ON THE MINIMAL DETECTABLE BIASES OF GPS PHASE AMBIGUITY SLIPS

P.J.G. Teunissen
Delft Geodetic Computer Centre (LGR)
Dept. of Geodetic Engineering, Delft University of Technology
Thijsseweg 11, 2629 JA Delft, The Netherlands

Abstract

Real-time estimation of parameters in dynamic systems becomes increasingly important in the field of high precision navigation. The real-time estimation requires real-time quality control of the models underlying the navigation system. The DIA-procedure for the detection, identification and adaption of model errors is particularly suited for the real-time validation of integrated navigation systems. The DIA-procedure is completely recursive and avoids the explicit use of a parallel bank of augmented navigation filters. Real-time testing in the DIA-procedure is done with uniformly-most-powerful-invariant test statistics. In the present contribution the test statistic used in the DIA-procedure for validating GPS phase ambiguities is given and its power is discussed using the concept of minimal detectable biases (MDB). The high power of our test statistic is proven through the size of the MDB. It is also shown that the MDB of our test statistic compares very favourable to the MDB that follows when testing for GPS phase ambiguity slips is done on a single-channel basis.

I. INTRODUCTION

The objective of the present paper is to illustrate the power of the test statistics as used in the DIA-procedure [1-5]. In order to do so, we have chosen to apply the DIA-procedure to a simplified partially constant state-space model. The structure of the partially constant state-space model is chosen such that it resembles one of the simpler models of kinematic GPS positioning. To facilitate the discussion we will therefore use the terminology of GPS and speak of pseudo ranges, phase measurements and phase ambiguities. Applications of the partially constant state-space model are however not restricted to GPS only. The model is in fact applicable to all problems for which parts of the state vector can be considered constant or only slowly changing in time. This may for instance be the case with instrumental parameters.

Our working hypothesis will be based on the assumption that during the kinematic survey an m-number of pseudo ranges and an m-number of carrier phases are observed at every epoch k. The observables are assumed to be uncorrelated and their variances are assumed to be time-invariant. The observables are also assumed to be Gaussian distributed. The variance of the pseudo ranges is denoted as σ_1^2, and the variance of the carrier phases as σ_2^2. The m-number of unknown carrier phase ambiguities are collected in the state vector ∇. The state vector ∇ is constant in time in case phase ambiguity slips are absent. The remaining unknown parameters [such as (relative) coordinates of mobile receiver and (relative) receiver clock errors] are collected in the n-state vector x_k. In order to keep our model as simple as possible, we will assume that no dynamic model is introduced for the

---

In: Proceedings DGPS'91. First Int. Symp. Real Time Differential Applications of the Global Positioning System, Vol. 2, pp. 679-686.

time-varying state vector x_k . The 2m-by-n (linearized) design matrix of the state vector x_k which contains a.o. the receiver-satellite geometry, is denoted as $(A_k^*, A_k^*)^*$.
The above model constitutes our working hypothesis H_o. Under this hypothesis, recursive least-squares filtering provides us at every epoch k with the state vector estimates $\hat{x}_k$ and $\hat{\nabla}_k$, and corresponding covariance matrices $Q_{\hat{x}_k}$, $Q_{\hat{\nabla}_k}$, and $Q_{\hat{x}_k \hat{\nabla}_k}$. If H_o is true, the estimators produced by the navigation filter are optimal with well defined statistical properties. Misspecifications in the working hypothesis H_o will however invalidate the results of filtering and thus also any conclusion based on them. It is therefore of importance to have ways to verify the validity of H_o . This is done in real-time with the DIA-procedure. Since the objective of the present contribution is to illustrate the power of our test statistics as used in the DIA-procedure, we will restrict our attention in the following to the identification step of the DIA-procedure. It is therefore assumed that the overall model testing has been performed and that detection of unspecified model errors indeed took place. Identification implies then that a search among candidate hypotheses is done for the most likely alternative hypothesis and their most likely time of occurrence. Since only phase ambiguity slips are considered in the present contribution, each member of the class of alternative hypotheses describes one particular misspecification in the mean of ∇.

II. THE TEST STATISTIC $\underline{t}_{(i)}^{l,k}$ FOR GPS PHASE AMBIGUITY SLIPS

It will be clear that misspecifications in the mean of ∇ can only be tested if the navigation system provides for redundant information. It is namely this surplus of information which enables one to test whether the data can be considered to be statistically consistent with the assumed model. If we assume rank deficiencies to be absent, the overall redundancy of our model with pseudo ranges and carrier phases equals $k(2m-n)-m$. At every epoch we have a redundancy of $m-n$ (we assume that $m \geq n$) due to the fact that both pseudo ranges and carrier phases are observed. This makes for a redundancy of $k(m-n)$ after k number of epochs. In addition, it is also assumed under H_o that all the phase ambiguities remain constant. This takes care of the additional redundancy of $(k-1)m$. Under the assumption that sufficient redundancy is available, the test statistic as used in the DIA-procedure for the identification of a slip in the i^{th} phase ambiguity reads as

$$(1) \qquad \boxed{\underline{t}_{(i)}^{l,k} = \frac{c_i^* Q_{\hat{\nabla}_{l-1}}^{-1} [\underline{\hat{\nabla}}_{l-1} - \underline{\hat{\nabla}}_k]}{(c_i^* Q_{\hat{\nabla}_{l-1}}^{-1} [Q_{\hat{\nabla}_{l-1}} - Q_{\hat{\nabla}_k}] Q_{\hat{\nabla}_{l-1}}^{-1} c_i)^{1/2}}}$$

where c_i is the canonical unit vector with a 1 as its i^{th}-element. Note that the test statistic depends on the filtering results of the two epochs l and k. Therefore a *testmatrix* with the test statistics $\underline{t}_{(i)}^{l,k}$ as its elements is formed for each particular alternative hypothesis H_{a_i}. This is shown in figure 1a. An important aspect of the above test statistic is the choice for l, which is the time that the slip(s) are assumed to have started, and its relation with $k \geq l$, which is the time the testing is performed. The simplest choice would be to set l equal to the running time index k. In this case we speak of *local identification*, since no information is taken into account that has a bearing on data collected after epoch l. We speak of *global identification* if $k > l$. The power in case of global testing is of course higher than in case of local testing. However with k being larger than l one may be confronted with a possible delay in time of identification. Therefore in order to bound the possible delay, a *moving window* of length N is introduced by constraining l to $k-N+1 < l \leq k$. With this window the time of delay is at most equal to $N-1$. The corresponding testmatrix is shown in figure 1b. The choice of the window length N depends on the testing power that is required in a particular application. The choice of N is therefore typically a problem one should take into account when designing the navigation filter. Instead of constraining l to $k-N+1 < l \leq k$, it may be advantageous for some applications to constrain l even to $k-N+1 < l \leq k-M$. This is shown in figure 1c. The rationale behind this constraint is that the test statistic may be too insensitive for identification purposes if $l > k-M$. But again it is stressed that this choice should be based on the power required by the particular application.

$$
\begin{array}{cc} & \xrightarrow{k} \\ l\downarrow & \begin{bmatrix} t^{1,1} & t^{1,2} & t^{1,3} & t^{1,4} \\ & t^{2,2} & t^{2,3} & t^{2,4} \\ & & t^{3,3} & t^{3,4} \\ & & & t^{4,4} \end{bmatrix}_{H_a} \end{array}
\qquad
\begin{bmatrix} t^{1,1} & t^{1,2} & \bullet & \bullet \\ & t^{2,2} & t^{2,3} & \bullet \\ & & t^{3,3} & t^{3,4} \\ & & & t^{4,4} \end{bmatrix}_{H_a}
\qquad
\begin{bmatrix} \bullet & t^{1,2} & \bullet & \bullet \\ & \bullet & t^{2,3} & \bullet \\ & & \bullet & t^{3,4} \\ & & & \bullet \end{bmatrix}_{H_a}
$$

(a) (b) (c)

Figure 1: Testmatrix with (a) no window, (b) a moving window with $k-2 < l \leq k$ and (c) a moving window with $k-2 < l \leq k-1$.

Our identification step based on the test statistic (1) can now be described as follows. At the time of testing k one first determines per alternative hypothesis H_{a_i} the value of l for which $|t_{(i)}^{l,k}|$ is at a maximum. This value of l would then be the most likely starting time of the slip if the corresponding alternative hypothesis would be true. In order to find both the most likely alternative hypothesis and most likely value of l, the values $max_l|t_{(i)}^{l,k}|$ for the different alternative hypotheses H_{a_i}, $i = 1, 2, ..$, are compared. The maximum of this set identifies then both the most likely time of occurrence and most likely alternative hypothesis. After this the significance of the identified slip has to be tested. Since the test statistic (1) is standard normally distributed under H_o, the identified slip is considered likely enough to have occurred if $|t_{(i)}^{l,k}| \geq N_{1/2\alpha}(0,1)$.

The above expression (1) for the test statistic simplifies in case the design matrices A_k are square. When the design matrices are square, then $Q_{\hat{\underline{\nabla}}_k} = \frac{1}{k}(\sigma_1^2+\sigma_2^2)I$, and (1) reduces to

$$
(2) \qquad \underline{t}_{(i)}^{l,k} = \left[\frac{k(l-1)}{N(\sigma_1^2+\sigma_2^2)}\right]^{1/2} \left[\hat{\underline{\nabla}}_{l-1}^{i} - \hat{\underline{\nabla}}_{k}^{i}\right],
$$

with $\hat{\underline{\nabla}}_k^i$ being the i^{th} – *element* of the vector $\hat{\underline{\nabla}}_k$. Note that in this case the test statistic is simply a scaled version of the difference of the estimated phase ambiguities at times $l-1$ and k. The overall redundancy in this case equals $(k-1)m$, and is due to the assumed constancy of the phase ambiguities under the working hypothesis H_o .

III. THE MDB OF THE PHASE AMBIGUITY SLIP

The with the test statistic $\underline{t}_{(i)}^{l,k}$ corresponding *minimal detectable bias* (MDB) of a slip in the i^{th} phase ambiguity follows as [2-6]

$$(3) \qquad \boxed{|\nabla_{(i)}^{l,k}| = \left[\frac{\lambda(\alpha=\alpha_o,1,\gamma=\gamma_o)}{c_i^* Q_{\hat{\nabla}_{l-1}}^{-1}[Q_{\hat{\nabla}_{l-1}} - Q_{\hat{\nabla}_k}]Q_{\hat{\nabla}_{l-1}}^{-1} c_i}\right]^{1/2}}$$

with $\lambda(\alpha = \alpha_o, 1, \gamma = \gamma_o)$ being the inverted power function in symbolic notation. The above MDB describes the minimal size of the ambiguity slip of the i^{th} phase observable that can be detected and identified with a probability γ_o at a level of significance α_o. The MDB's are therefore an important diagnostic measure for inferring how well particular model errors can be detected and identified. And by propagating the MDB's through the navigation filter operating under H_o , it becomes possible to diagnose the expected biases in the filtered states [4,6].

In order to illustrate the importance of the MDB-concept and the high detection -and identification power of our test statistic, we will give a practically useful approximation to (3). In order to obtain this approximation, we will assume that the design matrices A_k are time-invariant, i.e. they are assumed to be constant. This of course impairs our generality somewhat. But the assumption is still considered to be realistic enough because of the relatively slow change of the receiver-satellite geometry in case of GPS. With the design matrices being constant, the square of the MDB for ambiguity slips can be shown to read as

$$(4) \qquad \boxed{|\nabla_{(i)}^{l,k}|^2 = \frac{1}{N}\frac{k}{(l-1)}\frac{(\sigma_1^2+\sigma_2^2)\lambda_o}{1+[\sigma_1^2/\sigma_2^2]c_i^* P_B c_i}},$$

with λ_o being the reference value $\lambda(\alpha_o, 1, \gamma_o)$ of the noncentrality parameter, and where $P_B = B[B^*B]^{-1}B^*$ is the orthogonal projector that projects onto the range space of B which equals the orthogonal complement of the range space of A. From an analysis of (4) some important conclusions can be drawn:

1. The case $m = n$ and $l = k$:

First assume that $m = n$ and that detection, identification and adaptation is done instantaneously. Then the projector P_B is identically zero, and $N = 1$ and $l = k$. Expression (4) reduces then to

$$(5) \qquad |\nabla_{(i)}^{k,k}|^2 = \frac{k}{(k-1)}(\sigma_1^2+\sigma_2^2)\lambda_o.$$

This shows since $\sigma_2 \ll \sigma_1$, that for k large enough the MDB's are approximately equal to $\sigma_1\lambda_o^{1/2}$ and *independent* of the receiver-satellite geometry. With $\alpha_o = 0.001$ and $\gamma_o = 0.80$ this amounts to an identifiable size of an ambiguity slip of approximately four times the standard deviation of the pseudo range. Thus in this case one can conclude that detection and identification of realistic ambiguity slips is virtually impossible.

2. The case $m = n$ and $l = k - N + 1$:

Let us now, in order to improve the power of identification, include a window in our testing procedure. But we still assume that $P_B = 0$. In that case expression (4) becomes

$$(6) \qquad |\nabla_{(i)}^{l,k}|^2 = \frac{1}{N(1-N/k)}(\sigma_1^2+\sigma_2^2)\lambda_o.$$

This result shows that with the inclusion of a window of length N, the MDB's are approximately equal to $\sigma_1[\lambda_o/N]^{1/2}$ for k large enough. If in a particular application the required level of the MDB is given, expression (6) can be used directly to obtain the correct window length N.

3. The case $m > n$ and $l = k$:

Let us now assume that detection, identification and adaptation is done instantaneously, and that $m > n$ and therefore $P_B \neq 0$. Expression (4) becomes then

$$(7) \qquad |\nabla_{(i)}^{k,k}|^2 = \frac{k}{(k-1)} \frac{(\sigma_1^2 + \sigma_2^2)\lambda_o}{1 + [\sigma_1^2/\sigma_2^2] c_i^* P_B c_i}.$$

In this case the MDB does depend on the receiver-satellite geometry. Note that (7) reduces to (5) if $c_i^* P_B c_i = 0$. This happens when the canonical unit vector c_i is orthogonal to the range space of matrix B, or, in other words, when c_i lies in the range space of matrix A. In the context of single point positioning based on pseudo ranges only, this would mean that the receiver-satellite geometry is such that an error in the i^{th} pseudo range would pass the tests unnoticed. This shows that it is of importance from a designing point of view to know in advance what the (approximate) receiver-satellite geometry will look like during navigation.

Another important conclusion that can be drawn from (7) is that, relative to our previous results, a drastic decrease takes place in the level of the MDB's when $c_i^* P_B c_i \neq 0$. This is due to the very large amplification factor σ_1^2/σ_2^2 with which $c_i^* P_B c_i$ is multiplied in the denominator of (7). A crude but useful approximation of (7) can be obtained in the following way. Since P_B is an orthogonal projector of rank $m-n$, we have $trace(P_B) = m - n$. Hence, since P_B is an $m - by - m$ matrix, $(m-n)/m$ can be considered a crude average value of the diagonal elements of P_B and therefore of $c_i^* P_B c_i$. With this approximation and using the fact that $\sigma_2 \ll \sigma_1$, the following crude but useful approximation to the MDB can be obtained for k large enough,

$$(8) \qquad |\nabla_{(i)}^{k,k}|^2 = \sigma_2[m\lambda_o/(m-n)]^{1/2}.$$

With $\alpha_o = 0.001$, $\gamma_o = 0.80$, $m = 8$ and $n = 4$ this amounts to an identifiable size of an ambiguity slip of approximately six times the standard deviation of the phase observable. This small value of the MDB indicates that the introduction of a window will not be necessary for most applications, and that instantaneous local testing will suffice.

IV. A COMPARISON BETWEEN $\underline{t}_{(i)}^{l,k}$ AND SINGLE-CHANNEL TESTING

As was pointed out earlier the test statistics used in our DIA-procedure are uniformly-most-powerful-invariant test statistics. Without going into the theoretical details, this implies somewhat loosly formulated that our test statistics have the property of correctly detecting and identifying model errors with the highest possible probability. In other words, the sizes of the MDB's of our test statistics are the smallest possible. No other test statistics exist with smaller MDB's, provided of course that one starts from identical assumptions. In order to illustrate this, we shall compare our one-dimensional test statistic $\underline{t}_{(i)}^{l,k}$ with a test statistic $\underline{\tilde{t}}_{(i)}^{l,k}$ that is based on single-channel monitoring.

Let us assume that the i^{th} phase observable is subject to a slip in its ambiguity at epoch l.

It would seem reasonable then to confront the estimate of the i^{th} ambiguity at the time of testing k with the estimate of the i^{th} ambiguity at the epoch just before the slip started to occur, $l-1$. Hence, it seems reasonable to look at the difference $\hat{\underline{\nabla}}^i_{l-1} - \hat{\underline{\nabla}}^i_k$, since one may expect that the slip should show up in this difference. In order to obtain a test statistic from this difference, it is standardized by dividing it by its standard deviation. Since the variance of the difference equals $c_i^*[Q_{\hat{\nabla}_{l-1}} - Q_{\hat{\nabla}_k}]c_i$, the following test statistic is obtained

$$\tilde{t}^{l,k}_{(i)} = \frac{c_i^*[\hat{\nabla}_{l-1} - \hat{\nabla}_k]}{(c_i^*[Q_{\hat{\nabla}_{l-1}} - Q_{\hat{\nabla}_k}]c_i)^{1/2}}. \tag{9}$$

This test statistic is just like our uniformly-most-powerful-invariant (UMPI) test statistic (1) standard normally distributed under H_o. Upon comparing (9) with (1) we note that the two test statistics are very much alike except for the presence of the covariance matrix $Q_{\hat{\nabla}_{l-1}}$ in (1). In (9) the covariance matrix $Q_{\hat{\nabla}_{l-1}}$ is absent, implying that testing is done directly on the basis of the estimation results that correspond with the i^{th} channel. Hence, in this case testing is done on a single-channel basis. In (1) however the presence of the covariance matrix $Q_{\hat{\nabla}_{l-1}}$ implies that also the information content of the covariances between the channels is taken into account. The two test statistics are therefore only identical when the covariance matrix $Q_{\hat{\nabla}_{l-1}}$ is diagonal. This is the case when the design matrices A_k are square and thus when m equals n. In general however the two test statistics differ, which implies that they will also differ in their power of testing. If we assume like we did in our derivation of (4) that the design matrices A_k are time-invariant, then the MDB of the test statistic (9) can be shown to read as

$$|\tilde{\nabla}^{l,k}_{(i)}|^2 = \frac{1}{N}\frac{k}{l-1}\frac{(\sigma_1^2+\sigma_2^2)\lambda_o}{(1-[\sigma_1^2/(\sigma_1^2+\sigma_2^2)]c_i^*P_Bc_i)^{-1}}. \tag{10}$$

When we compare this result with that of (4) some important remarks can be made. First note that both MDB's are affected in an identical manner by the introduction of a window with length N. Also note that both MDB's are equal in case P_B vanishes identically. This is of course due to the fact that both test statistics (1) and (9) become identical when the design matrices A_k are square. In this case both test statistics simplify to (2). The two MDB's (4) and (10) also happen to coincide when $P_B \neq 0$, but either $c_i^*P_Bc_i = 0$ or $c_i^*P_Bc_i = 1$ holds. This shows that the two MDB's are identical when the c_i-vector lies either in the range space of A or in its orthogonal complement. If the c_i-vector lies in the range space of A then both MDB's are dominated by σ_1^2 and therefore take on large values. If on the other hand the receiver-satellite geometry is such that the c_i-vector lies in the range space of B then both MDB's are dominated by σ_2^2 and therefore take on very small values. Thus there do exist situations where the MDB of the single-channel test statistic $\tilde{t}^{l,k}_{(i)}$ coincides with the MDB of our UMPI test statistic (1). In general however, the two MDB's differ when $P_B \neq 0$. And most importantly this difference can reach very large values indeed. In order to show this, we will look at the ratio of the two MDB's. If we divide (10) by (4), one can show that the following result holds true

$$\frac{|\tilde{\nabla}^{l,k}_{(i)}|^2}{|\nabla^{l,k}_{(i)}|^2} = 1 + [(\sigma_2^2/\sigma_1^2) + (\sigma_2^2/\sigma_1^2)^2]^{-1}c_i^*P_Ac_ic_i^*P_Bc_i. \tag{11}$$

This result can now be used to bound the ratio of the squares of the MDB's from above and below. If we denote the angle between the two vectors c_i and P_Ac_i as α_i, then $c_i^*P_Ac_i =$

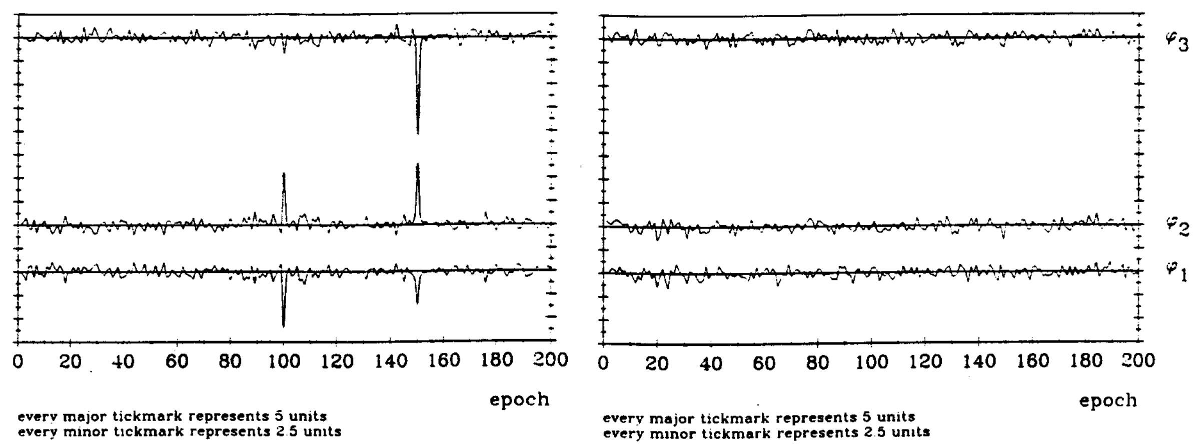
$cos^2\alpha_i$ and $c_i^* P_B c_i = sin^2\alpha_i$. Hence, $c_i^* P_A c_i c_i^* P_B c_i = \frac{1}{4} sin^2 2\alpha_i$. From this follows that (11) is bounded as

$$l \leq \frac{|\tilde{\nabla}^{l,k}_{(i)}|^2}{|\nabla^{l,k}_{(i)}|^2} \leq 1 + \frac{1}{4}[(\sigma_2^2/\sigma_1^2) + (\sigma_2^2/\sigma_1^2)^2]^{-1}. \tag{12}$$

This result shows since $\sigma_2 \ll \sigma_1$ that the ratio can reach very large values indeed. This illustrates the superiority in terms of power of our test statistic over the single-channel test statistic (9). A somewhat crude but useful average value of the ratio can be obtained if we use the approximation $c_i^* P_A c_i = n/m$. Then, since $\sigma_2 \ll \sigma_1$,

$$\frac{|\tilde{\nabla}^{l,k}_{(i)}|^2}{|\nabla^{l,k}_{(i)}|^2} \doteq 1 + \frac{\sigma_1^2}{\sigma_2^2}\frac{n}{m}\frac{m-n}{m}. \tag{13}$$

In order to illustrate numerically the lack of power of the single-channel test statistic as compared to the power of our test statistic $\underline{t}^{l,k}_{(i)}$, a simple kinematic GPS survey based on both pseudo ranges and carrier phases was simulated ($\sigma_1 = 3m, \sigma_2 = 3mm$). Two ambiguity slips of 10cm were introduced in the data. One slip was introduced at epoch 100 in channel 1, and the other slip was introduced at epoch 150 in channel 3. The same data was processed twice. Once it was processed using the DIA-procedure with the uniformly-most-powerful-invariant test statistic $\underline{t}^{l,k}_{(i)}(l = k)$, and once it was processed using the single-channel test statistic. In both cases testing was done with a window length of one. The sample values of the two test statistics are plotted in figure 2 for the first three channels.



(a) (b)

Figure 2: a) The UMPI test statistic $\underline{t}^{l,k}_{(i)}(l = k)$ for $i = 1,2,3$; b) The single-channel test statistic $\underline{\tilde{t}}^{l,k}_i(l = k)$ for $i = 1,2,3$.

Figure 2a shows the results of the DIA-procedure. This figure shows that the sample values of our test statistic first exceed the critical value of $N_{1/2\alpha}(0,1) = 3.29$ ($\alpha = 0.001$) at epoch 100. In this case the sample values of all three test statistics $\underline{t}^{l,k}_{(i)}(l = k)$, $i = 1,2,3$, exceed the critical value. The largest sample value is that of $\underline{t}^{l,k}_{(1)}(l = k)$ showing that the correct channel is identified. After the identification at epoch 100, adaptation of the recursive navigation filter was performed to eliminate the presence of biases. That adaptation was done correctly is reflected in the sharp decrease in the sample values of the test statistics in figure 2a. The second instant that the sample values exceed the critical value occurs at epoch 150. And in this case identification and adaptation is again performed successfully. Now the third channel is identified as the one containing the ambiguity slip. Figure 2b shows the sample values of the single-channel test statistic for the first three channels. The results shown are based on exactly the same data as was used for figure 2a. The poor power of the single-channel test statistic is now clearly evidenced from the fact that it is unable to detect and identify the slips that occured at the epochs 100 and 150.

References

[1] Teunissen, P.J.G. and M.A. Salzmann (1988): Performance Analysis of Kalman Filters. Proceedings HYDRO88, Special Publication no. 23, Hydrographic Society, pp. 185-193.

[2] Teunissen, P.J.G. (1989): Quality Control in Integrated Navigation Systems. IEEE Aerospace and Electronic Systems Magazine, Vol. 5, No. 7, 1990, pp. 35-41.

[3] Teunissen, P.J.G. and M.A. Salzmann (1989): A Recursive Slippage Test for Use in State-Space Filtering. Manuscripta Geodaetica, Vol. 14, pp. 383-390

[4] Teunissen, P.J.G. (1990): An Integrity and Quality Control Procedure for Use in Multi-Sensor Integration. Proceedings of ION GPS-90. Colorado Springs, Colorado, USA, 19-21 September 1990. pp. 513-522.

[5] Teunissen, P.J.G. (1991): Some Aspects of Real-Time Model Validation Techniques for Use in Integrated Systems. Proceedings of the IAG-Symposium on Kinematic Systems in Geodesy, Surveying and Remote Sensing, Banff, Canada, 10-13 September 1990, pp. 191-200.

[6] Salzmann, M.A. (1991): MDB: A Design Tool for Integrated Navigation Systems. Bulletin Géodésique Vol. 65, No. 2, pp. 109-115.