_z contains all real-valued ambiguity vectors that will be mapped by S to the same integer vector $z \in Z^n$. This subset is referred to as the *pull-in region* of z (Jonkman, 1998; Teunissen, 1998b). It is the region in which all ambiguity 'float' solutions are pulled to the same 'fixed' ambiguity vector z. Using the pull-in regions, one can give an explicit expression for the corresponding integer ambiguity estimator. It reads

$$\check{a} = \sum_{z \in Z^n} z s_z(\hat{a}) \quad (6)$$

with the indicator function

$$s_z(\hat{a}) = \begin{cases} 1 \text{ if } \hat{a} \in S_z \\ 0 \text{ otherwise} \end{cases}. \quad (7)$$

Since the pull-in regions define the integer estimator completely, one can define classes of integer estimators by imposing various conditions on the pull-in regions. In Teunissen (1999) we defined one such class, which we called the class of admissible integer estimators. These integer estimators are defined as follows.

Definition The integer estimator $\check{a} = \sum_{z \in Z^n} z s_z(\hat{a})$ is said to be *admissible* if

(i) $\bigcup_{z \in Z^n} S_z = R^n$
(ii) $S_{z_1} \bigcap S_{z_2} = \emptyset, \quad \forall z_1, z_2 \in Z^n, z_1 \neq z_2$
(iii) $S_z = z + S_0, \quad \forall z \in Z^n.$

This definition is motivated as follows. Each one of the above three conditions describe a property of which it seems reasonable that it is possessed by an arbitrary integer ambiguity estimator. The first condition states that the pull-in regions should not leave any gaps and the second that they should not overlap. The absence of gaps is needed in order to be able to map any 'float' solution $\hat{a} \in R^n$ to Z^n, while the absence of overlaps is needed to guarantee that the 'float' solution is mapped to just one integer vector. Note that we allow the pull-in regions to have common boundaries. This is permitted if we assume to have zero probability that $\hat{a}$ lies on one of the boundaries. This will be the case when the probability density function (pdf) of $\hat{a}$ is continuous.

The third and last condition follows from the requirement that $S(x + z) = S(x) + z, \forall x \in R^n, z \in Z^n$. Also this condition is a reasonable one to ask for. It states that when the 'float' solution is perturbed by $z \in Z^n$, the corresponding integer solution is perturbed by the same amount. This property allows one to apply the *integer remove-restore* technique: $S(\hat{a} - z) + z = S(\hat{a})$. It therefore allows one to work with the fractional parts of the entries of $\hat{a}$, instead of with its complete entries.

With the division of R^n into mutually exclusive pull-in regions, we are now in the position to present the distribution of both the integer ambiguity estimator and the 'fixed' baseline estimator. These distributions follow ones the pull-in region of the chosen integer estimator has been defined. Three such examples of integer estimators are 'rounding','bootstrapping' and 'integer least-squares'. Since their pull-in regions differ, see Teunissen (1998a), also their probabilistic properties will differ. The probability of correct integer estimation of the bootstrapped estimator is given in Teunissen (1997) and that of the integer least-squares estimator in Hassibi and Boyd (1998) and Teunissen (1998b). In the following we will consider the probabilistic properties of the whole class of admissible integer estimators, of which 'rounding', 'bootstrapping' and 'integer least-squares' are special cases. We first consider the distribution of $\check{a}$. This distribution is of the *discrete* type and it will be denoted as $P(\check{a} = z)$. It is a probability mass function, having zero masses at nongrid points and nonzero masses at some or all grid points. If we denote the *continuous* probability density function of $\hat{a}$ as $p_{\hat{a}}(x)$, the distribution of $\check{a}$ follows as

$$P(\check{a} = z) = \int_{S_z} p_{\hat{a}}(x) dx \ , \ z \in Z^n. \quad (8)$$

This expression holds for any distribution the 'float' ambiguities $\hat{a}$ might have. In most GNSS applications however, one assumes the vector of observables y to be normally distributed. The estimator $\hat{a}$ is therefore normally distributed too, with mean a and vc-matrix $Q_{\hat{a}}$. Its probability density function reads

$$p_{\hat{a}}(x) = \frac{1}{\sqrt{\det(Q_{\hat{a}})}(2\pi)^{\frac{1}{2}n}} \exp\{-\frac{1}{2} \parallel x - a \parallel^2_{Q_{\hat{a}}}\} \quad (9)$$

with the squared weighted norm $\parallel . \parallel^2_{Q_{\hat{a}}} = (.)^T Q_{\hat{a}}^{-1} (.)$. Note that $P(\check{a} = a)$ equals the probability of *correct* integer ambiguity estimation. It describes the expected success rate of GNSS ambiguity resolution.

We are now in the position to determine the distribution of the 'fixed' baseline $\check{b}$. It will be denoted as $p_{\check{b}}(x)$. Once it has been determined, its peakedness can be studied and probabilistic statements such as

$$P(\check{b} \in T) = \int_T p_{\check{b}}(x)dx \quad , \quad T \subset R^o \tag{10}$$

can be made. The determination of $p_{\check{b}}(x)$ would be straightforward in case $\check{a}$ is deterministic. In that case normality would be preserved when propagating the normal distribution of $\hat{a}$ and $\hat{b}$ through (4). In our case however, $\check{a}$ is not deterministic but stochastic. We therefore also need to take the distribution of these integer ambiguities into account. As a result one obtains the probability density function of $\check{b}$ as

$$p_{\check{b}}(x) = \sum_{z \in Z^n} p_{\hat{b}|\hat{a}=z}(x) P(\check{a} = z). \tag{11}$$

This distribution describes the probabilistic properties of the GNSS baseline in case the integerness of the carrier phase ambiguities is included in the model. The distribution is clearly not normal. It is a weighted and infinite sum of conditional distributions. The weights are given by the probability masses of the distribution of $\check{a}$. The conditional distributions $p_{\hat{b}|\hat{a}=z}(x), z \in Z^n$ are translated copies of one another and are given as

$$p_{\hat{b}|\hat{a}=z}(x) = \frac{1}{\sqrt{\det Q_{\hat{b}|\hat{a}}}(2\pi)^{\frac{1}{2}o}} \cdot \exp\{-\frac{1}{2} \parallel x - b_{|\hat{a}=z} \parallel^2_{Q_{\hat{b}|\hat{a}}}\} \tag{12}$$

with the conditional mean $b_{|\hat{a}=z} = b - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1}(a - z)$ and the conditional vc-matrix $Q_{\hat{b}|\hat{a}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}}$.

In most practical applications of GNSS, the distribution $p_{\hat{b}|\hat{a}=a}(x)$ is used instead of the theoretically correct distribution (11). This approximation is only permitted however, when the estimated integer ambiguities can be considered sufficiently nonrandom. This follows from the limit

$$p_{\hat{b}|\hat{a}=a}(x) = \lim_{P(\check{a}=a) \to 1} p_{\check{b}}(x). \tag{13}$$

For this conditional distribution to be a good approximation, one thus has to make sure that the probability of correct integer estimation is sufficiently close to one.

3. An Optimal Integer Ambiguity Estimator

The distributional results presented so far hold for any admissible ambiguity estimator. Two examples of admissible ambiguity estimators are the ‘rounding’ estimator and the ‘bootstrapped’ estimator. The simplest way to obtain an integer vector from the real-valued ‘float’ solution is to round each of the entries of $\hat{a}$ to its nearest integer. The corresponding integer estimator reads therefore

$$\check{a}_R = ([\hat{a}_1], \ldots, [\hat{a}_n])^T \tag{14}$$

where ‘[.]’ denotes rounding to the nearest integer. The pull-in region of this integer estimator equals the multivariate version of a square.

Another relatively simple integer ambiguity estimator is the bootstrapped estimator. The bootstrapped estimator can be seen as a generalization of the previous estimator. It still makes use of integer rounding, but it also takes some of the correlation between the ambiguities into account. The bootstrapped estimator follows from a sequential conditional least-squares adjustment and it is computed as follows. If n ambiguities are available, one starts with the first ambiguity $\hat{a}_1$, and rounds its value to the nearest integer. Having obtained the integer value of this first ambiguity, the real-valued estimates of all remaining ambiguities are then corrected by virtue of their correlation with the first ambiguity. Then the second, but now corrected, real-valued ambiguity estimate is rounded to its nearest integer. Having obtained the integer value of the second ambiguity, the real-valued estimates of all remaining $n-2$ ambiguities are then again corrected, but now by virtue of their correlation with the second ambiguity. This process is continued until all ambiguities are considered. The components of the bootstrapped estimator $\check{a}_B$ are given as

$$\check{a}_B = ([\hat{a}_1], [\hat{a}_{2|1}], \ldots, [\hat{a}_{n|N}])^T \tag{15}$$

where the shorthand notation $\hat{a}_{i|I}$ stands for the ith least-squares ambiguity obtained through a conditioning on the previous $I = \{1, \ldots, (i-1)\}$ sequentially rounded ambiguities. The pull-in region of the bootstrapped estimator equals the multivariate version of a parallelogram.

Although various integer estimators exist which are admissible, some may be better than others. Having the problem of GNSS ambiguity resolution in mind, one is particularly interested in the estimator which maximizes the probability of correct integer estimation. This probability equals $P(\check{a} = a)$, but it will differ for different ambiguity estimators. In order to find the estimator which has the largest probability of correct integer estimation, we need to know which estimator maximizes $P(\check{a} = a)$. The answer to this question is given by the following theorem.

Theorem Let the integer least-squares estimator be defined as

$$\check{a}_{LS} = \arg\min_{z \in Z^n} \parallel \hat{a} - z \parallel^2_{Q_{\hat{a}}} \tag{16}$$

and the pdf of $\hat{a}$ be given as

$$p_a(x) = \sqrt{\det(Q_{\hat{a}}^{-1})} G(\parallel x - a \parallel^2_{Q_{\hat{a}}}) \tag{17}$$

where $G : R \mapsto [0, \infty)$ is decreasing and $Q_{\hat{a}}$ is positive-definite. Then

$$P(\check{a}_{LS} = a) \geq P(\check{a} = a) \tag{18}$$

for any admissible estimator $\check{a}$.

This theorem gives a probabilistic justification for using the integer least-squares estimator. For GNSS ambiguity resolution it shows, that one is better off using the integer least-squares estimator than any other admissible integer estimator. The theorem was introduced and proved in [*ibid*]. The family of distributions defined in (17), is known as the family of elliptically contoured distributions (Chmielewsky, 1981). Several important distributions belong to this family. The multivariate distribution can be shown to be a member of this family by choosing $G(x) = (2\pi)^{-\frac{n}{2}} \exp -\frac{1}{2}x, x \in R$. Another member is the multivariate t-distribution.

As a direct consequence of the above theorem we have the following corollary.

Corollary Let Σ be any positive-definite matrix of order n and define

$$\check{a}_\Sigma = \arg\min_{z \in Z^n} \parallel \hat{a} - z \parallel^2_\Sigma . \quad (19)$$

Then $\check{a}_\Sigma$ is admissible and

$$P(\check{a}_{LS} = a) \geq P(\check{a}_\Sigma = a). \quad (20)$$

In order to prove the corollary, we only need to show that $\check{a}_\Sigma$ is admissible. Once this has been established, the stated result (20) follows from the theorem. The admissibility can be shown as follows. The first two conditions of the definition are satisfied, since (19) produces—apart from boundary ties—a unique integer vector for any 'float' solution $\hat{a} \in R^n$. And since $\check{a}_\Sigma = \arg\min_{z \in Z^n} \parallel \hat{a} - u - z \parallel^2_\Sigma + u$ holds true for any integer $u \in Z^n$, also the integer remove-restore technique applies.

As the corollary shows, a proper choice of the data weight matrix is also of importance for ambiguity resolution. The choice of weights is optimal when the weight matrix equals the inverse of the ambiguity vc-matrix. A too optimistic precision description or a too pessimistic precision description, will both result in a less than optimal ambiguity success rate.

Another aspect made clear by the corollary, is the relation between 'integer rounding' and 'integer least-squares'. One of the simplest choices for Σ would be a diagonal matrix. In that case $\parallel \hat{a} - z \parallel^2_\Sigma$ reduces to a sum of squares and $\check{a}_\Sigma$ becomes the integer estimator that follows from a rounding to the nearest integer of the entries of $\hat{a}$. Thus $\check{a}_\Sigma = [\hat{a}]$, where '[.]' denotes the operation of componentwise rounding, and

$$P(\check{a}_{LS} = a) \geq P([\hat{a}] = a). \quad (21)$$

We can generalize this result to a whole class of integer estimators based on rounding, when the choice $\Sigma = (Z^T D Z)^{-1}$ is made, where D is a diagonal matrix with positive entries and Z is an admissible ambiguity transformation. Ambiguity transformations are said to be admissible when all the entries of both Z and its inverse are integer (Teunissen, 1995). For this particular choice of Σ, we have $\parallel \hat{a} - z \parallel^2_\Sigma = (Z\hat{a} - u)^T D(Z\hat{a} - u)$, with $u = Zz \in Z^n$. Hence, when parametrized in u, $\parallel \hat{a} - z \parallel^2_\Sigma$ again reduces to a sum of squares. Thus $\check{u} = [Z\hat{a}]$ and $\check{a}_\Sigma = Z^{-1}[Z\hat{a}]$. In this case the integer estimator is computed by first transforming the 'float' solution, then applying the componentwise rounding scheme, followed by the back-transformation. For the probability of correct integer estimation, we thus have

$$P(\check{a}_{LS} = a) \geq P(Z^{-1}[Z\hat{a}] = a) \quad (22)$$

for any admissible ambiguity transformation Z.

Note that (21) is a special case of (22). The choice $Z = I_n$, however, is usually not the best one. That is, if one insists on using the integer estimator based on rounding, one can often improve upon the ambiguity success rate by choosing an appropriate transformation matrix Z. This is particularly true in case of GNSS, when the DD ambiguities are used. Since the equality in (22) will hold true in case the vc-matrix of $Z\hat{a}$ is diagonal, an ambiguity transformation should be used that results in a close to diagonal form as possible. This is achieved when using the decorrelation process of the LAMBDA method (Teunissen, 1993). Hence, when one decides to use the integer estimator based on rounding, one should at least decorrelate the ambiguities first, before applying the integer rounding scheme. In this way one will obtain a success rate which is higher than the one obtained without using the decorrelation process.

Summary

When evaluating the quality of the estimated real-valued GNSS parameters, such as the 'fixed' GNSS baselines or atmospheric delays, it is not enough to simply assume that the estimated integer ambiguities are deterministic variates. Not only are the estimated integer ambiguities random by definition, their probabilistic properties also depend on the chosen method of integer estimation as governed by their respective pull-in regions. In this contribution, these probabilistic properties were given for the whole class of admissible integer estimators, of which 'rounding', 'bootstrapping' and 'integer least-squares' are special cases. Within this class, assuming that the distribution of the 'float' solution belongs to the family of elliptically contoured distributions, the largest ambiguity success rate is obtained by the integer least-squares estimator. This therefore also holds true for the special case of a normally distributed 'float' solution, provided the ambiguity variance-covariance matrix is taken as the corresponding weight matrix.

References

Boon, F. and B. Ambrosius, Results of real-time applications of the LAMBDA method in GPS based aircraft landings, Proceedings KIS97, pp. 339–345, 1997.

Boon, F., P. J. de Jonge, and C. C. J. M. Tiberius, Precise aircraft positioning by fast ambiguity resolution using improved troposphere modelling, Proceedings ION GPS-97, Vol. 2, pp. 1877–1884, 1997.

Chmielewsky, M. A., Elliptically symmetric distributions: A review and bibliography, *Internat. Statist. Rev.*, **49**, 67–74, 1981.

de Jonge, P. J. and C. C. J. M. Tiberius, *The LAMBDA Method for Integer Ambiguity Estimation: Implementation Aspects*, Delft Geodetic Computing Centre, LGR Series, No. 12, 45 pp., Delft University of Technology, 1996a.

de Jonge, P. J. and C. C. J. M. Tiberius, Integer estimation with the LAMBDA method, Proceedings IAG Symposium No. 115, 'GPS trends interrestrial, airborne and spaceborne applications', edited by G. Beutler *et al.*, pp. 280–284, Springer Verlag, 1996b.

de Jonge, P. J., C. C. J. M. Tiberius, and P. J. G. Teunissen, Computational aspects of the LAMBDA method for GPS ambiguity resolution, Proceedings ION GPS-96, pp. 935–944, 1996.

Han, S., Ambiguity resolution techniques using integer least-squares estimation for rapid static or kinematic positioning, Symposium Satellite Navigation Technology: 1995 and beyond, 10 p., Brisbane, Australia, 1995.

Hassibi, A. and S. Boyd, Integer parameter estimation in linear models with applications to GPS, IEEE Trans. Signal Processing, Nov. 1998, **46**(11), 2938–2952, 1998.

Hofmann-Wellenhof, B., H. Lichtenegger, and J. Collins, *Global Positioning System: Theory and Practice*, fourth edition, Springer Verlag, 1997.

Jonkman, N. F., *Integer GPS-Ambiguity Estimation without the Receiver-Satellite Geometry*, Delft Geodetic Computing Centre, LGR Series, No. 18, 95 pp., Delft University of Technology, 1998.

Leick, A., *GPS Satellite Surveying*, 2nd edition, John Wiley & Sons, 560 pp., New York, 1995.

Parkinson, B. and J. J. Spilker (eds.), *GPS: Theory and Applications*, Vols. 1 and 2, AIAA, Washington D.C., 1995.

Strang, G. and K. Borre, *Linear Algebra, Geodesy, and GPS*, 624 pp., Wellesley-Cambridge Press, 1997.

Teunissen, P. J. G., *Least-squares estimation of the integer GPS ambiguities*, Invited Lecture, Section IV Theory and Methodology, IAG General Meeting, Beijing, China, August 1993. Also in: LGR Series, No. 6, Delft

Geodetic Computing Centre, 1993.
Teunissen, P. J. G., The invertible GPS ambiguity transformations, *Man. Geod.*, **20**, 489–497, 1995.
Teunissen, P. J. G., Some remarks on GPS ambiguity resolution, *Artificial Satellites*, **32**(3), 119–130, 1997.
Teunissen, P. J. G., A class of unbiased integer GPS ambiguity estimators, *Artificial Satellites*, **33**(1), 3–10, 1998a.
Teunissen, P. J. G., On the integer normal distribution of the GPS ambiguities, *Artificial Satellites*, **33**(2), 49–64, 1998b.
Teunissen, P. J. G., A theorem on maximizing the probability of correct integer estimation, *Artificial Satellites*, **34**(1), 3–10, 1999.
Teunissen, P. J. G. and A. Kleusberg (eds.), *GPS for Geodesy*, 2nd enlarged edition, 650 pp., Springer Verlag, 1998.
Tiberius, C. C. J. M. and P. J. de Jonge, Fast positioning using the LAMBDA method, Proceedings DSNS-95, Paper 80, 8 p., 1995.
Tiberius, C. C. J. M., P. J. G. Teunissen, and P. J. de Jonge, Kinematic GPS: performance and quality control, Proceedings KIS97, pp. 289–299, 1997.

P. J. G. Teunissen (e-mail: P.J.G.Teunissen@geo.tudelft.nl)