ARTIFICIAL SATELLITES, Vol. 32, No. 2 - 1997

INTERNAL RELIABILITY OF SINGLE FREQUENCY GPS DATA

P.J.G. Teunissen
Delft Geodetic Computing Centre (LGR)
Faculty of Geodesy
Delft University of Technology
Thijsseweg 11
2629 JA Delft, The Netherlands
Fax: ++ 31 15 278 3711
e-mail: P.J.G.Teunissen@geo.tudelft.nl

ABSTRACT: In this contribution we describe the internal reliability of single- frequency GPS data. This is done for outliers in the code data and for cycle slips in the phase data. The internal reliability will be given for three different GPS single-baseline models. They are the geometry-free model and two variants of the geometry-based model, the roving-variant and the stationary-variant.

1. INTRODUCTION

The quality of estimation is often measured by means of the precision of the estimators. Precision, however, measures only one aspect of the estimator, namely the amount of expected variability in samples of the estimator around its mean. This measure may suffice in case the estimator is unbiased. But since this depends on the validity of the model used, the unbiasedness of the estimator cannot be guaranteed. The purpose of model testing is therefore to minimize the risk of having a biased solution. But like any result of estimation, also the outcomes of statistical tests are prone to errors. It depends on the strength of the model how much confidence one can have in the outcomes of these statistical tests. A measure for this confidence is provided by the theory of reliability as introduced by Baarda (1968).

Internal reliability, as represented by the Minimal Detectable Biases (MDB's), describes the size of the model errors which can just be detected with the appropriate test statistics. In this contribution, closed form expressions will be given for two type of MDB's, the ones that correspond with outliers in GPS code data and the ones that correspond with cycle slips in GPS phase data. The models considered are of the single-baseline type, separated by a short distance only, using single-frequency GPS data. They form the common mode of operation in most local surveying applications. Within this class, three different single-baseline models will be considered. They are the geometry-free model and two variants of the geometry-based model, namely the roving-variant and the stationary-variant. Both variants make an explicit use of the receiver-satellite geometry. The only difference between the two variants is whether or not the baseline is assumed stationary over the observation time span.

The general concept of the minimal detectable biases is easiest explained if we start from the null-hypothesis H_0 and alternative hypothesis H_a

$$H_0: \; E\{y\} = Ax \;,\; D\{y\} = Q_y \text{ and } H_a: \; E\{y\} = Ax + b_y \;,\; D\{y\} = Q_y \qquad (1)$$

where $E\{.\}$ and $D\{.\}$ are, respectively, the expectation and dispersion operator, y is the $m-$vector of normally distributed observables, A is the $m \times n$ design matrix, x is the n- vector of unknown parameters, Q_y is the variance matrix of the observables and b_y is the bias that describes the model error. It will be assumed that the model error is one-dimensional, that is, the bias b_y can be parametrized by means of a single parameter ∇ as $b_y = c_y\nabla$, where the m-vector c_y is assumed known and the scalar ∇ unknown. The vector c_y specifies the type of model error. The test statistic for testing H_0 against H_a is given as

$$w = \frac{c_y^T Q_y^{-1} P_A^{\perp} y}{\sqrt{c_y^T Q_y^{-1} P_A^{\perp} c_y}} \qquad (2)$$

where $P_A^{\perp} = I_m - P_A$ and P_A is the least-squares projector. The least-squares projector projects onto the range space of A and along its orthogonal complement. The test statistic w has a standard normal distribution under H_0 and a mean shifted normal distribution under H_a.

Two type of errors can be made when testing the null hypothesis, a type I error and a type II error. A type I error is made when the null hypothesis is rejected while it is true. The probability of making a type I error is referred to as the level of significance and it is usually denoted as α. A type II error is made when the null hypothesis is accepted while it is false. The probability of a type II error is denoted as β. Its complement $\gamma = 1 - \beta$ is referred to as the detection power of the test. The non-zero mean of w under H_a can be computed once the level of significance α_0 and the detection power γ_0 are chosen. In case of a two-sided test it reads $\sqrt{\lambda_0} = \sqrt{c_y^T Q_y^{-1} P_A^{\perp} c_y} \mid \nabla \mid$, where $\lambda_0 = \lambda(\alpha_0, \gamma_0)$ follows from the normal distribution. For instance, for $\alpha_0 = 0.001$ and $\gamma_0 = 0.80$, we get $\lambda_0 = 17.075$. Once λ_0 is known, the corresponding size of the bias can be computed as

$$\mid \nabla \mid = \sqrt{\frac{\lambda_0}{c_y^T Q_y^{-1} P_A^{\perp} c_y}} \qquad (3)$$

This is the celebrated MDB. It is the minimal size of the bias that can be detected with the test statistic w, when the level of significance and the power are set at, respectively, α_0 and γ_0. Apart from the chosen level of significance and the power, the MDB depends on the vector c_y, the design matrix A and the variance matrix Q_y. In section 2 we specify the variance matrix and construct the design matrix for the three different single-baseline models. In section 3 the least-squares projector is derived. It is given in a form which facilitates the computation of the MDB's. In sections 4 and 5 we give the closed form expressions for the MDB's. In section 4 they are given for outliers in the code data and in section 5 for cycle slips in the phase data.

2. CONSTRUCTING THE DESIGN MATRIX

In this section we will construct the design matrix for the following three single-baseline models: the geometry-free model, the roving-receiver geometry-based model and the stationary-receiver geometry-based model. When tracking satellite s at epoch t using two receivers i and j, the single-difference (SD) observation equations for single-frequency phase and code read, see e.g. Hofmann-Wellenhof et al. (1994), Leick (1995), Teunissen and Kleusberg (1996)

$$\begin{array}{rcl} \phi^s_{ij}(t) & = & dt_{ij}(t) + \delta_{ij}(t) + \rho^s_{ij}(t) - I^s_{ij}(t) + T^s_{ij}(t) + \delta m^s_{ij}(t) + \phi_{ij}(t_0) + \lambda N^s_{ij} + n^s_{\phi,ij}(t) \\ p^s_{ij}(t) & = & dt_{ij}(t) + d_{ij}(t) + \rho^s_{ij}(t) + I^s_{ij}(t) + T^s_{ij}(t) + dm^s_{ij}(t) + n^s_{p,ij}(t) \end{array} \quad (4)$$

where $\phi^s_{ij}(t)$ is the SD phase observable expressed in units of range rather than cycles, $p^s_{ij}(t)$ is the SD code observable, $dt_{ij}(t)$ is the unknown relative receiver clock error, $\delta_{ij}(t)$ and $d_{ij}(t)$ are the carrier phase and code equipment delays, $\rho^s_{ij}(t)$ is the unknown SD receiver-satellite range, $I^s_{ij}(t)$ and $T^s_{ij}(t)$ are the ionospheric and tropospheric delays, $\delta m^s_{ij}(t)$ and $dm^s_{ij}(t)$ are phase and code multipath terms, $\phi_{ij}(t_0)$ is the relative receiver non-zero initial phase offset, N^s_{ij} is the integer carrier phase ambiguity that corresponds with the wavelength λ, and $n^s_{\phi,ij}(t)$ and $n^s_{p,ij}(t)$ are, respectively, the noises of phase and code.

In the following we will assume that the two type of atmospheric delays can either be corrected for using an a priori model or that they are sufficiently small to be neglected. This is a realistic assumption if we consider sufficiently short baselines with a not too big height difference between the two receiver antennas. This is the typical environment for local surveying applications. We also assume multipath to be absent. Thus sufficient precautions are assumed taken with respect to antenna siting and shielding. Finally we assume that over the observation time span the same m satellites are tracked ($s = 1,\ldots,m$). This too is a realistic assumption if we consider short time spans only. By lumping the nonseparable parameters together as $dt_{\phi_k} = dt_{ij}(t_k)+\delta_{ij}(t_k)$, $dt_{p_k} = dt_{ij}(t_k)+d_{ij}(t_k)$ and $\alpha^s_{ij} = \phi_{ij}(t_0) + \lambda N^s_{ij}$, the $2m$ SD observation equations for epoch t_k can be written in compact form using vector notation as

$$\begin{array}{rcl} \phi_k & = & e_m dt_{\phi_k} + \rho_k + \alpha + n_{\phi_k} \\ p_k & = & e_m dt_{p_k} + \rho_k + n_{p_k} \end{array} \quad (5)$$

where e_m is an m-vector having all its entries equal to one.

The two clock terms dt_{ϕ_k} and dt_{p_k} can be eliminated by using double-differences (DD) instead of single-differences. There are many different ways in which double-differences can be formed. Each of the m satellites for instance, can be taken as a reference satellite. This already gives m different ways of forming double- differences. The DD-transformation that takes the first satellite as reference reads $[-e_{m-1}, I_{m-1}]$, while the DD-transformation that takes the last satellite as reference reads $[I_{m-1}, -e_{m-1}]$. Although the DD-transformation itself is not unique, the adjustment results will be unique if a proper care is taken of the correlation that is introduced by the DD-process. We can therefore take any one of the admissible DD-transformations. Let D^T be such an $(m-1)\times m$ matrix that transforms single-differences into double-differences. The $2(m-1)$ DD observation equations for epoch t_k follow then from (5) as

$$\begin{array}{rcl} D^T\phi_k & = & r_k + a + D^T n_{\phi_k} \\ D^T p_k & = & r_k + D^T n_{p_k} \end{array} \quad (6)$$

where $r_k = D^T\rho_k$ contains the DD ranges and $a = D^T\alpha$ contains the DD ambiguities. The DD ambiguities are integers now, since the non-zero initial phase offset has been eliminated by the double- differencing as well. The DD observation equations can be written in a more compact form as

$$y_k = (e_2 \otimes I_{m-1})r_k + (c_1 \otimes I_{m-1})a + n_k \tag{7}$$

where y_k contains the DD phase and code data of epoch t_k, n_k contains the corresponding DD noise terms and where $\otimes$ denotes the Kronecker product. This set of DD observation equations can now be used as starting point to construct the appropriate design matrices.

Geometry-free model: In this model the observation equations are not parametrized in terms of the baseline components. Instead, they remain parametrized in terms of the unknown DD receiver-satellite ranges. This implies that the observation equations remain linear and that the receiver-satellite geometry is not explicitly present in these equations. Hence, this model permits both receivers to be either stationary or roving. Since the equations as given by (7) are the ones that belong to the geometry-free model, we get for k epochs

$$y = [I_k \otimes (e_2 \otimes I_{m-1})]r + [e_k \otimes (c_1 \otimes I_{m-1})]a + n \tag{8}$$

where for $i = 1, \ldots, k$, the y_i, r_i and n_i are collected in, respectively, y, r and n. The design matrix is of order $2(m-1)k \times (m-1)(k+1)$ and since it is of full rank, the redundancy follows as $(m-1)(k-1)$. Thus in order to have redundancy, we need to use more than one epoch of data while tracking two or more satellites. For every additional epoch the redundancy increases by $(m-1)$.

Roving-receiver geometry-based model: In case of the geometry- based model, the observation equations are parametrized in terms of the unknown baseline components. Since these equations are nonlinear, a linearization of the DD receiver-satellite ranges with respect to the baseline components is needed

$$\Delta r_k = D^T G_k \Delta b_k \quad \text{with} \quad G_k = [\frac{\partial \rho_k}{\partial b_k}] \tag{9}$$

The geometry of the SD relative receiver-satellite configuration is captured by the $m \times 3$ matrix G_k. It is well-known that due to the high altitude orbits of the GPS satellites, the receiver- satellite geometry changes only slowly with time. We will therefore assume that G_k is a time-invariant matrix, $G_k = G =$ constant. This approximation is allowed for short time spans, in particular since we restrict our attention to the computation of the MDB's only. With $G_k = G$ and $b = (b_1^T, \ldots, b_k^T)^T$, it follows from (9), when we omit the Δ-symbol for notational convenience, that $r = (I_k \otimes D^T G)b$. Substitution into (8) gives the DD observation equations as

$$y = [I_k \otimes (e_2 \otimes D^T G)]b + [e_k \otimes (c_1 \otimes I_{m-1})]a + n \tag{10}$$

The design matrix is of order $2(m-1)k \times (3k + m - 1)$ and since it is of full rank, the redundancy follows as $(m-1)(2k-1) - 3k$. Thus in order to have redundancy for a single epoch, more than four satellites need to be tracked. For every additional epoch the redundancy increases by $(2m-5)$. Note that the design matrix of the geometry-free model follows from that of the roving-receiver geometry-based model, when the matrix D^TG is replaced by the unit matrix I_{m-1}.

Stationary-receiver geometry-based model: When the two receivers are stationary, the k baselines b_k collapse to one single baseline b^*. Since in that case $b = (e_k \otimes I_3)b^*$ it follows that $r = (e_k \otimes D^TG)b^*$. Substitution into (8) gives the DD observation equations as

$$y = [e_k \otimes (e_2 \otimes D^TG)]b^* + [e_k \otimes (c_1 \otimes I_{m-1})]a + n \tag{11}$$

The design matrix is of order $2(m-1)k \times (m-4)$ and since it is of full rank, the redundancy follows as $(m-1)(2k-1)-3$. When compared to the previous model the redundancy has increased by $3(k-1)$, which equals the number of baseline components that have been constrained.

3. THE LEAST-SQUARES PROJECTOR

Now that we know the structure of the three design matrices, we can start to construct the least-squares projector P_A. It is needed for computing the MDB's. The least-squares projector is given as

$$P_A = A(A^TQ_y^{-1}A)^{-1}A^TQ_y^{-1} \tag{12}$$

Thus in order to construct it we need both A and Q_y. The variance matrix of the observables is assumed to be given as

$$Q_y = I_k \otimes Q \quad \text{with } Q = \text{blockdiag}(\sigma_\phi^2 D^TD, \sigma_p^2 D^TD) \tag{13}$$

where σ_ϕ^2 and σ_p^2 are the SD variances of, respectively, the phase and code data. The design matrix itself follows from the results of the previous section as

$$A = [M_k, N_k] \text{ with } M_k = I_k \otimes M \ , \ N_k = e_k \otimes N \tag{14}$$

where

$$\begin{cases} \text{geometry} - \text{free}: & M = e_2 \otimes I_{m-1} & N = c_1 \otimes I_{m-1} \\ \text{roving} - \text{receiver}: & M = e_2 \otimes D^TG & N = c_1 \otimes I_{m-1} \\ \text{stationary} - \text{receiver}: & M = 0 & N = (e_2 \otimes D^TG, c_1 \otimes I_{m-1}) \end{cases}$$

We will now decompose the least-squares projector step-by-step in order to obtain a form which can be used for the computation of the MDB's. Since the range space of $[M_k, N_k]$ equals that of $[M_k, P_{M_k}^\perp N_k]$, where $P_{M_k}^\perp$ projects orthogonally onto the orthogonal complement of the range space of M_k, we have the projector decomposition

$$P_A = P_{[M_k,N_k]} = P_{M_k} + P_{P_{M_k}^\perp N_k} \tag{15}$$

When using (13), we also have $P_{M_k} = I_k \otimes P_M$ and $P_{P_{M_k}^\perp N_k} = P_{e_k} \otimes P_{P_M^\perp N}$. Substitution into (15) while using $P_{P_M^\perp N} = P_{[M,N]} - P_M$, gives the decomposition

$$P_A = P_{e_k}^\perp \otimes P_M + P_{e_k} \otimes P_{[M,N]} \tag{16}$$

This decomposition clearly shows the time-dependent and the time- invariant contribution of the design matrix. A further decomposition is possible if we consider the entries of the matrix $[M, N]$. For both the roving-receiver and stationary-receiver geometry-based model we have $[M,N] = [e_2 \otimes D^T G, c_1 \otimes I_{m-1}]$. For the geometry-free model we simply have to replace $D^T G$ by I_{m-1}. Since the range space of $[M, N]$ equals that of $[c_1 \otimes I_{m-1}, c_2 \otimes D^T G]$, where the columns of the last matrix are partitioned in two mutually orthogonal sets, we have $P_{[M,N]} = P_{c_1 \otimes I_{m-1}} + P_{c_2 \otimes D^T G}$, which can be decomposed further as $P_{[M,N]} = P_{c_1} \otimes I_{m-1} + P_{c_2} \otimes P_{D^T G}$. Substitution into (16) gives

$$P_A = P_{e_k}^{\perp} \otimes P_M + P_{e_k} \otimes [P_{c_1} \otimes I_{m-1} + P_{c_2} \otimes P_{D^T G}] \tag{17}$$

The advantage of this decomposition is that the contribution of P_{c_1} vanishes when outliers are considered, while the contribution of P_{c_2} vanishes when cycle slips are considered. Note that the projector is still expressed in the DD receiver-satellite geometry. Since this obscures our interpretations in the following sections somewhat, we prefer to express the projector directly in the SD-matrix G, rather than in its SD counterpart $D^T G$. This is possible if we make use of $P_{D^T G} = D^T P_{[G,e_m]} D^{+T}$, where D^+ is the pseudo-inverse of D. As a result we get

$$P_A = P_{e_k}^{\perp} \otimes P_M + P_{e_k} \otimes [P_{c_1} \otimes I_{m-1} + P_{c_2} \otimes D^T P_{[G,e_m]} D^{+T}] \tag{18}$$

This is the decomposition that will be used in the next two sections for deriving the outlier and cycle slip MDB's. It holds true for all three single-baseline models. Only P_M varies for the three models and in addition $P_{[G,e_m]}$ needs to be replaced by I_m when the geometry-free model is considered. For the projector P_M we have

$$\begin{cases} \text{geometry-free}: & P_M = \frac{1}{1+\epsilon} e_2 \otimes (I_{m-1}, \epsilon I_{m-1}) \\ \text{roving-receiver}: & P_M = \frac{1}{1+\epsilon} e_2 \otimes D^T (P_{[G,e_m]}, \epsilon P_{[G,e_m]}) D^{+T} \\ \text{stationary-receiver}: & P_M = 0 \end{cases} \tag{19}$$

with the phase-code variance ratio $\epsilon = \sigma_\phi^2 / \sigma_p^2$.

4. OUTLIERS IN THE CODE DATA

In this section we will restrict our attention to outliers in the code data. The outlier MDB's will be derived for the geometry-free model, the roving-receiver geometry-based model and the stationary-receiver geometry-based model. In order to compute the MDB we first need to specify the appropriate c_y-vector. For a code outlier at epoch l $(1 \le l \le k)$ in the range to satellite $i \in \{1, \ldots, m\}$, the c_y-vector takes the form

$$c_y = c_l \otimes d_2 \;\text{ with }\; d_2 = c_2 \otimes D^T c_i \tag{20}$$

where c_l, c_2 and c_i are canonical unit vectors of appropriate dimension, having the 1 as their, respectively, lth, 2^{nd} and ith entry. The vector c_l selects the appropriate epoch, the vector c_2 selects the vector of code data of that epoch and the vector c_i selects the satellite to which the range error is supposedly made. The vector $D^T c_i$ describes how an outlier in a SD observable affects a DD observable. This vector will become a vector of all 1's in case the outlier occurs in the range to a reference satellite. Otherwise it will remain a canonical unit vector.

With (18), (19) and (20) we are now in the position to consider each of the three single-baseline models individually.

Geometry-free model: Due to the absence of the receiver-satellite geometry we have to set $P_{[G,e_m]}$ equal to I_m. Using (18), (19) and (20), and performing the necessary multiplications according to (3), the outlier MDB follows as

$$| \nabla_p | = \sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}(\frac{1+k\epsilon}{1+\epsilon})]}} \tag{21}$$

The MDB is a function of the code precision, σ_p, the phase-code variance ratio, ϵ, the number of satellites tracked, m, and the number of epochs used, k. It is independent, however, of the epoch the outlier occurs, l, and, of course, of the receiver-satellite geometry. At first instant it may seem curious that the MDB of the geometry-free model is still dependent on the number of satellites tracked. This is due, however, to the correlation introduced by the double-differencing process.

Note, since in practice the phase-code variance ratio is very small indeed (e.g. $\epsilon = 10^{-4}$), that the precision of the phase data has no significant impact on the value of the MDB. Also note that due to the absence of redundancy, the MDB becomes infinite in case $k = 1$ or $m = 1$. Outlier detection in the code data is thus only possible when more than one satellite is tracked over more than one epoch. Smaller outliers can be detected when more satellites are tracked and more epochs are used. The smallest possible MDB equals $| \nabla_p | = \sigma_p \sqrt{\lambda_0}$.

The roving-receiver geometry-based model: In this case the receiver-satellite geometry is explicitly used. The baselines, however, are nonstationary. Using (18), (19) and (20), and performing the necessary multiplications according to (3), the outlier MDB follows as

$$| \nabla_p | = \sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}(\frac{1+k\epsilon}{1+\epsilon})] + \frac{1}{k}(\frac{1+k\epsilon}{1+\epsilon})[1 - c_i^T P_{[G,e_m]} c_i]}} \tag{22}$$

Compare this result with that of (21). Again note that the precision of the phase data has no significant impact on the value of the MDB. Also note that the impact of the receiver-satellite geometry is felt through the projector $P_{[G,e_m]}$. Due to its inclusion, the MDB of (22) is in general smaller than that of (21). The two MDB's are identical, however, in case satellite redundancy is absent. That is, when only four satellites are tracked ($m = 4$). In that case the projector $P_{[G,e_m]}$ reduces to the identity matrix.

Due to the inclusion of the receiver-satellite geometry, outlier detection is possible now with only one epoch of data. To get an indication of the corresponding value of the MDB, the following average can be used. Since the trace of a projector is equal to its rank and since the rank of the projector $P_{[G,e_m]}$ equals 4, it follows that the average value of a diagonal entry of this projector equals $\frac{4}{m}$. Hence, if we use the approximations $\epsilon = 0$ and $c_i^T P_{[G,e_m]} c_i = \frac{4}{m}$, it follows that

$$| \nabla_p | = \sigma_p \sqrt{\frac{\lambda_0}{1 - \frac{1}{m}(1+\frac{3}{k})}}$$

This shows that for $k = 1$ and $\lambda_0 = 17.075$, the MDB equals about six times the SD standard deviation of code when eight satellites are tracked. The value of the MDB goes up to about nine times the SD standard deviation of code, when only five satellites are tracked.

The stationary-receiver geometry-based model: In this case the receiver-satellite geometry is still explicitly used, but now the projector P_M vanishes. Using (18), (19) and (20), and performing the necessary multiplications according to (3), the outlier MDB follows as

$$| \nabla_p | = \sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}] + \frac{1}{k}[1 - c_i^T P_{[G,e_m]} c_i]}} \tag{23}$$

Compare this result with that of (22). Due to the fact that the baseline is assumed stationary now, the MDB of (23) will of course be smaller than that of (22). Note, however, since ϵ is very small, that these two MDB's will not differ much in practice. Hence, we may conclude that one's ability to detect outliers in the code data will not be improved by much when the baseline is assumed stationary instead of moving.

5. CYCLE SLIPS IN THE PHASE DATA

We will now consider cycle slips in the phase data. They will be expressed in units of range rather than in units of cycles. In order to compute the MDB we first need to specify the appropriate c_y-vector. For a phase-slip at epoch l $(1 \leq l \leq k)$ in the range to satellite $i \in \{1, \ldots, m\}$, the c_y-vector takes the form

$$c_y = s_l \otimes d_1 \quad \text{with} \quad d_1 = c_1 \otimes D^T c_i \tag{24}$$

where c_1 and c_i are canonical unit vectors of appropriate dimension, having the 1 as their first and ith entry respectively. The slip vector s_l is a k-vector having zero's as its first $l-1$ entries and 1's otherwise. Again we consider the three different single-baseline models separately.

Geometry-free model: Using (18), (19) and (24), and performing the necessary multiplications according to (3), the cycle slip MDB follows as

$$| \nabla_\phi | = \sigma_p \sqrt{\frac{(1+\epsilon)\lambda_0}{[l-1][1-\frac{l-1}{k}][1-\frac{1}{m}]}} \tag{25}$$

As with the outlier MDB's, the cycle slip MDB is a function of the code precision, σ_p, the phase-code variance ratio, ϵ, the number of satellites tracked, m, and the number of epochs used, k. In contrast with the outlier MDB however, the cycle slip MDB now also depends on the moment the slip started to occur. Note that the MDB becomes infinite in case $l = 1$. This reflects the situation that cycle slips cannot be found when they already commence with the first epoch. In that case the slip cannot be separated from the corresponding phase ambiguity itself.

Also note, since ϵ is very small, that the precision of the phase data has no significant impact on the value of the MDB. Hence the cycle slip MDB is predominantly governed by the poor precision of the code data. This has an important impact on one's ability to detect cycle slips with the geometry-free model. Let us first consider the case that $l = k$. It corresponds with the situation that the cycle slip occurs at the last epoch of the data set. In that case the smallest possible value of the MDB reads $| \nabla_\phi | = \sigma_p \sqrt{(1+\epsilon)\lambda_0}$. Hence, in this case one cannot expect to find slips as small as one cycle. Let us now consider the case that $l < k$. If we denote the time window between k and l as $N = k - l + 1$, it follows that in this case the smallest possible value of the MDB reads $| \nabla_\phi | = \frac{\sigma_p}{\sqrt{N}}\sqrt{(1+\epsilon)\lambda_0}$. This shows that now sufficiently small slips can be found provided the time window N is large enough.

The roving-receiver geometry-based model: With the receiver- satellite geometry included, the cycle slip MDB follows from (18), (19), (24) and (3) as

$$| \nabla_\phi | = \sigma_\phi \sqrt{\frac{(1+\epsilon)\lambda_0}{[l-1][1-\frac{l-1}{k}][(1-\frac{1}{m})\epsilon + (1 - c_i^T P_{[G,e_m]} c_i)]}} \tag{26}$$

Compare this result with that of (25). The impact of the receiver-satellite geometry is felt through the presence of the projector $P_{[G,e_m]}$. This impact is absent in case only four satellites are tracked. In that case the projector reduces to the identity matrix and (26) becomes identical to the MDB of the geometry-free model. Hence, satellite redundancy is needed per se in order to get a sufficiently small MDB when $l = k$. In that case it is the precision of phase, instead of code, that governs the value of the MDB. To get an approximate value for the MDB when $l = k$, we use the approximations $\epsilon = 0$ and $c_i^T P_{[G,e_m]} c_i = \frac{4}{m}$. This gives

$$| \nabla_\phi | = \sigma_\phi \sqrt{\frac{\lambda_0}{(1-\frac{1}{k})(1-\frac{4}{m})}}$$

This shows that for $k = 2$, $m = 5$ and $\lambda_0 = 17.075$, the MDB equals about thirteen times the SD standard deviation of phase, which is about a quarter of a full wavelength.

The stationary-receiver geometry-based model: In this case the projector P_M should be set to zero. From (18), (19), (24) and (3), the cycle slip MDB follows then as

$$| \nabla_\phi | = \sigma_\phi \sqrt{\frac{\lambda_0}{[l-1][1-\frac{l-1}{k}][1-\frac{1}{m}]}} \tag{27}$$

Compare this result with that of (26). Due to the fact that the baseline is assumed stationary now, the MDB of (27) will of course be smaller than that of (26). Thus also in this case one can expect to be able to detect sufficiently small cycle slips when $l = k > 2$. When we compare (27) with its counterpart (25) of the geometry-free model, we note that the ratio of the two MDB's is about equal to the square-root of the phase-code variance ratio. Hence in practice, the cycle slip MDB of the stationary-receiver geometry- based model will be about one hundred times smaller than its counterpart of the geometry-free model. Also note that (27) is independent of the receiver-satellite geometry, which is quite remarkable. Although it is a consequence of the fact that matrix G_k was assumed time-invariant, it does imply that the MDB is not significantly influenced by the receiver-satellite geometry itself. Hence, in thi s case it is not so much the geometric distribution of the satellites that counts, but more the number of satellites that are tracked.

MDB	geometry-free	roving baseline	stationary baseline
$\mid \nabla_p \mid$	$\sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}]}}$	$\sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}]+\frac{1}{k}\delta}}$	$\sigma_p \sqrt{\frac{\lambda_0}{[1-\frac{1}{m}][1-\frac{1}{k}]+\frac{1}{k}\delta}}$
$\mid \nabla_\phi \mid$	$\sigma_p \sqrt{\frac{\lambda_0}{[l-1][1-\frac{l-1}{k}][1-\frac{1}{m}]}}$	$\sigma_\phi \sqrt{\frac{\lambda_0}{[l-1][1-\frac{l-1}{k}]\delta}}$	$\sigma_\phi \sqrt{\frac{\lambda_0}{[l-1][1-\frac{l-1}{k}][1-\frac{1}{m}]}}$

Table 1: MDB's for an outlier/slip at epoch l in code/phase based on k epochs, using single-frequency data, with the approximations $\epsilon = \frac{\sigma_\phi^2}{\sigma_p^2} \approx 0$ and $G_k \approx G$ ($\lambda_0 = 17.075$ for $\alpha = 0.001$ and $\gamma = 0.80$, and $\delta = 1 - c_i^T P_{[G,c_m]} c_i$ with the average value $1 - \frac{4}{m}$).

6. SUMMARY

In this contribution we derived closed form expressions for the single- frequency minimal detectable biases of outliers in the code data and of cycle slips in the phase data. The MDB's were given for three different single-baseline models: the geometry-free model, the roving-receiver geometry-based model and the stationary-receiver geometry-based model. For an easy reference the results are summarized in table 1. The entries in the table are based on the approximation of neglecting the very small phase-code variance ratio ϵ.

It was shown that all outlier MDB's are insensitive to the precision of the phase data. They are predominantly governed by the precision of the code data. With the geometry-free model single-epoch based outlier detection is not possible. It is possible, however, when use is made of the receiver-satellite geometry, provided satellite redundancy is present. This is true for the roving-variant and the stationary-variant.

It was also shown that there is practically no difference between the MDB's of the roving-variant and the stationary-variant. Constraining the baseline to be stationary does therefore not improve one's ability to detect outliers in the code data.

Two out of the three cycle slip MDB's were shown to be governed by the high precision of the phase data. In these cases sufficiently small slips can be detected, even when using the smallest possible time window of $k - l + 1 = 1$. The exception occurs with the geometry-free model. Then it is not the precision of phase, but the precision of code that governs the MDB. This implies that small slips cannot be found, unless a sufficiently large time window is used. In the absence of satellite redundancy, this same situation occurs also when using the roving-variant. However, this will not happen with the stationary-variant. Finally it was shown that the cycle slip MDB is not significantly influenced by the receiver-satellite geometry when the baseline is stationary.

REFERENCES

Baarda, W. (1968): *A testing procedure for use in geodetic networks*, Netherlands Geodetic Commission, Publications on Geodesy, New Series, Vol. 2, No. 5.

Hofmann-Wellenhof, B., B.H. Lichtenegger, J. Collins (1994): *GPS Theory and Practice.* Third edition, Wien: Springer-Verlag.

Leick, A. (1995): *GPS Satellite Surveying.* Second edition, John Wiley, New York.

Teunissen, P.J.G. (1985): Quality control in geodetic networks. Chapter 17 in *Optimization and design of geodetic networks*, E.W. Grafarend, F. Sanso (Eds.), Springer-Verlag, Heidelberg, New York.

Teunissen, P.J.G., A. Kleusberg (1996): GPS observation equations and positioning concepts. Chapter 5 in: *GPS for Geodesy*, A. Kleusberg, P.J.G. Teunissen (Eds.), Lecture Notes in Earth Sciences, Vol. 60, Springer-Verlag, Heidelberg, New York.

Received: May 5, 1997
Reviewed: August 31, 1997, by W. Pachelski
Accepted: September 5, 1997