CONSEQUENCES OF THE CROSS-CORRELATION MEASUREMENT TECHNIQUE[1]

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ABSTRACT - *The use of dual frequency GPS observations offers several advantages over the use of single frequency observations. Unfortunately, the use of the second GPS frequency (L2) is hindered by the encryption of the known P-code into a secret Y-code under Anti-Spoofing (AS). Civilian users therefore have to resort to special measurement techniques, termed codeless and semi-codeless, to circumvent the encryption and make observations on the L2 frequency. Cross-correlation is one such measurement technique employed by some of the leading GPS equipment manufacturers.*

In this contribution we will elaborate on the stochastic properties of cross-correlation reconstructed observables and derive an appropriate stochastic model for this technique. We will present empirical evidence to support this model. In addition we will show that the use of such a model is of importance for both ambiguity estimation and positioning. Not taking into account the cross-correlation stochastics will affect the ambiguity estimation and also result in non-optimal position estimators.

1 - INTRODUCTION

The processing of GPS data requires the specification of an observation model, consisting of a functional and a stochastic model. The GPS functional model is (sufficiently) well known and documented. The same however can not be said of the GPS stochastic model. In the many GPS textbooks available, one will usually find only a few comments, if any, on the stochastic model of the GPS observables. GPS receiver advertisements or data sheets are usually also vague in their specifications of the precision characteristics of the data outputted by the receivers.

Due to this lack of information in the public domain, most of us are probably inclined to start with the simplest stochastic model possible: a diagonal model in which the observations are simply weighted type-by-type (pseudo ranges and carrier phases). Such a model however may be an oversimplification that fails to do justice to the more complicated precision characteristics of the data. To demonstrate that a more elaborate model is generally needed and that an oversimplified model can have an adverse effect on the parameter estimation, we will focus in this contribution on the effects on the GPS observation precision characteristics of the cross-correlation measurement technique, a resort for dual frequency GPS receivers in the presence of Anti-Spoofing.

2 - PSEUDO RANGE AND CARRIER PHASE NOISE

The GPS stochastic model should reflect the noise properties of the observations. A first impression of the pseudo range and carrier phase noise can be obtained from appropriately constructed time series of data. For that purpose we will consider time series of double differenced (DD) pseudo range and carrier observations collected in a zero baseline set-up.

DD data are free from receiver and satellite clock effects. In addition, as a result of the zero-baseline set-up, the contributions of the receiver-satellite geometry and the environment (atmosphere

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and multipath) are also largely eliminated. The remaining small geometric and environmental effects are due mainly to the non-simultaneity of sampling of the two receivers involved.

In order to remove the residual geometric effects from the DD pseudo range observations the DD carrier phase observations are used as a reference. The DD phase observations exhibit the same geometric effects as the DD pseudo range observations, but the carrier phase noise is at least a factor of one hundred smaller than the pseudo range noise. The difference between DD pseudo range and carrier phase observations will therefore primarily be affected by the pseudo range noise. (Note that this approach does assume the carrier phase observations to be corrected for the unknown integer ambiguities. For our analysis the ambiguities were resolved reliably from a full hour of data).

As a reference for the DD carrier phase observations a second order polynomial is used. The geometric effects in the DD data change in principle only slowly over time. Hence, by fitting a low order polynomial through the carrier phase observations the geometric effects can largely be accounted for. Still, a polynomial is not nearly as good a reference for the DD carrier phase observations as the (unambiguous) DD carrier phases are for the DD pseudo range observations. Hence, the differences between the carrier phase observations and the polynomial can be expected to exhibit some small residual effects.

Figure 1 shows an example of three time series of DD pseudo range observations corrected with DD carrier phase observations. Each time series comprises of 3000 samples spaced by 1 second. The time series therefore cover a time span of 50 minutes.

The first time series of figure 1 pertains to L1-frequency observations. (Thus the DD pseudo range observations on the L1 frequency minus the unambiguous DD carrier phase observations on the L1 frequency). The second time series pertains to L2 frequency observations and the third time series pertains to differences between L1 and L2 frequency observations. The first two time series give an impression of the pseudo range noise on the L1 and L2 frequency, while the third time series gives an impression of the noise of the difference between the L1 and L2 frequency pseudo ranges.

Examination of the first two time series of figure 1 shows that the L2 frequency pseudo range observations are somewhat noisier than the L1 frequency observations. Moreover, the time series of L2 frequency pseudo ranges shows some small transient effects hinting at internal filtering by the receivers collecting the observations. In addition it appears from a comparison of the first two time series with the third time series that the noise in the differences between the L1 and L2 frequency pseudo range observations is of the same order of magnitude - or even somewhat smaller - than the noise in the L1 and L2 frequency pseudo range observations.

Figure 2 shows a similar example of three time series of DD carrier phase observations corrected with a second order polynomial. As can be seen from the figure, our comments on the noise in the time series of pseudo range observations apply to a large extent also to the noise in the time series of carrier phase observations.

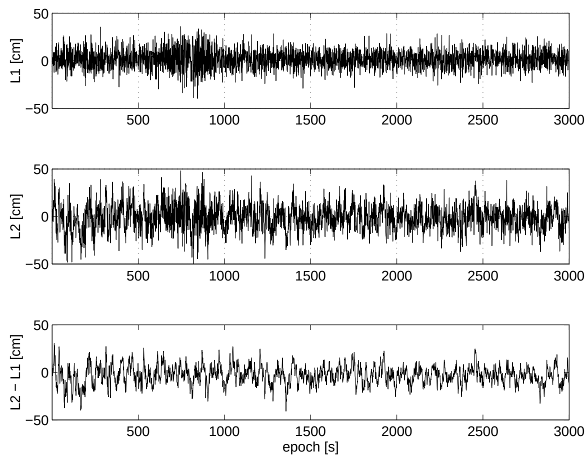

Figure 1. Time series of DD pseudo range observations corrected with unambiguous DD carrier phase observations; in centimeters. (top) L1 frequency observations; (middle) L2 frequency observations; (bottom) L2 minus L1 frequency observations.

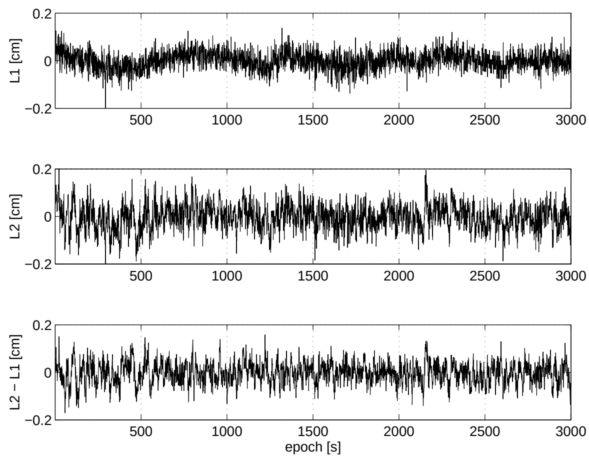

Figure 2. Time series of DD carrier phase observations corrected with 2nd order polynomial; in centimeters. (top) L1 frequency observations; (middle) L2 frequency observations; (bottom) L2 minus L1 frequency observations.

The relatively small noise component in the difference between the L1 and L2 frequency pseudo range and carrier phase observations is somewhat surprising and contrary to what one would expect if the L1 and L2 frequency observations outputted by the receivers would be uncorrelated. Under the assumption of uncorrelated observations one would expect the noise in the difference between the L1 and L2 frequency observations to be larger than the noise in either the L1 or the L2 frequency observations. Based on this observation and on our comments on the noise component in the time series of figure 1 and figure 2 we may conclude that the L1 and L2 frequency observations in our data sets are (positively) correlated. As will be shown in the next section this positive correlation is the hallmark of the cross-correlation reconstruction technique with which the observations were collected.

3 - CROSS-CORRELATION

GPS satellites transmit carrier waves on two frequencies (L1 and L2) that are modulated with, beside the navigation message, the Coarse Acquisition (C/A-) code (on L1 only) and the Precise (P-) code (on both L1 and L2). Pseudo ranges are measured by a GPS receiver by comparing a received C/A- or P-code sequence transmitted by a satellite with a receiver synthesised copy of the sequence. Carrier phases are measured by first demodulating the received signal, i.e. removing the code sequences, and subsequently comparing the demodulated carrier wave with a receiver synthesised copy. Hence, both pseudo range and carrier phase observations require, at least in principle, full knowledge of the codes with which the carrier waves are modulated.

In the presence of Anti-Spoofing (AS) an encrypting W-code is superimposed onto the P-code. The resulting code, termed the Y-code, is to be used exclusively by US military and other authorised users. Civilian users, like navigators and surveyors, can therefore under AS not directly access the L2 frequency for pseudo range and carrier phase observations.

To circumvent the P-code encryption, so-called codeless and semi-codeless techniques have been developed. An overview of these measurement techniques can be found in e.g. [*Ashjaee and Lorenz*, 1992], [*Dierendonck*, 1995] and [*Hofmann-Wellenhof et al*, 1997]. One such codeless technique is cross-correlation. It is based on the fact that under AS both the L1 and L2 frequency carrier waves are modulated coherently with the same Y-code. This allows the receiver to measure the difference between the L1 and L2 frequency pseudo ranges and the difference between the L1 and L2 frequency carrier phases without knowledge of the actual Y-code. Geodetic receivers that, to our knowledge, employ cross-correlation to provide dual frequency code and phase observations are the TurboRogue SNR-8000 [*Meehan et al.*, 1992] and the 4000 series (SSE/SSi) of Trimble [*Trimble*, 1994].

3.1 - pseudo range cross-correlation

The cross-correlation technique was implemented in the measurement process that formed the basis of the time series presented in section 2. We will therefore try to come up with a simple, but hopefully effective stochastic model for such reconstructed pseudo range and carrier phase observations.

In case of the pseudo range observations one can argue that using the cross-correlation technique not the L1 and L2 pseudo ranges (p_1 and p_2) are the independent observables, but rather the L1 pseudo range p_1 and the difference $\Delta p = p_2 - p_1$. The outputted L1 and L2 frequency pseudo range observations are then reconstructed as

$$p_1 = p_1 \quad \text{and} \quad p_2 = p_1 + \Delta p$$

If we now apply the error propagation law and assume p_1 and Δp to be uncorrelated with variances $\sigma^2_{p_1}$ and $\sigma^2_{\Delta p}$, the resulting stochastic model, represented by the variance matrix, reads

$$D\left\{\begin{bmatrix} p_1(i) \\ p_2(i) \end{bmatrix}\right\} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} D\left\{\begin{bmatrix} p_1(i) \\ \Delta p(i) \end{bmatrix}\right\} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \sigma^2_{p_1} & \sigma^2_{p_1} \\ \sigma^2_{p_1} & \sigma^2_{p_1} + \sigma^2_{\Delta p} \end{bmatrix}$$

where $D\{.\}$ denotes the dispersion operator. This matrix is clearly non-diagonal. Moreover the variance of the L2 frequency pseudo range observable is, as a consequence of the way in which the

L2 pseudo ranges are reconstructed from p_1 and Δp, by definition larger than the variance of the L1 frequency pseudo range observable.

3.2 - carrier phase cross-correlation

For the carrier phase observations one can argue, somewhat analogous to the pseudo range case, that using the cross-correlation technique not the L1 and L2 carrier phases (ϕ_1 and ϕ_2) are the independent observables, but rather the L1 carrier phase ϕ_1 and the difference of the two phases. This difference however, is now taken in the domain of cycles instead of in the range domain. In essence this means that the second independent observable is not ϕ_2 but rather the *widelane* carrier phase observable

$$\phi_w = \lambda_w (\frac{\phi_2}{\lambda_2} - \frac{\phi_1}{\lambda_1})$$

with the L1 and L2 frequency wavelengths, λ_1 and λ_2, and the widelane wavelength λ_w=$(1/\lambda_1$-$1/\lambda_2)^{-1}$. Hence, the outputted L1 and L2 carrier phases, when expressed in units of range rather than cycles, are reconstructed as

$$\phi_1 = \phi_1 \quad \text{and} \quad \phi_2 = \lambda_2 (\frac{\phi_1}{\lambda_1} + \frac{\phi_w}{\lambda_w})$$

If we now apply the error propagation law and assume ϕ_1 and ϕ_w to be uncorrelated, the resulting variance matrix becomes

$$D\{\begin{bmatrix} \phi_1(i) \\ \phi_2(i) \end{bmatrix}\} = \begin{bmatrix} 1 & 0 \\ \lambda_2 / \lambda_1 & \lambda_2 / \lambda_w \end{bmatrix} D\{\begin{bmatrix} \phi_1(i) \\ \phi_w(i) \end{bmatrix}\} \begin{bmatrix} 1 & \lambda_2 / \lambda_1 \\ 0 & \lambda_2 / \lambda_w \end{bmatrix} = \begin{bmatrix} \sigma^2_{\phi_1} & (\lambda_2 / \lambda_1)\,\sigma^2_{\phi_1} \\ (\lambda_2 / \lambda_1)\,\sigma^2_{\phi_1} & (\lambda_2 / \lambda_1)^2\,\sigma^2_{\phi_1} + (\lambda_2 / \lambda_w)^2\,\sigma^2_{\phi_w} \end{bmatrix}$$

Again this matrix is clearly non-diagonal. And again the variance of the L2 frequency carrier phase observations is by definition larger than the variance of the L1 frequency carrier phase observation.

4 - CONSEQUENCES

The L1 and L2 frequency pseudo range and carrier phase observations are not stochastically independent if the cross-correlation technique is used. This implies that a user has two options available if he wants to apply a proper weighting to his data. Either he uses the receiver outputted data directly, in which case he will have to work with the non-diagonal variance matrices given above, or he back transforms his data first to the original uncorrelated observables, in which case he can again work with diagonal variance matrices. Both approaches will give identical results, provided the correct stochastic models are used. Different, and in fact less precise results will be obtained when the first approach is used while it is assumed that the data are correlation free.

When one writes down the observation equations corresponding to the second, back transformation approach, one will note that one of the cross-correlation observables, the difference between the L1 and L2 frequency pseudo ranges Δp, has no model parameters, other than the atmospheric delays, in common with the other observables. Hence, when the atmospheric delays are assumed absent, these differences, which are not correlated with the other observables, will not contribute at all to the estimation of the unknown parameters. For the surveyor's short baseline case, typically upto 10 to 15 kilometres, we may thus conclude that the observed differences Δp could just as well be absent. As indicated in the previous paragraph, this same conclusion can also be phrased for the L2 frequency pseudo range observable reconstructed with cross-correlation.

In fact, the conclusion can be formulated in even stronger terms. When L2 frequency pseudo range observations are included, while using a diagonal variance matrix, results are obtained which are less precise than the results obtained based on excluding these L2 frequency data. We thus end up with a remarkable situation: more precise estimates for the unknowns can be obtained by *not* using one of the receiver-outputted observables. To illustrate this surprising result we will first show examples of DD ambiguity estimation with and without the L2 frequency pseudo ranges and then consider the consequences of omitting the L2 frequency pseudo range observations for differential positioning.

4.1 - effect on ambiguity estimation

Figure 3 represents time series of L1 ambiguities estimated epoch-by-epoch from zero baseline data *with* L2 frequency pseudo range observations (figure 3 top) and *without* L2 frequency pseudo range observations (figure 3 bottom). On comparing the time series it is clear that the noise in the ambiguities estimated with the L2 frequency pseudo range observations is indeed considerably larger than the noise in the ambiguities estimated without the L2 frequency pseudo ranges. In addition it appears from figure 3 that the noise in the ambiguities estimated with the L2 frequency pseudo range observations increases notably towards the end of the observation span. This in fact is another consequence of the cross-correlation reconstruction technique. Application of cross-correlation results namely in a relatively small signal-to-noise ratio (SNR), see [*Ashjaee and Lorenz*, 1992]. The small SNR gives rise to a quite pronounced elevation dependence of the noise in the cross-correlation reconstructed pseudo range and carrier phase observations. And, as one satellite of the pair considered for figure 3 sets during the observation span, this may well explain the progressive increase in noise in the ambiguities estimated with the L2 frequency pseudo range observations.

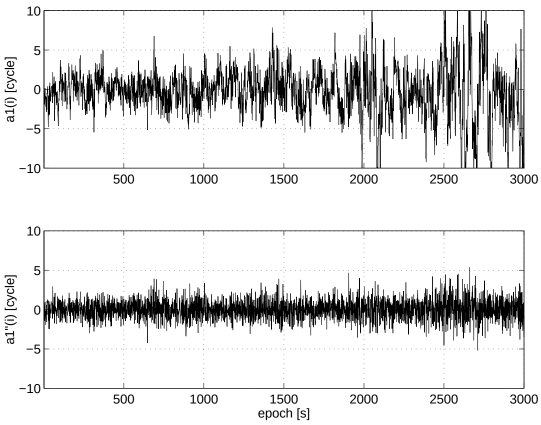

Figure 3. Time series of L1 frequency carrier phase ambiguities in cycles estimated *with* L2 pseudo range observations (top) and *without* L2 frequency pseudo range observations (bottom).

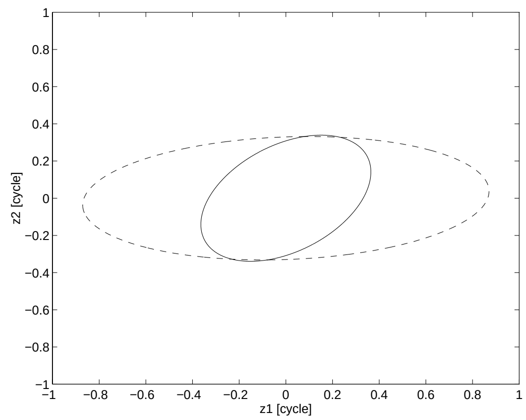

Figure 4. 95% Empirical standard ellipses of dual frequency ambiguities in cycles estimated *with* L2 pseudo range observations (dashed) and *without* L2 frequency pseudo range observations (full). (Note that the ellipses are LAMBDA transformed and that the transformed ambiguities are indicated as z1 and z2).

In figure 4 an example is shown of the empirical precision of L1 and L2 frequency ambiguities estimated *with* L2 frequency pseudo range observations (dashed ellipse) and *without* L2 frequency pseudo range observations (full ellipse). The empirical precision is completely determined by the data themselves, and is represented by means of 95% standard ellipses. (Note that a decorrelating LAMBDA transformation, see [*Teunissen*, 1995], was used to reduce the elongation of the standard ellipses). As with the time series of L1 frequency ambiguity estimates, it is obvious from a comparison of the standard ellipses in figure 4 that *not* using the cross-correlation reconstructed L2 frequency pseudo range observations yields much better results.

4.2 - effect on differential positioning

Figure 5 and figure 6 represent scatter plots of a short 3 meter baseline estimated epoch-by-epoch (over 1500 samples) with both L1 and L2 frequency pseudo range observations (figure 5) and with L1 frequency pseudo range observations (figure 6). Hence, figure 5 represents the case in which correlation is neglected, while figure 6 represents the case in which it is properly taken into account. As only pseudo range data are used to compute the baseline estimates, this example is representative for Differential

GPS positioning (DGPS). Note that the computed baseline coordinates were differenced with precise reference coordinates; the latter correspond with the origin of both figures.

One can see that, for such a short interstation distance, the spread in estimated coordinates is of the order of several decimetres, (mainly caused by the pseudo range measurement noise). However, the cloud of estimated positions in figure 5 is larger than the cloud of estimated positions in figure 6. This again illustrates that cross-correlation reconstructed L2 frequency pseudo range observations should not be used for the estimation of the unknown parameters of the short baseline model.

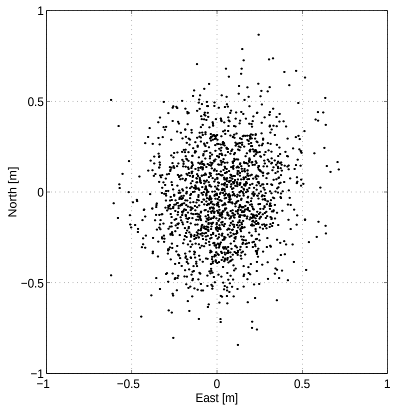

Figure 5. 1680 Estimates of a short 3 meter baseline using both L1 and L2 frequency pseudo range observations to 7 satellites.

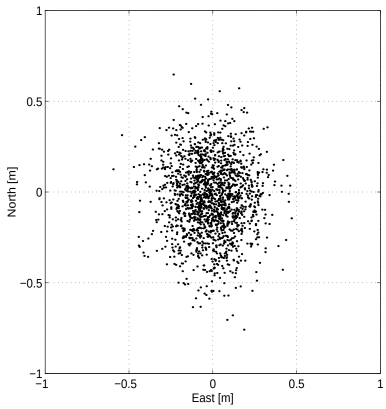

Figure 6. 1680 Estimates of a short 3 meter baseline using only L1 frequency pseudo range observations to 7 satellites.

5 - CONCLUSION

In this contribution we considered the consequences of the cross-correlation measurement technique on GPS positioning under Anti-Spoofing (AS), by analyzing the stochastic model of cross-correlation reconstructed observations. At present the dual civil and military role of GPS is recognized and plans are being developed for a civil signal on a second frequency. The navigator and surveyor will therefore eventually have full access to both the L1 and the L2 frequency. Implementation details of the second civil signal are however not all clear yet. Moreover, it is anticipated that this new signal will not be (operationally) available until the second half of the next decade. Meanwhile, AS has been employed on block II satellites continuously since January 31, 1994 and it will remain on as it is judged to be of critical importance to the US military [NRC, 1995].

AS remaining on and the second civil coded signal not yet being realised leaves civilian users no other choice than to turn to codeless and semi-codeless reconstruction techniques in order to gain access to the L2 frequency. And although it is possible to obtain L2 frequency observations with these reconstruction techniques, this does certainly not mean that they constitute a full dual frequency GPS positioning capability when AS is on. On the contrary, as shown in this contribution, depending on the reconstruction technique used and the functional model specified, the reconstructed observations may turn out to be of little or no value.

6 - REFERENCES

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