NONLINEAR ADJUSTMENT

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1. Introduction

Nonlinear optimization, nonlinear least-squares and densities of nonlinear estimators are a trilogy of problems that are intimately related in the framework of *nonlinear inversion or adjustment* of geodetic data. Usually the description of physical phenomena proceeds through models in which a mapping, A, is defined, from a set of parameters, N, to a set of experimental outcomes, M. M is supposed to contain the image of the map A. Obtaining the image $y = A(x) \in M$ of $x \in M$ in e.g. a least-squares sense, is solving the *inverse problem.* The inverse problem is said to be linear if $A(\alpha_1 x_1 + \alpha_2 x_2) = \alpha_1 A(x_1) + \alpha_2 A(x_2), \forall \alpha_1, \alpha_2 \in R; x_1, x_2 \in N$. Almost no geodetic inverse problem is truely linear. A consequence of nonlinearity is that the inverse problem increases in complexity. These complications manifest themselves: a) in the problem of finding the numerical estimates of the parameters x; and b) in the problem of finding the a posteriory probability density function of the nonlinear estimators.

The numerical estimation of parameters is typically a problem of optimization. The estimation of parameters requires frequently the maximization or minimization of an objective function. Typical objective functions are risk functions, robust loss functions, posteriory density functions, likelihood functions and (weighted and unweighted) sums of squares. In general no direct methods exist for estimation in nonlinear models. For these cases the nonlinear problem is attacked iteratively: at each step the solution of a linear problem, in terms of (the Fréchet) derivatives, is constructed.

It will be clear that a numerical parameter estimation or inversion procedure is incomplete without an analysis of the uncertainties in the results. That is, it is not enough to compute the nonlinear parameter estimates and state that they are the solution to the inverse problem. Knowledge of the a posteriori probability density functions of the nonlinear estimators is needed in order to infer the quality of the results obtained. For linear models a rather complete theory of inference exists. Unfortunately the results which hold true for linear models do not carry over to the nonlinear case. That is, although some exact methods for deriving the distribution of nonlinear estimators exist, these methods are in general very difficult to apply in practice.

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From the above it will be clear that the solution of the nonlinear inverse problem is not as straightforward as it is for the linear case. It is therefore expedient to have ways of assessing the amount of nonlinearity in nonlinear models and methods to prove whether a linear(ized) model is a sufficient approximation. Within the context of nonlinear inversion the objectives of the SSG 4.120 are therefore to examine how performance measures of estimators vary when the actual model differs from the assumed model, to evaluate the performance of numerical methods for computing the desired estimates and to devise, test and evaluate methods which are robust with respect to certain departures from the assumed model. This has led to the following research topics of the SSG:

1. Non-linear statistical inference:
Evaluation of the consequences of non-linearity for the linear inference procedures customarily used (e.g. practical measures of non-linearity; distributional properties of non-linear estimators and statistics). Further development of the differential geometric theory of nonlinear inference.
2. Numerical methods for nonlinear geodetic optimization:
Methods for computing estimates in nonlinear models are usually iterative in nature. An evaluation of the performance of the various iteration methods when applied to geodetic models is needed (e.g., local and global convergence proofs; rates of convergence).
3. Perturbation analysis for linear inference:
This includes, but goes beyond, the classical problem of hypothesis testing. The objective here is to analyze and describe the sensitivity of linear inference procedures for perturbations in the assumed linear(ized) model (e.g., influence of perturbations in functional and stochastic model on the estimators, test statistics, variance components estimation, reliability and precision measures).
4. Robust statistics:
Study to what extent and under which circumstances methods of robust statistics can compete with or complement the more traditional inference procedures in case of geodetic adjustments.

2. Review of SSG-research

Although it is rather difficult to classify the results of the activity of the members of the studygroup in definite categories, the main results will be presented according to the following classification: numerical and analytical methods for nonlinear geodetic optimization; nonlinear statistical inference; perturbation analysis for linear inference; robust statistics.

In general nonlinear inversion problems have to be attacked iteratively. The best known iterative techniques are: the Steepest-Ascent (Descent) method, the (Quasi-)Newton methods, the Conjugate Direction methods and the Trust Region methods. From a theoretical as well as a practical point of view it is important to have available verifyable local and global convergency theorems, practical estimates of the rates of convergence and suitable termination criteria. In [Adamczewski and Vo Hung Dang, 1987], [Vo Hung Dang, 1988] it was shown that the rate of convergence for the computation of geodetic networks is favourably effected by an ordering of the parameters based on the topology of the network. In [Blaha, 1987], [Blaha and Bisette, 1989] the resolution of a nonlinear parametric adjustment model is addressed through an isomorphic geometrical set-up with tensor structure. The geometrical set-up leads to the solution of modified normal

equations which contains second order partial derivatives. This approach shortens the convergency process as compared to standard methods. It would be interesting to compare the method with the classical Newton process.
In [Grafarend et al., 1989][Grafarend and Schaffrin, 1989a,b] an analytical solution of the nonlinear resection problem is given. Conditions for the uniqueness of the solution are given using concepts from differential geometry.
The theory of the Symmetric Helmert Transformation as introduced in [Teunissen, 1985] is discussed and extended in [Teunissen, 1988], [Krarup, 1988], [Koch, 1989a,b] and [Wolf, 1989]. In [Teunissen, 1988] a computational efficient two-step procedure for ruled-type manifolds is introduced. The method has also found its application in studies of the electromagnetic field of the brain [J.C. de Munck, 1989]. In [Koch, 1989a,b] an alternative solution method of the 1D Symmetric Helmert transformation based on Bayesian principles is presented. And in [Wolf, 1989] it is shown that the eigenvalue approach of [Teunissen, 1985] is equivalent in the 1D-case to the approach based on a formula of R. Schumann. A generalization of the Symmetric Helmert transformation to the N-dimensional case is given in [Krarup, 1988]. Based on differential geometric concepts this study presents a detailed analysis of the properties of the stationary or critical points.

Generally there are three approaches that one can follow to estimate the probabilistic properties of nonlinear estimates. The first approach relies on results from asymptotic theory. The central idea of asymptotic theory is that when the number of observations is large and errors of estimation corresponding small, simplifications become available that are not available in general. The rigorous mathematical development involves limiting distributional results and is closely related to the classical limit theorems of probability theory. Unfortunately, since the theory is based on the assumption that the number of observations increase indifinitely, the results obtained up to now cannot satisfy all the requirements of application in practice.
An alternative way to estimate the distribution of nonlinear estimators would be to rely on Monte Carlo methods. One replicates the series of experiments as many times as one needs, each time with a new sample drawn from the parent distribution and so obtains the relevant distributional properties by averaging over all replications. A drawback of this technique is however that it may become computationally demanding for large scale inverse problems.
A third approach that comes to mind to compute the distribution of nonlinear estimators is based on the fundamental relations that define distribution functions. If the parent density is given, then theoretically at least, one can find both the cumulative and density distribution of the nonlinear estimator. The practical problem with this method is, however, that in general one cannot easily evaluate the complicated integrals and inverses of the nonlinear maps involved. Instead of aiming at a complete description of the distribution, one could restrict oneselves to some of the moments of the distribution. The complexity of these computations depends very much on the nature of the parent distribution and the nonlinear maps involved. But in general they can become quite complicated, especially in the multivariate case. If in a particular problem it is impossible to apply the above mentioned analytical methods, the next one thing one can try to do is to make use of approximations based on a suitable Taylor expansion. In this way appropriate approximations to the first two moments and density of nonlinear least-squares estimators were obtained in [Pazman, 1987; Jeudy, 1988; Teunissen, 1988a, 1988b].

An analytical expression for the first moment of the nonlinear least-squares estimators of the parameters of the 2D symmetric Helmert transformation is given in [Teunissen, 1989a]. Nonlinearity diagnostics for the nonlinear inversion of geodetic and geophysical data based on concepts from differential goemtry were developed in [Teunissen, 1989b]. Quadratic approximations for geodetic adjustment models and the solution of the geodetic boundary value problem are developed in [Bähr, 1988], [Heck, 1988], [Heck, 1989].

The objective of perturbation studies in linear(ized) models is to analyze the influence of departures from the underlying assumptions. In [Hahn, Van Mierlo, 1987] and [Van Mierlo, Hahn, 1987] the consequences of changes in the weight matrix is analyzed in detail. In [Borre, Lauritzen, 1989] the concept of conjugate curvature is introduced and its relation to the principal and normal curvatures of [Krarup, 1982], [Teunissen, 1984] is shown. The concept of conjugate curvatures enables one to give a geometric description of the procedure of simultaneous estimation of components of both the functional and stochastic model. In [Van Mierlo, 1989] teststatistics are proposed for the case that the intersection of the nominal and the alternative models is not equal to the nominal models.

The objective of robust statistics is to find procedures of inference that are less sensitive to hypothized perturbations in the assumed mathematical model. In [Schaffrin, 1989] less sensitive tests are obtained by introducing a priori random characteristics on the linear hypotheses. The implication of methods of robust estimation for photogrammetry and deformation models are studied in [Kubik et al. 1987a,b] and [Caspary and Borutta, 1987]. In [Kampmann, 1988] an approximate testprocedure based on the properties of the least absolute value estimator is derived. A deficiency of most robust procedures is still the lack of a proper reliability description. An attempt to include reliability indicators into robust estimation is made in [Borutta, 1988]. The robustification of general prediction methods is treated in [Schaffrin, Grafarend, 1987], [Schaffrin, 1989].

3. Outlook

From the previous brief review follows that a whole variety of problems have been studied that in one way or the other relate to what one could call the "Theory of Geodetic Inference". Although the various topics which have been studied cover a very wide spectrum and differ considerably in complexity, one important unifying point of view seems to underly all the research reported. The general research trend is namely to focus on departures from the assumptions on which the "classical" mathematical models and corresponding inference procedures are based, to formulate diagnostics that identify the influence of the assumed departures and to devise procedures of inference that are either most-sensitive or least-sensitive to these departures.

Up to now most of the research has been directed towards the more or less classical Gauss-Markov type models. It seems expedient, however, in view of "dynamic" applications such as for instance kinematic positioning, navigation and digital fotogrammetry, to extend this field so as to include the rich and diverse univariate and multivariate state-space filtering techniques as well.

This field also offers the possibility to strengthen the link with the more classical inference procedures. Let us therefore in conclusion entertain the hope that this challenge will not remain unanswered.

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