

On the selection of frequencies for long baseline GALILEO ambiguity resolution

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Introduction

The European GALILEO system is currently in its definition phase. During this phase, which will be completed in December 2000, a decision should be taken on the carrier frequencies to be used for this new GNSS (Global Navigation Satellite System). This decision not only includes the number of frequencies, but also the frequencies themselves. There is no doubt among potential users of the system that GALILEO will transmit at least at two carrier frequencies: ionospheric delays, which are frequency-dependent and an important error

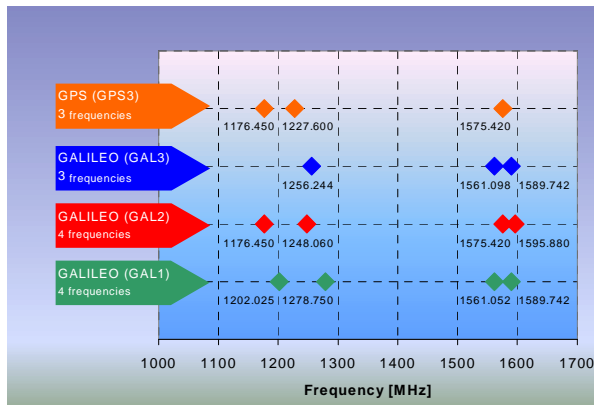


Figure 1: Some of the proposed GALILEO and modernized GPS frequencies.

source, can be removed by forming a linear combination of observations, made at two different frequencies. However, additional frequencies are more than welcome. In fact, a number of proposals were already made in recent years that include three and even four carrier frequencies, see Figure 1 and the November 1999 issue of GPS World, [3]. These frequencies will facilitate fast resolution of the integer ambiguities of the carrier observations. Carrier observations are used for very precise (cm-level or better) relative positioning applications. These observations, however, contain a constant unknown bias of an integer number of cycles, which is known as the integer ambiguity. GNSS data processing for precise relative positioning therefore consists of the three steps shown in Figure 2. In the first step, baseline components and ambiguity parameters are estimated. The ambiguities from this step are not integer, but real valued. The results from this step are therefore generally referred to as the float solution. In the second step, the integer ambiguities are determined. Once they have been resolved, the carrier observations will start acting as very precise pseudo range measurements. In the third and final step, known as the fixed solution, the GNSS observations are processed again, but this time with the ambiguities fixed to their integer values. As a result, the estimated baseline components will have a very high precision.

Research in recent years revealed that the more frequencies are available, the faster the integer ambiguities can be resolved. This is important in particular for real-time applications. Real-time kinematic (RTK) positioning using GPS is already very popular and it can be assumed that the

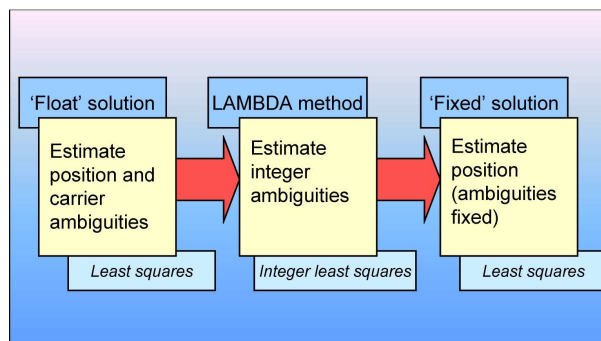


Figure 2: The three steps involved in GNSS data processing for precise relative positioning and the corresponding optimal estimation methods.

market potential for this kind of techniques is enormous, in particular if fast and reliable ambiguity resolution can be guaranteed. Applications range from surveying and mapping to machine guidance and from hydrography to positioning of low Earth orbiting satellites.

In order to ensure reliable ambiguity resolution, a sufficiently large probability of correct integer ambiguity estimation is needed. This probability, also referred to as the success rate, depends on a number of factors:

1. The observation equations (functional model), which describe the relationship between observations and parameters to be estimated.
2. The observation variance-covariance matrix (stochastic model), which captures correlation and measurement uncertainty.
3. The method used to estimate the integer ambiguities.

One can distinguish between two types of functional models, geometry-free and geometry-based. The geometry-based model is most widely used for GNSS positioning applications. In this model the observation equations are parametrized and linearized in terms of the unknown station coordinates. In the geometry-free model, however, the observations are parametrized in terms of the unknown receiver-satellite ranges, see [5]. It is the simplest model that still allows for integer ambiguity resolution. The model is linear and since the receiver-satellite geometry is excluded, it does not require any information with regard to e.g. satellite positions. For the present study this is an important property, since the GALILEO orbits have not yet been defined. Therefore, the analyses done in this contribution will be based on the geometry-free model.

The stochastic model consists of the a priori precision of code and carrier observations. Here we will assume, unless stated otherwise, that the standard deviations of the *undifferenced* observations are 25 cm and 2.5 mm for code and carrier respectively. Thus, for double differences, they are twice as large. Time correlation and correlation between observations are assumed absent.

A number of methods exist for estimating the integer ambiguities, such as simple rounding, sequential rounding and integer least-squares. It can be shown that integer least-squares, as implemented in the LAMBDA-method, [6], results in the highest possible success-rates. To compute success-rates, no actual data are required. All one needs is the covariance matrix of the estimated real-valued ambiguities. This covariance matrix depends on both the functional and the stochastic model, but not on the observations themselves. As such, the success-rate is an important design parameter, which can be used not only as a planning tool, comparable to the popular DOP (Dilution of Precision) values, but also as a system design tool, which

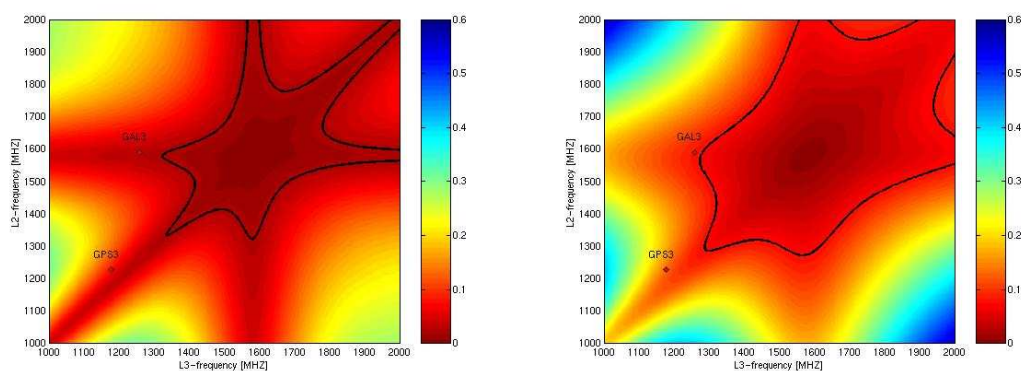


Figure 3: Success-rates for instantaneous long-baseline ambiguity resolution for the geometry-free model as function of two of the three carrier frequencies. One frequency was kept fixed to 1580 MHz. Standard deviation of the code observations was 25 cm (left) and 10 cm. The contour line depicts the success-rate of the current dual-frequency GPS.

allows for a well-founded selection of carrier frequencies for future GNSS's.

In this contribution we will concentrate on success-rates for long-baseline GALILEO applications, i.e., baselines for which ionospheric effects cannot be ignored. As it will turn out, it is not always possible to reliably resolve all integer ambiguities. Therefore, we also

consider the case of partial ambiguity resolution and its influence on the estimated DD range parameter.

Full ambiguity resolution

In this section we will consider long baseline ambiguity success-rates for triple-frequency GNSS's, assuming all ambiguities have to be resolved. This allows for a comparison of the success-rates of a proposed triple-frequency GALILEO (GAL3) and modernized GPS (GPS3), see Figure 1. We will also investigate if these two triplets are the best possible choices or if other sets of three frequencies would result in higher success-rates. This will be done by keeping one frequency fixed to 1580 MHz, which is close to one of the proposed GALILEO and GPS frequencies, and let the other two frequencies vary between 1000 and 2000 MHz. The results for all these triple-frequency systems and for a single observation epoch are shown in Figure 3 for a standard deviation of the code observations of 25 and 10 cm. Also shown in this figure is the contour line corresponding to the success-rate of the current dual-frequency GPS. From Figure 3 we may conclude that the success-rate depends on the selected frequencies. All triple-frequency success-rates within the dual-frequency GPS contour line of success-rates are smaller than those dual-frequency success-rates. Thus, adding a third frequency not always results in higher success-rates. As we can also see from Figure 3, for the proposed GALILEO and modernized GPS frequencies, the success-rates lie outside the contour line, but better combinations of three frequencies are possible. For example, selecting the current GPS L2 frequency as second frequency and the third frequency around 1000 MHz results in a higher success-rate. We also notice the increase in success-rate when the precision of the code observations is improved from 25 to 10 cm. However, even for a standard deviation of 10 cm, the success-rates are less than 50% for all possible triplets. It can therefore be concluded that reliable instantaneous full ambiguity resolution is not possible for the frequencies considered.

Instead of keeping the number of observation epochs fixed to one epoch and computing the corresponding success-rate, one may also compute the number of epochs required to attain a predefined success-rate. The required number of epochs for the four scenarios of Figure 1 and for a standard deviation of the code observations of 10 cm are shown in Figure 4. It becomes clear from this figure that, for a given success-rate, the proposed GAL3 requires a much longer observation period than GPS3.

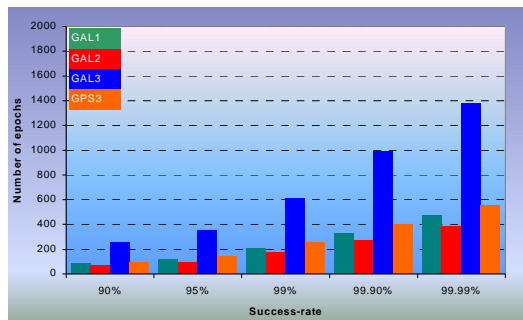


Figure 4: Number of epochs required to attain a pre-defined success-rate for the four systems of Figure 1. Standard deviation of the code observations was 10 cm.

longer observation period than GPS3. For the considered standard deviation of the code observations it is 2.5 times longer (for a standard deviation of 25 cm, not shown here, it is 5.3 times longer). For the two four-frequency systems, GAL1 and GAL2, the required number of epochs is of the same order of magnitude. This number, although still large, is smaller than for the triple-frequency systems. The general conclusion that can be drawn from Figure 4 is that, for the geometry-free model, fast full ambiguity resolution is not possible.

Partial ambiguity resolution

Although one usually aims at resolving all integer ambiguities simultaneously ('full ambiguity resolution'), it could happen – as we have seen in the above examples – that this requires too many epochs of data. In that case, one might consider as an alternative the resolution of only a subset of the ambiguities. Fewer epochs will then be needed for partial ambiguity resolution to be successful.

To see this procedure at work, we again take a triple-frequency case as an example. Two frequencies are set at fixed values (1200 and 1600 MHz), while the third frequency is varied

between 800 and 2000 MHz. Figure 5 shows the single-epoch standard deviations (in cycles) of the three LAMBDA-transformed ambiguities as function of the varying third frequency. As the figure shows, two of the three ambiguities have a (relatively) high precision, whereas the precision of the third ambiguity is rather poor.

Fixing the ambiguities that have a good precision requires only a moderate number of epochs. This would be the case when only two ambiguities, corresponding to the blue and yellow lines in Figure 5, are fixed. Also resolving the third ambiguity, depicted by the red line, however, results in a significant increase in the number of epochs, due to its poor precision. Or, equivalently, the single-epoch success-rate for partial ambiguity resolution will be much

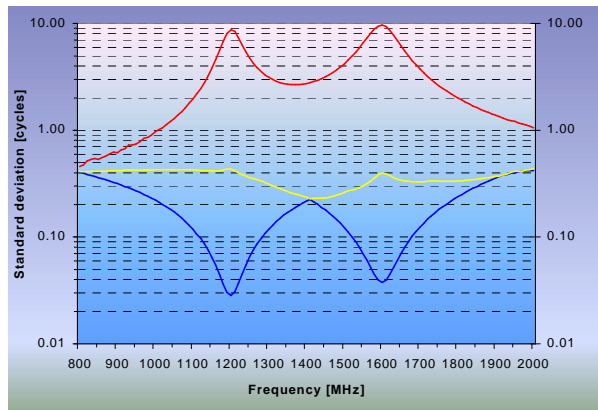


Figure 5: Single-epoch standard deviations (cycles) of the three decorrelated LAMBDA transformed ambiguities as function of the third frequency. The first two frequencies are fixed at 1200 and 1600 MHz.

higher than for full ambiguity resolution.

Since this example made clear that partial ambiguity resolution can be successful even when full ambiguity resolution is not, we will now re-examine our four earlier cases and determine their potential for partial ambiguity resolution. In all four cases, the poorest determined (transformed) ambiguity is excluded from the resolution process. For the two triple-frequency cases, partial ambiguity resolution is thus based on the best two ambiguities, while for the other two it is based on the best three ambiguities. The results are shown in Figure 6.

Upon comparing these results with the full ambiguity resolution results of Figure 4, we immediately notice the dramatic reduction in the number of required epochs. Very fast to reasonably fast partial ambiguity resolution is now possible in all four cases. The variations observed in the required number of epochs are due to the differences in frequency allocation, the chosen measurement precision and the value of the success-rate aimed at. Also note that cases which were best in the full ambiguity resolution set-up, may not remain best when partial ambiguity resolution is considered. This is due to the differences in the absolute versus relative frequency allocations of the four cases.

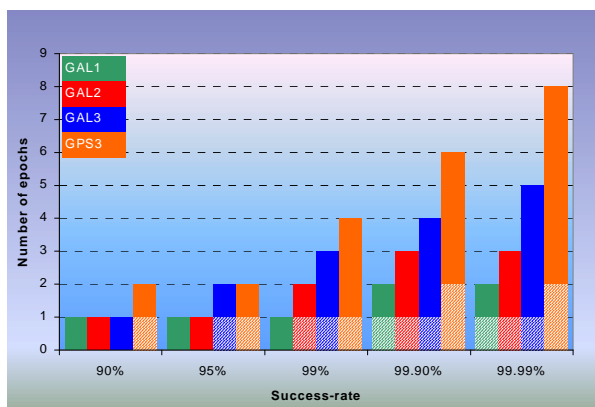


Figure 6: Number of epochs required to attain a pre-defined success-rate for the four systems of Figure 1 when fixing the best n_f-1 (n_f – number of frequencies) ambiguities. Code standard deviation was 25 and 10 cm. The results for a code precision of 10 cm are shown as dashed columns and only when they differ from the 25 cm results.

Does partial ambiguity resolution make sense?

Resolving only a subset of the ambiguities also implies, however, that not all of the carrier phase data will exhibit the property of precise pseudo ranges. The precision improvement in the DD ranges due to partial ambiguity resolution will therefore always be smaller than it would have been in case of full ambiguity resolution. In fact, the precision improvement could even be so small, that the float solution reaches the same level of precision nearly as fast. In that case partial ambiguity resolution would not buy us much. For deciding whether partial ambiguity resolution makes sense, an

evaluation of the precision of the fixed DD ranges is therefore needed as well. These results are shown in Figure 7 for a 99.99% success-rate.

From the figure the following conclusions can be drawn. GAL3 clearly exhibits the poorest performance. It has the largest standard deviation of the fixed DD range and the second longest resolution time. Although its resolution is as fast or almost as fast as that of GAL1 and GAL2, the precision level of its fixed DD range is significantly poorer. GAL3 is also outperformed by GPS3. Although its resolution is faster than that of GPS3, the precision of the latter is significantly better. And this remains true even when the number of GAL3 epochs is increased to the GPS3 number of epochs.

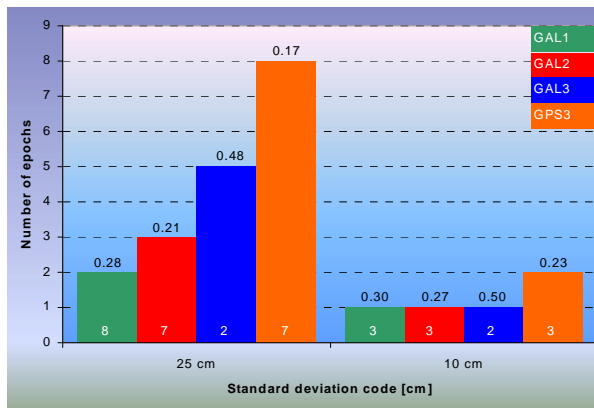


Figure 7: Required number of epochs to attain a success-rate of 99.99% for the best n_f-1 (n_f – number of frequencies) ambiguities of the four systems of Figure 1. The number on top of each column denotes the standard deviation (in meters) of the corresponding estimated DD range, the number at the bottom is the factor by which the height of the column has to be multiplied to obtain the number of epochs required for the float solution reach the same DD range precision.

times, but also narrow the gap between the fixed and float solution. As Figure 7 shows, this time gain is reduced considerably in case the code standard deviation decreases from 25 to 10 cm. In this case the float solution only needs a few epochs more than the fixed solution to obtain the same precision for the DD range.

Summary and concluding remarks

Important factors for ambiguity resolution are: frequency allocation, code precision and satellite geometry. In this contribution, the receiver-satellite geometry was discarded, since attention was restricted to the geometry-free model. This is an important restriction, since the receiver-satellite geometry is known to have a significant impact on ambiguity resolution performance. Besides the frequency allocation, the level of the code precision is of importance too. The impact of the code precision is such that differences between the options become less pronounced when more precise code measurements are used. In that case also the time gain between the float and fixed solution reduces. Full ambiguity resolution, however, will remain problematic with the geometry-free model.

For the frequency allocation, both the relative and absolute frequency values are of importance. Their effect on partial ambiguity resolution is however contrary to their effect on the precision of the fixed DD range and on full ambiguity resolution. Assuming partial ambiguity resolution to be successful, one can improve the precision of the fixed DD range by using lower frequencies, with a larger spacing between them. This implies that when two out of the three frequencies are close, which is beneficial for the first level of partial ambiguity resolution, the two frequencies are best chosen at the lower end of the spectrum. This explains the significant difference in performance between GPS3 and GAL3. The frequency allocation of GAL3 is unfortunate, since its two closest frequencies are at the high end of the spectrum, while its lowest frequency is still larger than the two lowest of GPS3.

There is not too much difference between the two four-frequency options GAL1 and GAL2. GAL1 is slightly faster than GAL2, but has a somewhat poorer precision of the fixed DD range. Both options are however faster than GPS3, although this difference gets less pronounced the better the code precision becomes. The two four-frequency options are therefore particularly helpful in case of a relatively poor code precision.

The level of the code precision is, however, not only of importance for deciding whether or not a fourth frequency is needed, but also for deciding whether partial ambiguity resolution makes sense at all. Using more precise code observations will not only reduce the resolution

An even worse triple-frequency option would be when all frequencies are chosen close together. Although this is beneficial for both levels of partial ambiguity resolution, it would need a much larger compensation – in terms of smaller frequencies – to get the precision of the fixed DD range to an acceptable level. The two triplets 1202.025, 1248.060, 1278.750 and 1561.052, 1589.742, 1595.880 MHz of the GAL1 and GAL2 options, for instance, would both have an excellent partial ambiguity resolution performance, but a very poor precision of the fixed DD range, with the first triplet slightly outperforming the second. When including two of the three GPS3 frequencies, as is the case with GAL2, a better triple-frequency option would therefore be to locate the third frequency closer to the midpoint and thus replace GPS3's 1227.600 by GAL1's 1278.750 MHz.

Finally, it was shown that the two four-frequency options GAL1 and GAL2 perform better than GAL3 and GPS3, respectively. A further improvement can be obtained, however, when it would be possible to choose the fourth frequency below the lowest of GPS3.

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See also www.geo.tudelft.nl/mgp for more papers on these topics.