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QUALITY CONTROL IN KINEMATIC DATA PROCESSING

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Abstract

The concepts of quality control in kinematic data processing are discussed. Particular attention is paid to testing procedures. A distinction is made between global and local model testing. Slippage-type tests are discussed in detail. Furthermore a method for the estimation of variance components is described. The interdependence of testing procedures and filter adaptation is pointed out and is illustrated by an example.

1 Introduction

Land vehicle navigation systems are supposed to provide the user with an optimal estimate of position at every instant. These position estimates are often obtained using kinematic data processing algorithms. The problems encountered in data processing for land navigation purposes are to a large extent similar to those found in other fields where kinematic data processing is applied, e.g. aircraft navigation and offshore surveying.

In general two characteristics of kinematic data processing are that the object (vehicle) of which the position has to be estimated is moving and that usually more than one positioning system is used. This inevitably leads us to the concept of integrated navigation systems.

Before any computations can be performed at all, models underlying the moving system have to be specified. These are functional models describing the non-random characteristics of the dynamics (or kinematic behaviour) of the system and the relation of the observables with the system parameters. Stochastic models describing random disturbances in dynamic and measurement models have to

be designed. However, any misspecification in the (functional or stochastic) model will invalidate the results of the (position) estimation and thus also any conclusion based on these results. It is crucial therefore to have ways to validate the assumed model. This brings us into the realm of *quality control.*

We assume that the kinematic data processing is based on the well-known Kalman filter (see, e.g., Gelb [1974]). The rationale of using Kalman filters is that only then optimal use is made of all present and past information and that a much more enhanced quality description of the system is feasible than with single fix processing.

Quality control and Kalman filtering are general concepts and thus independent of a particular application. Hence we have no particular sensor combination in an integrated navigation system in mind. Possible positioning measurements are, for instance, map matching procedures, dead reckoning ((differential) odometers and compass), and radiopositioning systems (GPS, Loran-C). The particular characteristics of land vehicle navigation enter in the design of the dynamic system model (which parameters are of interest and what is their relation in time?) and in the design of the measurement models for the various sensors to be implemented. Once the dynamic and measurement models have been specified the quality control procedures outlined in this paper can be applied.

The contents of this paper is as follows. In the following section we briefly introduce the Kalman filter. Then we discuss the concepts of quality control in Section 3. Testing procedures to detect possible misspecifactions in the model are discussed in Section 4. In Section 5 the adaptation of the stochastic model is discussed. The close relation between model adaptation and testing procedures is discussed in Section 6 and is illustrated with an example. Finally some concluding remarks are given.

2 The Kalman filter

In this section no derivation of the Kalman filter algorithm is given. The literature abounds with numerous derivations based on different concepts, and we merely point out that the Kalman filter can also be derived using the concept of least squares. The reason we make this observation is that the theory of quality control is well established for least squares adjustment [Teunissen, 1985]. This observation greatly facilitates the implementation of quality control techniques in the Kalman filter.

Consider the following linear dynamic system with dynamic model and measurement model given as

$$\underline{x}_k = \Phi_{k,k-1}\underline{x}_{k-1} + \underline{d}_k \quad (1)$$

$$\underline{y}_k = A x_k + \underline{e}_k \quad (2)$$

where an underscore indicates the random character of a variable; the n-vector x_k denotes the system state, a set of parameters that completely describes the system at time k; $\Phi_{k,k-1}$ is the state transition matrix that describes the non-random transition of the state from time $k-1$ to k; $\underline{d}_k$ is an unobservable random disturbance vector that models the uncertainty in the state transition at time k (the sample of $\underline{d}_k$ is taken to be equal to the zero vector); $\underline{y}_k$ is the m_k-vector of observables at time k; $\underline{e}_k$ is m_k-vector of measurement noise; and A_k is the $m_k \times n$ designmatrix that relates the mean of $\underline{y}_k$ to the statevector x_k. With the customary assumptions the Kalman filter algorithm reads [Kalman, 1960; Gelb, 1974]

$$\hat{\underline{x}}_{k|k-1} = \Phi_{k,k-1}\hat{\underline{x}}_{k-1|k-1} + \underline{d}_k \tag{3}$$

$$P_{k|k-1} = \Phi_{k,k-1}P_{k-1}\Phi^*_{k,k-1} + Q_k \tag{4}$$

$$\hat{\underline{x}}_{k|k} = \hat{\underline{x}}_{k|k-1} + K_k(\underline{y}_k - A_k\hat{\underline{x}}_{k|k-1}) \tag{5}$$

$$P_k = (I - K_kA_k)P_{k|k-1} \tag{6}$$

$$K_k = P_{k|k-1}A^*_k(R_k + A_kP_{k|k-1}A^*_k)^{-1} \tag{7}$$

where P_k, Q_k, and R_k are the covariance matrices of the system state, disturbances, and observables respectively.

Equations (3) and (4) constitute the *time update* and equations (5) to (7) the *measurement update.* An important role in the process of verifying the validity of the assumed mathematical model is played by the so-called *predicted residual.* The predicted residual is defined as the difference between the actual system output and the predicted output based on the predicted state

$$\underline{v}_k = \underline{y}_k - A_k\hat{\underline{x}}_{k|k-1} \; . \tag{8}$$

The predicted residual represents the new information brought in by the latest observable $\underline{y}_k$. Under the working hypothesis that the mathematical model is specified correctly, the predicted residual has well defined statistical properties:

$$\underline{v}_k \sim N(0, Q_{v_k}) \quad , \tag{9}$$

where

$$Q_{v_k} = (R_k + A_kP_{k|k-1}A^*_k) \; . \tag{10}$$

Note that the predicted residual and its covariance matrix are available during each measurement update.

Knowledge of the distribution of the predicted residual can be used for testing the validity of the assumed mathematical model.

3 Quality Control

In the introduction we established why quality control is of the utmost importance in kinematic data processing. Additionaly it can be remarked that in general dynamic systems are probably more prone to system failures, which we will call model misspecifications, than static systems. Furthermore such misspecifications (e.g., outliers in the data; sensor failures) have to be detected in *real-time* in order to prevent degradation of the positioning estimation result. In general one only has two or three positioning related measurement systems available. This renders a correct detection of a misspecification and decision regarding the cause of the misspecification rather doubtful if one restrics oneself to a single positioning update. Besides an unmodelled trend (e.g. a "soft" sensor failure) might be impossible to detect using quality control procedures based on a single fix only. We therefore introduce methods that take more than one epoch into account in order to facilitate the detection of unmodelled trends.

Another aspect of quality control we will consider is the establishment of a correct stochastic model. As noted before, the estimation results are only optimal if the model is correct and an improved estimate of the stochastic model will certainly enhance the estimation results.

We will now discuss the term *quality* in some more detail. In geodetic literature the term quality comprises two aspects: *reliability* and *precision*. Precision is a measure for the spread of the estimation results due to the stochastic model and is usually represented by a covariance matrix.

Reliability is concerned with the effects of possible misspecifications of the model on the estimation results. More precisely reliability describes the ability of the redundant observations to check model errors. Only if precision and reliability are taken into account simultaneously one can speak of quality control.

The concept of reliability only has actual meaning if one performs tests for possible misspecifications in the model. Thus testing procedures form the basis of any reliability theory. This means, for example, that an integrated system with redundant measurements is still unreliable if the redundancy of the system is not used to test for possible misspecifications.

Because one usually has some ideas on which misspecifications might occur one can check beforehand (i.e. in the design phase) if the tests related to these misspecifications lead to an sufficiently reliable system.

In this paper we will examine two aspects of quality control more closely. Firstly we describe test statistics to detect misspecifications in the model, in particular misspecifications in the mean of the predicted residual (8). That is, we will only consider slippage tests. Secondly we discuss a method to estimate the variance levels of the different observation types and the dynamic noise.

4 On Testing Procedures and Reliability

For the testing procedures considered the null hypothesis is that the models (1) and (2) and the associated stochastic models are specified correctly. We make a distinction between *local* model testing and *global* model testing. We speak of local model testing when the tests performed at time k only depend on the predicted state at time k and the observations at time k. If the test takes more than one epoch into account we speak of global model testing. From this definition it follows that in contrast with the global tests, the local tests can be executed in real-time and thus corrective action can be taken immediately.

Furthermore we will consider two specific types of tests. *Overall* model tests do not specify any particular alternative hypothesis to be tested against the null hypothesis. Rejection of an overall model test may indicate an unspecified failure of the system model. Because one is generally interested in the cause of the rejection one will try to specify several misspecifications as alternative hypotheses. Our null hypothesis and alternative hypotheses can be specified in terms of the predicted residuals. We will first consider local tests. The following two hypotheses are considered:

$$\begin{aligned} H_{0\_k}: \quad \underline{v}_k &\sim N(0, Q_{v\_k}) \\ H_{A\_k}: \quad \underline{v}_k &\sim N(\nabla v_k, Q_{v\_k}) \end{aligned} \tag{11}$$

We will assume that the m_k-vector ∇v_k can be parametrized as

$$\underset{m_k \times 1}{\nabla v_k} = \underset{m_k \times b_k}{C_{v\_k}} \underset{b_k \times 1}{\nabla_k} , \tag{12}$$

with the full rank matrix C_{v_k} known, the b_k-vector ∇_k unknown, and $b_k \leq m_k$.

It is well-known [Graybill, 1976; Teunissen, 1985; Koch, 1988] that the *local overall model* LOM test statistic can be written as

$$\boxed{\underline{T}^k = \underline{v}_k^* Q_{v\_k}^{-1} \underline{v}_k} \quad . \tag{13}$$

where the test statistic $\underline{T}^k$ has the following distributions under H_{0_k} and H_{A_k}:

$$\begin{aligned} H_{0\_k}: \quad \underline{T}^k &\sim \chi^2(m_k, 0) \\ H_{A\_k}: \quad \underline{T}^k &\sim \chi^2(m_k, \lambda_k) \end{aligned} \tag{14}$$

where

$$\lambda_k = \nabla_k^* C_{v\_k}^* Q_{v\_k}^{-1} C_{v\_k} \nabla_k$$

is the noncentrality parameter of the test.

The test of size α is now as follows: Reject H_{0_k} if and only if T^k satisfies $T^k \geq \chi^2(\alpha : m_k)$ where $\chi^2(\alpha : m_k)$ is the upper α probability point of the central

χ^2-distribution with m_k degrees of freedom. If the hypothesis H_{0_k} is rejected, one can search for possible misspecifications of the model.

A special case arises if one considers one-dimensional misspecifications. Then the number of columns of the matrix C_{v_k} is chosen equal to one and the matrix becomes a vector, which we denote as c_{v_k}. In this case the test statistic can be written as [Teunissen and Salzmann, 1989]:

$$\boxed{\underline{t}^k = \frac{(c_{v\_k}^* Q_{v\_k}^{-1} \underline{v}_k)^2}{c_{v\_k}^* Q_{v\_k}^{-1} c_{v\_k}}} \quad . \tag{15}$$

The lower case kernel letter t will be used for our one-dimensional slippage test statistics. This test statistic can be used to identify particular one-dimensional misspecifications in H_{0_k}, such as a slippage in the mean of the predicted state, a slippage in the mean of the observables, or a slippage in the mean of a combination of observables and the predicted state. Hence we call (15) a *local slippage* (LS) test statistic.

For every test procedure one has to define the level of significance (α). The level of significance is the probability that one rejects the null hypothesis H_{0_k} even when H_{0_k} is true. Additionaly one also wants to control the error of accepting H_{0_k} when the alternative hypothesis H_{A_k} is true. This is done by choosing the power of the test $(1-\beta)$, where β is the probability of accepting H_{0_k} when H_{A_k} is true. By fixing α and β the noncentrality parameter λ of the (one-dimensional) test can be determined.

Once the noncentrality parameter has been fixed, we can compute the size of the errors ∇. These are called *marginal detectable errors* (MDE's) and can be computed for each one-dimensional alternative hypothesis as

$$|\nabla_k| = \sqrt{\frac{\lambda_k}{c_{v\_k}^* Q_{v\_k}^{-1} c_{v\_k}}} \quad . \tag{16}$$

The MDE's give the magnitude of the misspecifications that can be found with probability β performing the one-dimensional tests. The MDE's are a measure of the so-called internal reliability.

One can obtain a best estimator of ∇_k under H_{A_k} as

$$\underline{\hat{\nabla}}_k = [c_{v\_k}^* Q_{v\_k}^{-1} c_{v\_k}]^{-1} c_{v\_k}^* Q_{v\_k}^{-1} \underline{v}_k \quad . \tag{17}$$

If one, for instance, suspects sensor failures or outlying observations one can follow the *datasnooping* approach [Baarda, 1968; Teunissen, 1985] by choosing m_k number of vectors c_{v_k} of the form

$$\underset{m_k \times 1}{c_i} = \underset{1}{(0 \cdots 1 \cdots 0)^*}_{\substack{\\ i \\ m_k}} \quad , \tag{18}$$

for $i = 1, \ldots, m_k$.

The test statistics given above are easily executed in a Kalman filter environment. This follows since the predicted residual $\underline{v}_k$ and its covariance matrix Q_{v_k} are readily available during each measurement update. A disadvantage of the test statistics given above is, however, that they are local. It will be clear that observations taken after time k have no effect on these local tests at time k. Thus any misspecification in the mathematical model that may occur after time k has no effect on these tests. A somewhat similar situation exists for observations taken prior to time k. That is, although local model testing is dependent on the observations taken prior to time k, this dependency is rather weak, since misspecifications that occur prior to time k are only felt indirectly via the predicted state.

Therefore we turn to global test statistics that take more than one epoch into account. Global test statistics are based on a batch (or window) of predicted residuals $(\underline{v}_i,\ i = l, l+1, \ldots, k)$. It can be shown [Teunissen and Salzmann, 1989] that a test statistic similar to the LOM test statistic can be derived

$$\boxed{\underline{T}^{l,k} = \sum_{i=l}^{k} \underline{v}_i^* Q_{v_i}^{-1} \underline{v}_i} \,, \tag{19}$$

which under H_{0_k} is distributed as $\chi^2(\sum_{i=l}^{k} m_i, 0)$. Note that this test statistic reduces to the LOM test statistic (13) for $l = k$. The global recursive test statistic $\underline{T}^{l,k}$ can be used to perform an overall model test for detecting possible unspecified global model errors, and is consequently called *global overall model* (GOM) test statistic. It is seen that the GOM test statistic can be computed *recursively*. Hence, it can easily be implemented in Kalman filter software.

The LS test statistic can be generalized to more than one epoch as well, but this cannot be done so straightforwardly. This is due to the fact that a possible misspecification does not only influence the predicted residuals directly via the observation vector, but also indirectly via the predicted state. It is possible, however, to derive *recursive* forms for these so-called *global slippage* (GS) test statistics. The GS test statistic reads [Teunissen and Salzmann, 1989]:

$$\boxed{\underline{t}^{l,k} = \frac{[\sum_{i=l}^{k} c_{v\_i}^* Q_{v\_i}^{-1} \underline{v}_i]^2}{\sum_{i=l}^{k} c_{v\_i}^* Q_{v\_i}^{-1} c_{v\_i}}} \,, \tag{20}$$

Note that this test statistic reduces to the one-dimensional LS test statistic (15) for $l = k$. The one-dimensional global slippage test statistic $\underline{t}^{l,k}$ can be used to identify particular one-dimensional global misspecifications in H_{0_k}. Recursive schemes for the computation of the vectors c_{v_i}, $i = l, \ldots, k$ can be derived from the Kalman filter algorithm. In [Teunissen and Salzmann, 1989] recursive schemes are given for the following cases: a) a permanent slip in the state vector that starts at time

l; b) a single slip in the vector of observables at time l; and c) a sensor failure that starts at time l.

The advantage of global tests is that their detection power is considerably larger than that of local tests. Depending on the power of the various tests, the choice of the window length of the global tests should be made in the design phase of the filter. The price one has to pay is a small delay in the detection of a misspecification. In our opinion it is, however, more important to detect a possible misspecification with a small delay than not to detect it at all.

5 Estimation of Variance Components

In order to operate at an optimum the Kalman filter's functional and stochastical model have to be specified correctly. In practice the specification of the functional model poses the least problems. The dynamic behaviour of the object under consideration is usually quite well known. A good description of the measurement sensors (and their inherent biases) is generally available as well.

For the specification of the stochastic models (i.e. the matrices Q and R in Section 2) no standard recipes can be given. Besides modelling the truly random dynamic noise, the dynamic noise is often also used to model small non-random effects and thus the specification of the dynamic noise (Q) can be tedious. Furthermore the availability of several positioning sensors may require a more sophisticated stochastic model for the measurements, in which a variance component for each separate sensor type is introduced. If, for example, two positioning systems are available the stochastic model as introduced in Section 2 can be refined as:

$$\underline{e}_k \sim N(0, \sigma_1^2 R_1 + \sigma_2^2 R_2)$$

where σ_i^2 denotes the variance component for system i. Such a refinement is deemed sufficient for many applications. The variation of the precision of radiopositioning systems due to propagation effects and beacon positions relative to the user can, for example, to a large extent be described by a variable variance component.

Hence a technique to estimate the noise levels of the different components of the stochastic model would be more than welcome. The literature abounds with articles on so-called "adaptive filtering" techniques. These techniques are, however, often only useful under special conditions and/or difficult to implement in real-time systems.

Now let us assume that Q and R are linear functions of known matrices, i.e.

$$Q = \sum_{i=1}^{N} \sigma_i^2 Q_i \text{ and } R = \sum_{i=1}^{N} \sigma_i^2 R_i \qquad (21)$$

where N is the number of variance components. Bélanger [1974] showed that the correlation function of the predicted residuals can be written as a *linear* function of the variance components

$$E\{\underline{v}_k \underline{v}_{k-l}^t\} = \sum_{i=1}^{N} \sigma_i^2 A_i(k,l) \quad . \tag{22}$$

One arrives at (22) by straightforward application of the Kalman filter update equations ((3) to (7)) to the predicted residuals (8), starting at time $k = 0$. The matrices $A_i(k,l)$ can be computed in a recursive manner along with the Kalman filter. One can estimate the variance components in a recursive manner (using a separate (secondary) Kalman filter) and obtain *real-time* estimates for the variance components.

The computational cost associated with such an algorithm does not seem prohibitive for, e.g., high precision navigation applications. The algorithm to estimate the variance components is not described here any further.

6 Practical Considerations

In the previous sections we discussed testing procedures and briefly outlined a method for variance component estimation. Test statistics and variance component estimates depend on the predicted residuals. As it is clear that the predicted residuals are influenced by misspecifications in the model, it follows that the estimates of the variance components are also heavily dependent on these misspecifications if they are not corrected for. Good estimates of variance components can only be obtained from "clean" datasets, i.e. datasets with all misspecifications removed. On the other hand the test statistics are only optimal if the stochastic model is correct (note that the test statistics depend on the variance components through the covariance matrix of the predicted residual Q_{v_k}). The test statistics and the variance component estimates are thus correlated. Although this seems to lower the prospects for a combined testing procedure and variance component estimation technique, some remarks can be made. The test statistics are based on the predicted residuals at a single fix or on a small batch of predicted residuals. The variance component estimates on the other hand are based on a large number of predicted residuals. If only the largest misspecifications can be removed from the dataset (these will be detected even if the model is not totally correct) one will still obtain better variance component estimates. After a number of variance component estimate updates these will probably be close enough to the true variance levels to warrant optimal characteristics of the test statistics. This is an area of current research.

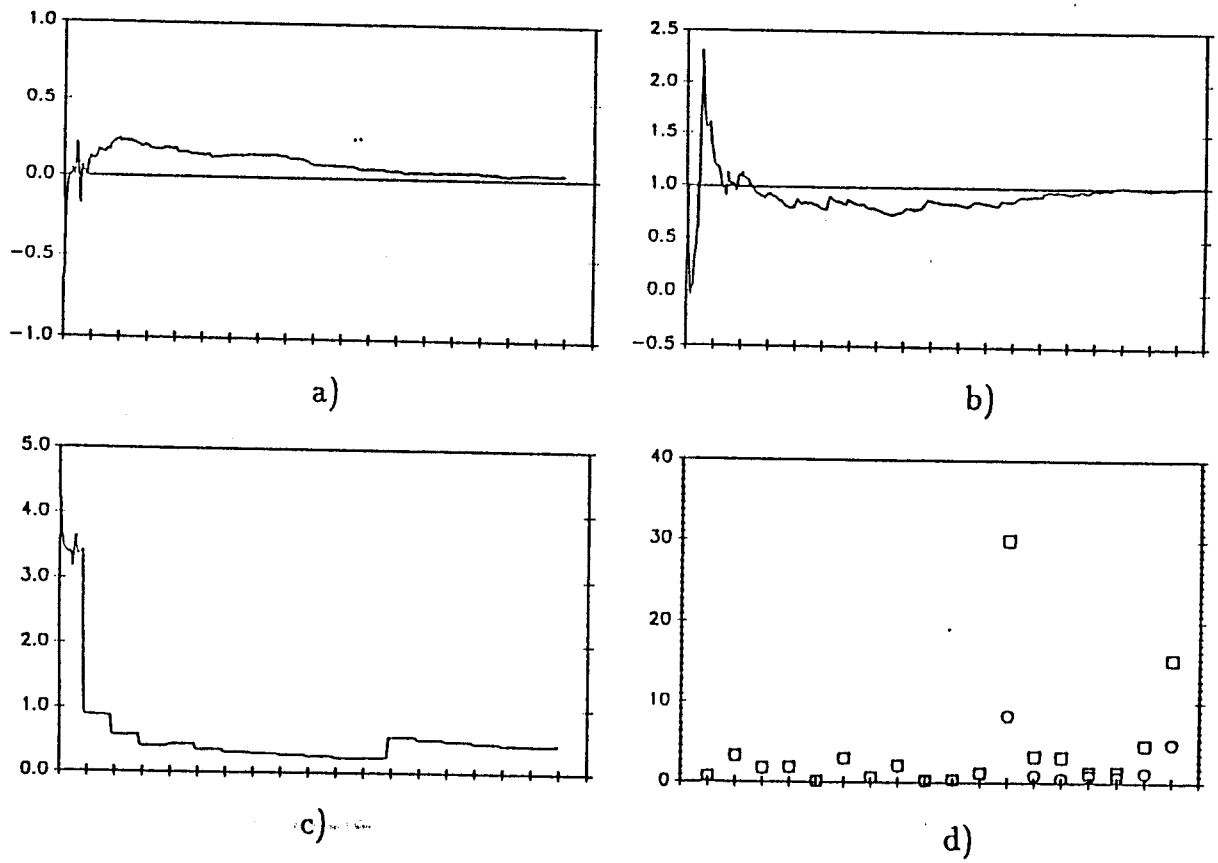

Figure 1. Simulation results; a) variance component estimate for the disturbances (dynamic noise); b) variance component estimate dead reckoning observables; c) variance component estimate position observable; d) LOM test statistics at fixes where a position observation is available, without reset (circles) and with reset (squares). A tickmark on the x-axis corresponds to 10 seconds.

Example

This example illustrates the dependency of the test statistics and variance component estimates. Observations are simulated for a vehicle running a straight track at 100 km/h. We assume dead reckoning observations available every second. A position update (e.g. GPS) is simulated every 10 seconds.

In our simulation we estimate variance components for the disturbances (dynamic noise), dead reckoning observables, and position observables. The a priori variance components used in the Kalman filter are specified correctly for the dead reckoning component and a factor four too large for the position observables. The a priori variance component for the dynamic noise is chosen in accordance with

a very rough track, and is thus specified far too large (an undisturbed track is simulated). Fig. 1a to 1c show the estimated variance components. The variance components for the dynamic noise (Fig. 1a) and dead reckoning observables (Fig. 1b) closely correspond to their expected values (0 and 1 respectively). The variance component of the position observables (Fig. 1c) seems to perform well, but displays a strange jump at the 12^{th} position fix. This is because we simulated an error of 100m in this position fix. It is clearly seen that a misspecification has an impact on the variance component estimate. What is worse, however, is that the misspecification is *not* detected. The circles in Fig. 1d give the sample values of the LOM test statistic. If the level of significance (α) is, e.g. 0.05, the critical value $\chi^2(0.05 : 4) = 9.49$ is never passed. Choosing a wrong a priori stochastic model thus impairs the performance of the tests.

If one had, for instance, reset the variance components to their estimated value at the 10^{th} position fix (where the variance estimate of the position observable is 0.237, remarkably close to its theoretical value of 1/4), the error would have been detected. The squares in Fig. 1d give the values of the LOM test statistic when a reset is applied at the 10^{th} position fix. Now the LOM test is clearly rejected at the 12^{th} fix ($30.4 \gg 9.49$). This example serves to demonstrate that careful tuning of the model (which can be achieved by applying methods described in this paper) is required to obtain optimal estimation results.

7 Concluding Remarks

In this paper we discussed some aspects of quality control in Kalman filtering. We paid particular attention to (slippage-type) test statistics. The test statistics derived are all functions of the predicted residuals. In Teunissen and Salzmann [1988, 1989] considerations regarding the implementation of global (slippage) tests are given. For the design of filters the detection power of the various tests should be taken into account. Reliability can be described by the marginal detectable errors.

To obtain optimal estimation results testing procedures and variance component estimation have to be applied simultaneously. The interdependence of the test statistics and the variance component estimates was discussed in Section 6 and was illustrated by an example. The adaptation of the filter model by variance component estimation and its relation with testing procedures is a field of current research.

References

Baarda W. (1968). A testing procedure for use in geodetic networks. *Neth. Geod.*

Comm., Publ. on Geodesy, New Series, Vol.2(5), Delft.

Bélanger P. (1974). Estimation of noise covariance matrices for a linear time-varying stochastic process, *Automatica* Vol.10, pp.267-275.

Gelb A. (ed.) (1974). *Applied Optimal Estimation.* MIT-Press, Cambridge MA.

Graybill F.A. (1976). *Theory and Application of the Linear Model.* Duxbury Press, North Scituate, MA.

Kalman R.E. (1960). A new approach to linear filtering and prediction problems. *ASME J. of Basic Engineering*, Vol. 82D, pp.34-45.

Koch, K.R. (1988). *Parameter Estimation and Hypothesis Testing in Linear Models.* Springer Verlag, Berlin.

Teunissen P.J.G. (1985). Quality control in geodetic networks. In: Grafarend, E. and F. Sansò (eds.), *Optimization and Design of Geodetic Networks*, Springer Verlag, Berlin, pp.526-547.

Teunissen P.J.G. and M.A. Salzmann (1988). Performance analysis of Kalman filters. Report 88.2, Section Mathematical and Physical Geodesy, Department of Geodesy, Delft University of Technology, Delft.

Teunissen P.J.G. and M.A. Salzmann (1989). A recursive slippage test for use in state-space filtering. Submitted for publication in *Manuscripta Geodaetica.*