TRIPLE-FREQUENCY IONOSPHERE-FREE PHASE COMBINATIONS FOR AMBIGUITY RESOLUTION

D. Odijk, P.J.G. Teunissen and C.C.J.M. Tiberius *

Abstract

Linear combinations of the carrier phase data which are independent of the ionospheric delays are referred to as ionosphere-free linear phase combinations. In the dual frequency case there exists only one such combination which at the same time ensures the integer estimability of the ambiguities. In the triple-frequency case there is a whole class of such linear combinations. We identify this class of linear combinations and determine their phase-only ambiguity resolution performance. The advantage of using carrier phase-only data is that ambiguity resolution will be freed from the potential presence of pseudorange multipath. We also identify an important pitfall when using ionosphere-free linear phase combinations. It is shown that not all such triple-frequency combinations permit a parametrization that retains the integer nature of the ambiguities. Results will be shown for triple-frequency Galileo as well as for modernized GPS.

1 Introduction

The ionosphere-free linear combination of L1 and L2 phase observables is often used in precise relative GPS positioning (cm accuracy or better) to process data of long baselines for which the (relative) ionospheric delays may not be neglected, and for which no a priori ionospheric information is available. In the literature it is often stated that this ionosphere-free phase combination is not suitable for fast GPS applications, claiming that the crucial integer property of the double-difference (DD) phase ambiguities is lost when forming the combination, see e.g. [*Hofmann-Wellenhof et al.*, 2001].

Although it is true that not all the original L1 and L2 ambiguities can be resolved, in this article it is shown that for the ionosphere-free combination it is possible to resolve a special integer linear combination of the L1 and L2 DD ambiguities. Also in the situation when phase observables at more than two frequencies are available, e.g. with a modernized GPS or Galileo system, it is possible to estimate integer ambiguity combinations, but since more than one ionosphere-free combination can be made, in that situation it becomes possible to estimate a whole class of integer combinations. In this contribution it is by means of planning computations shown which of the future GPS and Galileo ionosphere-free combinations are optimal for ambiguity resolution. In addition, the effect of ambiguity resolution on the precision of the coordinate parameters is also discussed.

It should be emphasized that in this contribution *only phase observations* are used and that the less precise code or pseudo-range data are not included. This immediately implies that the ionosphere-free combinations are not suitable for the ultra-fast instantaneous or single-epoch applications (at least two observation epochs are required in case of phase-only data). However, ambiguity resolution might still be beneficial in order to reduce the (long) time span, which would otherwise be necessary to obtain precise baseline coordinates based on the float ambiguities.

*Department of Mathematical Geodesy and Positioning, Delft University of Technology, The Netherlands, e-mail: d.odijk@geo.tudelft.nl

2 Ionosphere-free phase combinations

2.1 The dual-frequency case

Suppose we have phase observables available at two frequencies, denoted as Φ_f and Φ_g, in units of meters rather than cycles. In double-difference (DD) mode, their observation equations can in a compact way be written as:

$$\begin{cases} E\{\Phi_f\} &=& \rho \;+\; \lambda_f a_f \;-\; \imath_f \\ E\{\Phi_g\} &=& \rho \;+\; \lambda_g a_g \;-\; \imath_g \end{cases} \tag{1}$$

In these equations $E\{\cdot\}$ denotes the mathematical expectation, ρ the DD receiver-satellite range, λ_f and λ_g the wavelengths, a_f and a_g the integer DD phase ambiguities, and $\imath_f$ and $\imath_g$ the ionospheric delays. Note that when the observation equations above are linearized, the (baseline) coordinates, which are usually the parameters of interest, can be solved.

The ionospheric delay is dispersive, which means that the delay of Φ_g can be related to the delay of Φ_f via the known ratio of wavelengths of the two observables:

$$\imath_g \;=\; (\lambda_g^2/\lambda_f^2)\imath_f \tag{2}$$

When the two observables are ordered such that $\lambda_g > \lambda_f$, the wavelength ratio can be denoted as:

$$\frac{\lambda_g}{\lambda_f} \;=\; \frac{t}{n}, \qquad t > n \tag{3}$$

where both t and n are (positive) integers, since the wavelengths are derived from frequencies that are both derived from one nominal frequency. Using the wavelength ratio, the ionosphere-free linear combination of the two observables is obtained as:

$$\begin{aligned} E\{\Phi_{fg}\} &= \tfrac{t^2}{t^2-n^2}E\{\Phi_f\} \;-\; \tfrac{n^2}{t^2-n^2}E\{\Phi_g\} \\ &= \rho \;+\; \tfrac{t^2}{t^2-n^2}\lambda_f a_f - \tfrac{n^2}{t^2-n^2}\lambda_g a_g \;-\; \underbrace{\left(\frac{t^2}{t^2-n^2} - \frac{n^2}{t^2-n^2}\frac{t^2}{n^2}\right)}_{0}\imath_f \end{aligned} \tag{4}$$

where the coefficients are chosen such, that in the transformed observable the range ρ appears in the same way as in the original phase observation equations. Moreover, in this transformed observation equation it can be seen that the ionospheric delays are eliminated, and that a combined ambiguity term remains, which does not seem to be integer-valued. However, using $\lambda_g = \frac{t}{n}\lambda_f$ with t and n integers, it is possible to rewrite the ambiguity term, such that it has the integer property:

$$\boxed{E\{\Phi_{fg}\} \;=\; \rho \;+\; \underbrace{\frac{t}{t^2-n^2}\lambda_f}_{\lambda_{fg}}\underbrace{(ta_f - na_g)}_{a_{fg}}, \qquad a_{fg} \in Z} \tag{5}$$

where λ_{fg} denotes the artificial wavelength and a_{fg} the integer ambiguity of the ionosphere-free combination. So with the ionosphere-free phase combination it is possible to estimate integer ambiguities, in contrast to what is often stated in GPS literature.

A well-known consequence of taking the ionosphere-free combination is that the noise of the ionosphere-free observable is increased compared to the noise of the original phase observations. When it is assumed that the two original phase observables are uncorrelated and have the same precision, $\sigma_{\Phi_f} = \sigma_{\Phi_g} = \sigma_\Phi$ (in DD mode), the variance of the ionosphere-free combination is computed as follows:

$$\boxed{D\{\Phi_{fg}\} \;=\; \frac{t^4+n^4}{(t^2-n^2)^2}\sigma_\Phi^2} \tag{6}$$

where $D\{\cdot\}$ denotes the mathematical dispersion.

There is one issue about the wavelength ratio in equation (3) which needs to be addressed. It is often possible to divide numerator t and denominator n by the same integer, in order to obtain smaller entries. For ambiguity resolution it is very important that these smaller entries are indeed taken for t and n, since this results in a longer artificial wavelength. In fact, in order to obtain the smallest numerator and denominator possible, t and n should be divided by their *greatest common divisor*. This can be explained as follows. Denoting this greatest common divisor as c, we may write for the numerator and the denominator of the wavelength ratio $t = c \cdot t_c$ and $n = c \cdot n_c$, where $c \geq 1, \quad c \in N$. Inserting this in the equations (5) and (6), results in the following model for the ionosphere-free combination:

$$\left\{ \begin{array}{lllll} E\{\Phi_{fg}\} & = & \rho + \frac{c \cdot t_c}{c^2 t_c^2 - c^2 n_c^2} \lambda_f (c \cdot t_c a_f - c \cdot n_c a_g) & = & \rho + \frac{t_c}{t_c^2 - n_c^2} \lambda_f (t_c a_f - n_c a_g) \\ D\{\Phi_{fg}\} & = & \frac{c^4 t_c^4 + c^4 n_c^4}{(c^2 t_c^2 - c^2 n_c^2)^2} \sigma_\Phi^2 & = & \frac{t_c^4 + n_c^4}{(t_c^2 - n_c^2)^2} \sigma_\Phi^2 \end{array} \right. \tag{7}$$

So instead of the integer ambiguity combination $t a_f - n a_g$ the combination $t_c a_f - n_c a_g$ is resolved, which is also an integer, since t_c and n_c are integers as well. The (artificial) wavelength of the first combination is $\frac{t}{t^2 - n^2} \lambda_f$, whereas the wavelength corresponding to the second set is $\frac{t_c}{t_c^2 - n_c^2} \lambda_f$. Since it holds that $\frac{t_c}{t_c^2 - n_c^2} = c \cdot \frac{t}{t^2 - n^2}$ with $c \geq 1$, it follows that $\frac{t_c}{t_c^2 - n_c^2} \geq \frac{t}{t^2 - n^2}$, implying that the artificial wavelength of the second combination is longer than the wavelength of the first combination. It should be stressed that only the precision of the ambiguities is influenced by this longer wavelength. The precision of the ionosphere-free combination itself and the precision of the baseline coordinates turn out to be insensitive for leaving out the greatest common divisor or not, since for the variance factor of the ionosphere-free combination it holds that $\frac{t_c^4 + n_c^4}{(t_c^2 - n_c^2)^2} = \frac{t^4 + n^4}{(t^2 - n^2)^2}$.

As example, consider the current GPS L1 and L2 frequencies, which are 154 times respectively 120 times the nominal frequency of 10.23 MHz (see also Table 1). For the ionosphere-free combination L1/L2 however not the ratio 154/120 should be taken, but the ratio 77/60, since the greatest common divisor of 154 and 120 is 2. The mathematical model for the L1/L2 ionosphere-free combination reads:

$$\left\{ \begin{array}{lllll} E\{\Phi_{12}\} & = & \rho + \frac{154}{154^2 - 120^2} \lambda_1 (154 a_1 - 120 a_2) & = & \rho + \frac{77}{77^2 - 60^2} \lambda_1 (77 a_1 - 60 a_2) \\ D\{\Phi_{12}\} & = & \frac{154^2 + 120^4}{(154^2 - 120^2)^2} \sigma_\Phi^2 & = & \frac{77^4 + 60^4}{(77^2 - 60^2)^2} \sigma_\Phi^2 \end{array} \right. \tag{8}$$

So instead of the integer combination $154a_1 - 120a_2$, the combination $77a_1 - 60a_2$ should be resolved, since it has a wavelength which is a factor 2 longer than the wavelength of the first combination. The factor with which the standard deviation σ_Φ is multiplied, remains $\sqrt{\frac{154^2 + 120^4}{(154^2 - 120^2)^2}} = \sqrt{\frac{77^4 + 60^4}{(77^2 - 60^2)^2}} \approx 2.98$.

2.2 The triple-frequency case

When phase observables at three frequencies are available, there is unfortunately not one unique ionosphere-free combination to be made. Since for the elimination of the ionospheric delay two frequencies are sufficient, it is possible to draft *three* different dual-frequency ionosphere-free combinations for this purpose, instead of just one in the dual-frequency case, which may be processed together in one integral adjustment. On the other hand it is also possible to form just *one* truly triple-frequency ionosphere-free combination. In this subsection we will consider these alternatives in detail.

2.2.1 Forming three combinations of two frequencies

Suppose we have the following phase observables available, each at a different frequency: Φ_f, Φ_g, and Φ_h. Based on combinations of two observables, like was done in equation (5), three dual-frequency ionosphere-free combinations can be made: Φ_{fg}, Φ_{fh} and Φ_{gh}. These three combinations could be processed simultaneously in order to solve for the baseline coordinates and ambiguities.

However, one of the three dual-frequency combinations is unfortunately not independent, since it can be exactly constructed from the other two combinations. For example, when the wavelength ratios for the three observables are denoted as $\lambda_g/\lambda_f = t_g/n_g$ and $\lambda_h/\lambda_f = t_h/n_h$, it can be proved that Φ_{gh} can be written as a linear combination of Φ_{fg} and Φ_{fh}:

$$\Phi_{gh} \;=\; \frac{t_h^2(t_g^2-n_g^2)}{t_h^2(t_g^2-n_g^2)-t_g^2(t_h^2-n_h^2)}\Phi_{fg} \;-\; \frac{t_g^2(t_h^2-n_h^2)}{t_h^2(t_g^2-n_g^2)-t_g^2(t_h^2-n_h^2)}\Phi_{fh} \tag{9}$$

Processing the three dual-frequency combinations together, would yield a too optimistic precision of the unknown parameters, since it is assumed that there is more information present in the original phase observables than there really is. A more realistic result is obtained when *two* of the three ionosphere-free combinations are processed in one adjustment. For example, when we choose Φ_{fg} and Φ_{fh} as two independent combinations, the following functional model can be set up:

$$\begin{aligned} E\{\begin{bmatrix} \Phi_{fg} \\ \Phi_{fh} \end{bmatrix}\} \;&=\; \begin{bmatrix} \frac{t_g^2}{t_g^2-n_g^2} & -\frac{n_g^2}{t_g^2-n_g^2} & 0 \\ \frac{t_h^2}{t_h^2-n_h^2} & 0 & -\frac{n_h^2}{t_h^2-n_h^2} \end{bmatrix} E\{\begin{bmatrix} \Phi_f \\ \Phi_g \\ \Phi_h \end{bmatrix}\} \\ &=\; \begin{bmatrix} 1 \\ 1 \end{bmatrix}\rho \;+\; \begin{bmatrix} \lambda_{fg} & \\ & \lambda_{fh} \end{bmatrix}\begin{bmatrix} a_{fg} \\ a_{fh} \end{bmatrix}, \quad a_{fg},\; a_{fh} \;\in\; Z \end{aligned} \tag{10}$$

We could also have chosen two other combinations, e.g. Φ_{fg} and Φ_{gh}, to set up the functional model. For the coordinate solution (obtained after linearization of ρ) it makes fortunately no difference which combinations are selected, as long as these are *two* combinations for which a proper *stochastic* model or variance-covariance (vc-) matrix is used. Important is that the (mathematical) *correlation* between the ionosphere-free combinations is accounted for, since the two combinations are each based on a common phase observable. When it is assumed that the original phase observables are uncorrelated and have an equal precision, the vc-matrix of the combinations Φ_{fg} and Φ_{fh} reads:

$$D\{\begin{bmatrix} \Phi_{fg} \\ \Phi_{fh} \end{bmatrix}\} \;=\; \sigma_\Phi^2 \begin{bmatrix} \frac{t_g^4+n_g^4}{(t_g^2-n_g^2)^2} & \frac{t_g^2 t_h^2}{(t_g^2-n_g^2)(t_h^2-n_h^2)} \\ \frac{t_h^2 t_g^2}{(t_h^2-n_h^2)(t_g^2-n_g^2)} & \frac{t_h^4+n_h^4}{(t_h^2-n_h^2)^2} \end{bmatrix} \tag{11}$$

In contrast to the coordinate solution, for the ambiguity solution it makes a difference which two combinations are chosen, since different sets of ionosphere-free combinations introduce different sets of estimable integer ambiguities. In section 3 it is explained which combinations should be chosen for a future triple-frequency GPS system, while in section 4 for a triple-frequency Galileo these combinations are presented.

2.2.2 Forming one combination of three frequencies

Instead of using combinations of two frequencies, it is also possible to form ionosphere-free combinations that are linear combinations of all three observables. A triple-frequency phase observables which preserves the integer ambiguity property can be obtained as follows:

$$\left\{ \begin{aligned} E\{\Phi_{fgh}\} \;&=\; \frac{t_g^2+t_h^2}{(t_g^2-n_g^2)+(t_h^2-n_h^2)}E\{\Phi_f\} - \frac{n_g^2}{(t_g^2-n_g^2)+(t_h^2-n_h^2)}E\{\Phi_g\} - \frac{n_h^2}{(t_g^2-n_g^2)+(t_h^2-n_h^2)}E\{\Phi_h\} \\ &=\; \rho + \underbrace{\frac{1}{(t_g^2-n_g^2)+(t_h^2-n_h^2)}\lambda_f}_{\lambda_{fgh}} \underbrace{\left[(t_g^2+n_g^2)a_f - t_g n_g a_g - t_h n_h a_h\right]}_{a_{fgh}},\; a_{fgh}\in Z \\ D\{\Phi_{fgh}\} \;&=\; \frac{(t_g^2+t_h^2)^2+n_g^4+n_h^4}{[(t_g^2-n_g^2)+(t_h^2-n_h^2)]^2}\sigma_\Phi^2 \end{aligned} \right. \tag{12}$$

In order to make the wavelength of this combination as long as possible, the integer ambiguity a_{fgh} should be divided by the greatest common divisor of the three integers $(t_g^2 + n_g^2)$, $t_g n_g$ and $t_h n_h$, whereas the wavelength λ_{fgh} should be multiplied with it. Examples of other triple-frequency ionosphere-free combinations can be found in [*Han and Rizos*, 1999].

Although integer estimation is possible, these type of ionosphere-free combinations do not preserve the full information content in the three original phase observables. From a strict point of view namely, two frequencies are sufficient to eliminate the ionospheric delay, while the remaining frequency acts as redundant observable. In this case however, no redundant observable remains. Therefore, these type of triple-frequency ionosphere-free combinations are not discussed further in this paper.

3 The modernized GPS case

In this section we will take a close look at the ionosphere-free combinations of a modernized GPS with triple-frequency phase observables. In Table 1 the three GPS signals are summarized.

Table 1: Modernized GPS signals.

carrier signal	notation	frequency (MHz)	wavelength (cm)
L1	Φ_1	154 $\times$ 10.23 = 1575.420	19.03
L2	Φ_2	120 $\times$ 10.23 = 1227.600	24.42
L5	Φ_3	115 $\times$ 10.23 = 1176.450	25.48

From Table 1 the GPS wavelength ratios, divided by their greatest common divisors, follow as:

$$\frac{\lambda_2}{\lambda_1} = \frac{154}{120} = \frac{77}{60}, \qquad \frac{\lambda_3}{\lambda_1} = \frac{154}{115}, \qquad \frac{\lambda_3}{\lambda_2} = \frac{120}{115} = \frac{24}{23} \tag{13}$$

So for the ratio λ_2/λ_1 the greatest common divisor is 2, for λ_3/λ_1 it is 1, while for λ_3/λ_2 it is 5.

Using these wavelength ratios, three dual-frequency ionosphere-free combinations can be formed, which are besides the classical L1/L2 combination, the L2/L5 and L1/L5 combinations. In Table 2 these three ionosphere-free combinations are given, together with their artificial wavelengths, estimable integer ambiguity parameters and the factor with which the standard deviation of the original phase observables needs to be multiplied in order to get the standard deviation of the ionosphere-free combination.

Table 2: Possible dual-frequency ionosphere-free combinations for (modernized) GPS.

obs.	lin. comb.	wavelength	est. ambiguities	std. factor
L1/L2	$2.5457\Phi_1$ - $1.5457\Phi_2$	0.63 cm	$77a_1$ - $60a_2$	2.98
L2/L5	$12.2553\Phi_2$ - $11.2553\Phi_3$	12.47 cm	$24a_2$ - $23a_3$	16.64
L1/L5	$2.2606\Phi_1$ - $1.2606\Phi_3$	0.28 cm	$154a_1$ - $115a_3$	2.59

Instead of using two out of the three dual-frequency ionosphere-free combinations, as given in Table 2, one can also process just one dual-frequency combination in a modernized GPS situation. Instead of the current L1/L2 combination, one might take the ionosphere-free combination of the L2 and L5 phase observables, since it has a much longer wavelength (about 12 cm) than the current combination (0.63 cm), resulting in more precise ambiguities. However, the precision of the L2/L5 combination is much worse: it has a multiplication factor of almost 17 versus the well-known factor 3 of the current L1/L2 combination, which will have its deteriorating effect on the final baseline precision.

The full information content in the triple-frequency phase observables is preserved when two out of the three ionosphere combinations in Table 2 are processed in one adjustment. In that case the correlation between the two combinations needs to be taken into account, see Section 2.2. There are three two-combination sets possible, i.e. the L1/L2-L2/L5, L1/L2-L1/L5 or L1/L5-L2/L5 sets. For the purpose of ambiguity resolution these three sets are unfortunately not equivalent: in [*Teunissen and Odijk*, 2002] it is from a strict point of view shown that the L1/L2-L2/L5 set is admissible, while the other two sets are not, since their estimable ambiguity sets cannot be obtained from the ambiguities of the L1/L2-L2/L5 set using an *admissible* transformation. For example, the ambiguities of the L1/L2-L1/L5 set are transformed from the L1/L2-L2/L5 set as follows:

$$\begin{bmatrix} 77a_1 - 60a_2 \\ 154a_1 - 115a_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix}}_{Z} \begin{bmatrix} 77a_1 - 60a_2 \\ 24a_2 - 23a_3 \end{bmatrix} \tag{14}$$

In order for the above transformation to be admissible, the matrix between Z the two sets should fulfil two conditions [*Teunissen*, 1995]: i) it should have all integer entries, and ii) its determinant should equal ± 1. It can be seen from equation (14) that the first condition is fulfilled, though not the second, since the determinant of matrix Z is equal to 5. Hence, the transformation is not admissible. A similar conclusion reads for the L1/L5-L2/L5 set.

Of course, one should realize that when one has decided *not* to carry out ambiguity resolution and relies on the float ambiguity solution, the sketched pitfall does not hold. In that situation it is allowed to use the two other ionosphere-free sets, since the ambiguity parameterization does not affect the baseline solution.

4 The Galileo case

In a similar way as for modernized GPS, also for the future European Galileo system it is possible to investigate the ionosphere-free phase combinations and their estimable integer ambiguity sets. Since the Galileo frequencies and signals are still tentative [*Hein et al.*, 2001], in this paper two different triple-frequency scenarios for the Galileo carrier signals are considered, denoted as (a) and (b), see Table 3.

Table 3: Two possible Galileo scenarios.

scenario	carrier signal	notation	frequency (MHz)	wavelength (cm)
(a)	E1(-L1-E2)	Φ_1	154 $\times$ 10.23 = 1575.420	19.03
	E6	Φ_2	125 $\times$ 10.23 = 1278.750	23.44
	E5b	Φ_3	117.5 $\times$ 10.23 = 1202.025	24.94
(b)	E1(-L1-E2)	Φ_1	154 $\times$ 10.23 = 1575.420	19.03
	E5b	Φ_2	117.5 $\times$ 10.23 = 1202.025	24.94
	E5a	Φ_3	115 $\times$ 10.23 = 1176.450	25.48

Note that the only difference in both scenarios is that in scenario (a) the E6 signal is included, while in scenario (b) this carrier is replaced by the E5a signal. Moreover, note that this E5a signal overlays the GPS L5 signal, and that the E1-L1-E2 frequency equals the GPS L1 frequency. In the sequel, the E1-L1-E2 signal is denoted as E1.

4.1 Scenario (a): E1/E6/E5b

We first consider the ionosphere-free combinations for Galileo scenario (a). In this case the wavelength ratios are given as:

$$\frac{\lambda_2}{\lambda_1} = \frac{154}{125}, \qquad \frac{\lambda_3}{\lambda_1} = \frac{1540}{1175} = \frac{308}{235}, \qquad \frac{\lambda_3}{\lambda_2} = \frac{1250}{1175} = \frac{50}{47} \tag{15}$$

Using these ratios, in Table 4 the three dual-frequency ionosphere-free combinations are given.

Table 4: Possible dual-frequency ionosphere-free combinations for Galileo (a)

obs.	lin. comb.	wavelength	est. ambiguities	std. factor
E1/E6	$2.9312\Phi_1 - 1.9312\Phi_2$	0.36 cm	$154a_1 - 125a_2$	3.51
E6/E5b	$8.5911\Phi_2 - 7.5911\Phi_3$	4.03 cm	$50a_2 - 47a_3$	11.46
E1/E5b	$2.3932\Phi_1 - 1.3239\Phi_3$	0.15 cm	$308a_1 - 235a_3$	2.77

Compared to the GPS ionosphere-free combinations in Table 2, also in this case there are two combinations with a rather short wavelength and small precision factor, and one combination

(E6/E5b) with a longer wavelength, but large precision factor. However, the longest wavelength of about 4 cm is significantly smaller than the 12 cm of the GPS L2/L5 combination.

When all three Galileo (a) signals are used, like with GPS there is only one set of two dual-frequency ionosphere-free combinations which is optimal for ambiguity resolution, this is the E1/E6-E6/E5b set. The estimable ambiguities for the other two possible triple-frequency sets cannot be obtained from the ambiguity set of E1/E6-E6/E5b by an admissible transformation.

4.2 Scenario (b): E1/E5b/E5a

For the Galileo scenario (b) the three wavelength ratios read:

$$\frac{\lambda_2}{\lambda_1} = \frac{308}{235}, \qquad \frac{\lambda_3}{\lambda_1} = \frac{154}{115}, \qquad \frac{\lambda_3}{\lambda_2} = \frac{1175}{1150} = \frac{47}{46} \tag{16}$$

The three dual-frequency ionosphere-free combinations for this scenario are summarized in Table 5.

Table 5: Possible dual-frequency ionosphere-free combinations for Galileo (b)

obs.	lin. comb.	wavelength	est. ambiguities	std. factor
E1/E5b	$2.3932\Phi_1$ - $1.3239\Phi_2$	0.15 cm	$308a_1$ - $235a_2$	2.77
E5b/E5a	$23.7527\Phi_2$ - $22.7527\Phi_3$	12.60 cm	$47a_2$ - $46a_3$	32.89
E1/E5a	$2.2606\Phi_1$ - $1.2606\Phi_3$	0.28 cm	$154a_1$ - $115a_3$	2.59

Note from the table that for this scenario the E1/E5b combination appears, which also was one of the combination in scenario (a). Besides, the E1/E5a combination equals the GPS L1/L5 combination. The third dual-frequency combination, E5b/E5a, has not appeared so far. This is a combination with a wavelength of about 13 cm, which is much longer than the longest wavelength in Galileo scenario (a) and compares to the wavelength of the GPS L2/L5 combination. Its noise level is however very bad, considering the factor of about 33 with which the standard deviation needs to be multiplied.

For this scenario the set of two dual-frequency combinations which needs to be processed for optimal triple-frequency ambiguity resolution, is the E5b/E5a-E1/E5a set.

5 Evaluating ambiguity success-rates and baseline precision

In this section the expected performance of ambiguity resolution and the expected fixed baseline precision with the modernized GPS and Galileo ionosphere-free phase combinations are discussed. To measure the performance of ambiguity resolution the probability of correct integer estimation, that is the *ambiguity success-rate* [*Teunissen*, 1998], is used. Both ambiguity success-rate and baseline precision can be evaluated without collecting real observations, since they are only based on the assumptions as embedded in the mathematical model.

In all computations it is assumed that the DD ambiguities remain constant during the complete time span, such that advantage is taken from the changing receiver-satellite geometry. This receiver-satellite geometry was simulated for a location in the Netherlands at ($51°58'\ N$, $5°51'\ E$). To compute the positions of the GPS satellites and to simulate the positions of the Galileo satellites, a YUMA almanac was used, in the same way as was done for the GPS/Galileo computations as described in [*Eissfeller et al.*, 2001]. For both constellations 7 satellites were used, continuously tracked during a one hour time span from 03.00 to 04.00 UTC on January 19th, 2001, with a sampling-interval of 10 sec, and all satellites above 10° cut-off elevation. In the simulations also a tropospheric zenith delay parameter was introduced for the entire time span, for which the mapping coefficients were computed using the simple $1/\sin e$ mapping function, with e the elevation angle. The standard deviation of all phase observations was set at 2 mm (undifferenced).

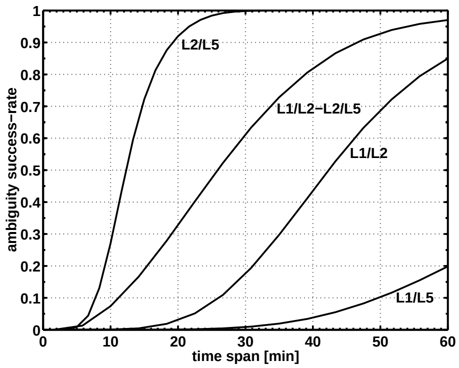

Figure 1: Ambiguity success-rates for the GPS ionosphere-free combinations.

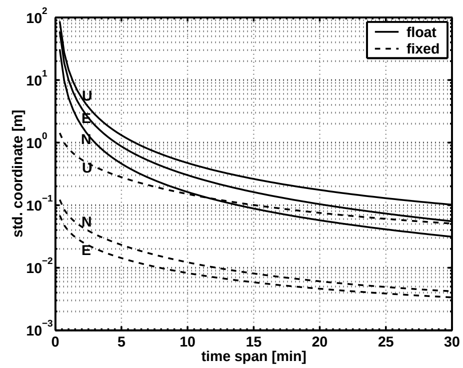

Figure 2: Baseline precision for the GPS L2/L5 ionosphere-free combination.

5.1 GPS results

In Figure 1 the ambiguity success-rates are plotted as function of the time span, for the GPS ionosphere-free combinations, as treated in Section 3. Besides the current L1/L2 combination, success-rate curves are plotted for the L1/L5 and L2/L5 dual-frequency combinations, plus for the combined L1/L2-L2/L5 set, which is the optimal triple-frequency set.

The figure shows several interesting things. Although integer parameterization is possible for the current L1/L2 ionosphere-free combination, even the use of a rather long time span of one hour does not seem to be sufficient to reliably resolve the integers. This is of course due to its relatively short wavelength (0.63 cm). With a modernized GPS system however, the L2/L5 combination seems to perform much better than the two other dual-frequency combinations, which could already be expected because of its relatively long wavelength. In this example after about 30 minutes the success-rate is very close to 1. This L2/L5 combination performs also much better than the triple-frequency L1/L2-L2/L5 set, for which the ambiguity success-rate after one hour is, although larger than for the current L1/L2 combination, not close enough to 1.

This small analysis suggests that in a modernized triple-frequency GPS situation it is better to use the L2/L5 combination only and not the integrated L1/L2-L2/L5 combinations. However, ambiguity resolution is not the end of the story, since one is usually interested in baseline coordinates estimated with the integer ambiguities fixed. From Section 3 we know that the noise level of the L2/L5 combination is about a factor 17, whereas for the current L1/L2 combination this is about 3, and this will have a proportional effect on the final baseline precision. To investigate to what size this large factor influences the level of the baseline precision, in Figure 2 for the L2/L5 combination the float and fixed baseline standard deviations (expressed in North, East and Up components) are plotted for the time span 30 minutes. At the end of this time span, in order for the ambiguity resolution to make sense, it is required that the fixed baseline precision is significantly better than its float counterpart, and that it is of an acceptable level. From the figure it can be inferred that after 30 minutes the float baseline precision is still at dm-level, while its fixed counterpart is much better, at sub-cm level, but only for the horizontal components. The precision of the fixed height component is only marginally better than its float counterpart (dm-level). This rather poor height precision is related to the estimation of a tropospheric zenith delay for the time span. Despite this, when one is mainly interested in the horizontal position it might be worthwhile to resolve the ambiguities using the L2/L5 combination only.

Although the fixed baseline precision using the L1/L2-L2/L5 set is better than using the L2/L5 set, when the same time span is used (since in the latter case one ionosphere-free observable less is available), there is no need to estimate the fixed baseline precision in the first case, since one has to wait so long before the ambiguity success-rate is close enough to 1. In Figure 1 one can see that for the example this takes more than one hour. Within this time span, the *float* baseline precision has already reached the sub-cm level, see Figure 3, which shows the float standard deviations as

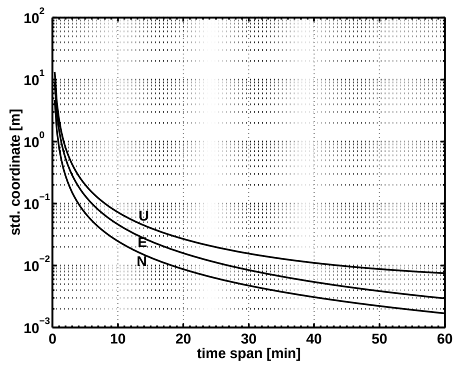

Figure 3: Float baseline precision for the set of L1/L2-L2/L5 ionosphere-free combinations.

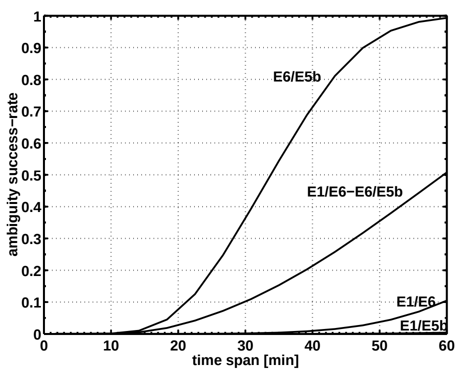

Figure 4: Ambiguity success-rates for the Galileo (a) ionosphere-free combinations.

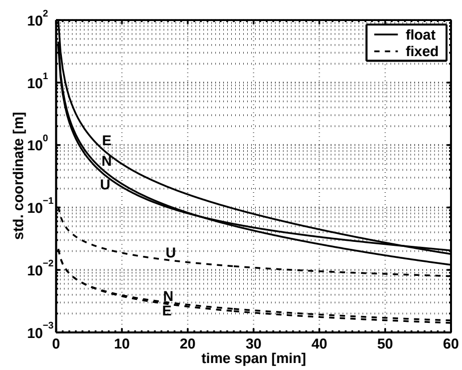

Figure 5: Baseline precision for the Galileo (a) E6/E5b ionosphere-free combination.

function of the time span, in this case 60 minutes.

5.2 Galileo results

In Figures 4 and 6 the ambiguity success-rates for the dual-frequency and optimal triple-frequency Galileo ionosphere-free combinations are plotted for the one hour time span, in the same manner as was done for the GPS combinations. A more or less similar behavior as with GPS is visible: the success-rate of the dual-frequency combination with the longest wavelength approaches 1 faster than all other combinations. For the Galileo (a) scenario this is the E6/E5b combination and for Galileo (b) this is the E5b/E5a combination. Comparing these two combinations, it is striking that the success-rates for both combinations have an approximately equal behavior in time. This similar behavior can be explained when the wavelengths of the combinations are considered, in relation to their noise level: the wavelength of the E5b/E5a combination (12 cm) is a factor 3 longer than for the E6/E5a combination (4 cm), but its precision level is a factor 3 worse than the level of E6/E5a (standard deviation factor 11 versus 33, see Tables 4 and 5). For the success-rates for both E6/E5b and E5b/E5a combinations however a much longer time span is required to be close to 1 than for the GPS L2/L5 dual-frequency combination. For the GPS combination this is for this example about 30 minutes, whereas for both Galileo combinations a time span about twice as long is required. For the horizontal baseline precision however, fixing of the integer ambiguities of both Galileo combinations still makes sense: it is at sub cm-level, whereas the float baseline precision lies only at sub dm-level, see Figures 5 and 7. The precision of the height component does not benefit much from ambiguity resolution using the one hour time span.

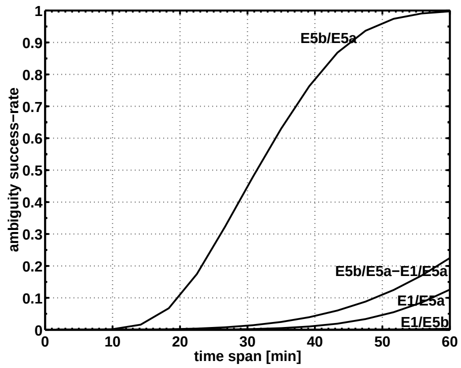

Figure 6: Ambiguity success-rates for the Galileo (b) ionosphere-free combinations.

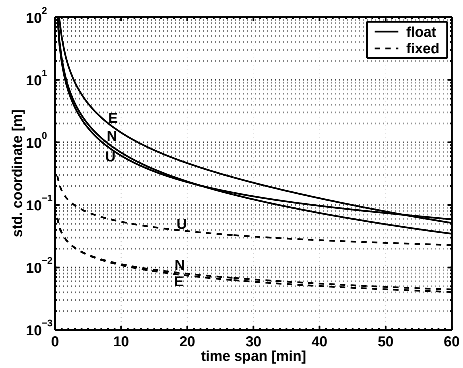

Figure 7: Baseline precision for the Galileo (b) E5b/E5a ionosphere-free combination.

6 Conclusions

In this article it has been shown that integer ambiguity resolution is possible for ionosphere-free combinations based on carrier phase-only data. Although not for the very fast applications, ambiguity resolution may improve the float (horizontal) baseline precision significantly, this is especially true for the L2/L5 combination in a modernized GPS situation. It is preferred to use this dual-frequency combination over the triple-frequency two-combination set L1/L2-L2/L5, since ambiguity resolution for this latter set requires a much longer time span and does then not improve the float baseline solution much. For Galileo, the dual-frequency ionosphere-free combinations E6/E5b in one scenario and E5b/E5a in another assumed scenario are expected to perform about the same for ambiguity resolution, though worse than the GPS L2/L5 combination. However, also ambiguity resolution for these Galileo combinations may result in much more precise final coordinate solutions than their ambiguity-float counterparts within the same time span.

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