GNSS Three Carrier Phase Ambiguity Resolution using the LAMBDA-method

Peter Joosten, Peter J.G. Teunissen and Niels Jonkman
Mathematical Geodesy and Positioning
Delft University of Technology
The Netherlands
Email: P.Joosten@geo.tudelft.nl

Abstract

The European second-generation satellite navigation system GNSS-2 (or ‘‘Galileo”) is intended to transmit signals on three navigational frequencies. It is expected that this will enhance possibilities in the area of high precision, real time navigation and positioning. This requires the use of phase observations, as well as a method to resolve the ambiguities which are inherent to these phase measurements. Although already a considerable amount of effort has been devoted to the development of methods to solve this ambiguity resolution problem in case of three frequencies, the well-known LAMBDA-method could be used without any modification, since this method does not have an inherent restriction on the number of frequencies. In this contribution, it will be shown that the LAMBDA-method can be used to resolve carrier phase ambiguities in the case of three frequency satellite navigation systems, in exactly the same way as it is currently being used in case of GPS.

Introduction

Precise navigation and fast surveying require the use of carrier phase measurements. Unfortunately GNSS carrier phase measurements are ambiguous, which implies that, in order to be able to utilize the phase measurements, these ambiguities need to be resolved successfully. One of the advantages of being able to use a third frequency would be that ambiguity resolution becomes feasible for longer baselines and/or shorter time-spans.

With the announced development of positioning systems using three frequencies (modernized GPS, Galileo), already some work has been carried out concentrating on the ambiguity resolution problem. Some of this work seems to indicate, that one would need to develop new methods of integer estimation and new algorithms to be able to deal with the availability of the additional third frequency. This is not true however. It is the purpose of this contribution to accentuate that the LAMBDA-method, being widely used for single- and dual-frequency GPS processing, is applicable and suitable for a three carrier system as well, without any modification to the method at all.

GNSS ambiguity estimation

Ambiguity resolution is the key to fast and high-precision relative positioning, both in the case of current GPS and future GNSS. The reason for this is that once the ambiguities have been resolved as integers, the phase measurements start to act as if they were high-precision range measurements, thereby allowing the parameters of interest (e.g. baseline-coordinates) to be estimated with a comparable high precision. Ambiguity resolution applies to a great variety of GPS models currently in use. They range from single-baseline models for kinematic positioning to multi-baseline models used as a tool for studying geodynamical phenomena. Models may have the relative satellite-receiver geometry included or excluded. Receivers may be stationary or in motion. Atmospheric delays may be included in the model as unknowns, making the model suitable for long baselines, or they may be excluded from the model, making the model suitable for relatively short baselines.

Ambiguity resolution is important for all these models, and it will be equally important for future GNSS models. Like the current GPS models, GNSS models can all be cast in the following conceptual frame of linear(ized) observation equations:

$$E\{y\} = Aa + Bb;\ D\{y\} = Q_y$$

where $E\{.\}$ and $D\{.\}$ denote the mathematical expectation and dispersion respectively and where y is the vector of observables, a and b are unknown parameter vectors, A and B are the corresponding design matrices, functionally relating the observations to the unknowns and Q_y is the variance-covariance (vc-)matrix of the observations. In this contribution we will assume that the entries in vector a are double differenced (DD) carrier phase ambiguities, expressed in units of cycles. They are known to be integers. The entries of vector b are the remaining unknown parameters, which can be baseline components (''coordinates'') and possibly atmospheric delay parameters (tropospheric and/or ionospheric delays).

As far as estimation of the above mentioned model is concerned, a three-step procedure may be followed. In a first step, the integer character of the ambiguities is discarded, and a standard least-squares adjustment is carried out. This provides us with real-valued estimates, referred to as the 'float solution', for a and b, together with their vc-matrix, denoted as:

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix};\ \begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix}$$

In a second step, the float ambiguity estimate $\hat{a}$ is used to compute the corresponding integer ambiguity estimate, here referred to as $\breve{a}$. In the third step this integer estimate is used to correct the float solution estimate $\hat{b}$. As a result one obtains the final baseline estimate:

$$\breve{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \breve{a})$$

The integer estimation part of this three-step procedure essentially boils down to applying a mapping function from the n-dimensional space of real values to the n-dimensional space of integer values. Several possibilities are available here, such as a simple rounding of the ambiguities, a conditional rounding of the ambiguities (bootstrapping), or an integer least-squares mapping of the ambiguities. These different methods of integer estimation will also have different probabilities for successful ambiguity resolution. These success-rates were derived in [7] and [8] while it was proven for the first time in [9] that the largest success-rate is obtained when using the integer least-squares estimator. The integer least-squares estimator is thus optimal in the sense that it maximizes the probability of correct integer ambiguity estimation. It is this optimal estimator which is efficiently mechanized in the LAMBDA-method.

The LAMBDA-method

The LAMBDA-method was introduced in [6] in 1993. The method can be characterized in brief as an application-independent method that produces an optimal integer ambiguity solution by means of efficiently solving the integer least-squares problem through the use of a decorrelation process. The integer least-squares problem which is solved by the LAMBDA-method reads as

$$\min \| \hat{a} - a \|^2_{Q_{\hat{a}}}\ ;\quad \text{with } \|.\|^2_{Q_{\hat{a}}} = (.)^T Q_{\hat{a}}^{-1} (.) \text{ and } a \in Z^n \text{ (n} - \text{dimensional integer space)}$$

The LAMBDA-method needs as input the ambiguity float vector $\hat{a}$ and its ambiguity vc-matrix $Q_{\hat{a}}$. As output it produces the integer least-squares ambiguity vector $\breve{a}$. This shows why the LAMBDA-method is independent of the application. One only needs the float solution together with its vc-matrix. At this stage of the computational process the underlying model is not relevant anymore. That is, it could be a single-, dual- or triple- frequency model, and it could be a GPS-, a Galileo-, or any other GNSS-model with all its varieties.

The ambiguity solution $\breve{a}$ is the integer vector nearest to the float solution $\hat{a}$, where nearness is measured in the metric of the vc-matrix of the float solution. The solution is computed by means of a discrete search in the so-called ambiguity search space. The search space is centered at the ambiguity float solution and its ellipsoidal shape is governed by the ambiguity vc-matrix. The computational efficiency of the LAMBDA-method stems from the fact that the search is not performed in the original DD ambiguity search space, but instead in a transformed search space. By means of the ambiguity decorrelation process the LAMBDA-method is capable of constructing a transformed search space with a more sphere-like shape. The transformed ambiguities are far less correlated and far more precise than the original DD ambiguities (see also figure 1). As a result a very fast search for the integer least-squares solution can be performed. This would not have been possible with the original DD ambiguity search space. This space is usually extremely elongated due to the high correlation and poor precision of the DD ambiguities. For more details on the method the reader is referred to [4].

In order to illustrate the decorrelation-step of the LAMBDA-method, we consider two examples of the three-frequency geometry-free GNSS model. The DD phase and pseudorange observation equations of this model are given as:

$$\phi_i = \rho - \left(\lambda_i / \lambda_1 \right)^2 I + \lambda_i a_i \; ; \quad p_i = \rho + \left(\lambda_i / \lambda_1 \right)^2 I$$

The ionospheric delay is denoted as *I*, the integer ambiguities as a_i and the corresponding wavelengths as λ_i. For both examples the standard deviations of the undifferenced phase and pseudorange observations are taken as resp. 3 mm and 30 cm.

In the first example we assume the ionospheric delay to be absent (referred to as the ionosphere-fixed model). This case applies therefore to short baselines only. When applying the LAMBDA-method to this model while using the above settings for the phase and pseudorange standard deviations, the following decorrelating ambiguity transformation is found:

$$Z = \begin{bmatrix} -3 & 4 & -1 \\ -2 & -2 & 5 \\ 1 & -1 & 0 \end{bmatrix}$$

The fact that this ambiguity transformation indeed achieves a significant decorrelation together with an improvement in precision can be seen when comparing the original ambiguity vc-matrix with its transformed counterpart. The entries of the two vc-matrices are given in the unit of cycles^2 and correspond to the use of a single epoch of data.

$$Q_{\hat{a}} = \begin{bmatrix} 3.3754 & 3.3136 & 2.6665 \\ 3.3136 & 3.2548 & 2.6184 \\ 2.6665 & 2.6184 & 2.1077 \end{bmatrix}; \quad Q_{\hat{z}} = Z Q_{\hat{a}} Z^* = \begin{bmatrix} 0.0894 & -0.0109 & 0.0014 \\ -0.0109 & 0.0243 & -0.0009 \\ 0.0014 & -0.0009 & 0.0031 \end{bmatrix}$$

In our second example we assume the ionospheric delay to be present and unknown (referred to as the ionosphere-float model). This case applies therefore to long baselines as well. When applying the LAMBDA-method to this model while again using the above settings for the phase and pseudorange standard deviations, we obtain as ambiguity transformation:

$$Z = \begin{bmatrix} -146 & 162 & -15 \\ -2 & 3 & -1 \\ 1 & -1 & 0 \end{bmatrix}$$

Again, by comparing the original ambiguity vc-matrix with its transformed counterpart, we can see that also in this case this ambiguity transformation achieves a significant decorrelation and improvement in precision.

$$Q_{\hat{a}} = \begin{bmatrix} 221.5435 & 221.1363 & 221.9237 \\ 221.1363 & 220.7326 & 221.5285 \\ 221.9237 & 221.5285 & 222.4594 \end{bmatrix}; \quad Q_{\hat{z}} = Z Q_{\hat{a}} Z^* = \begin{bmatrix} 149.9526 & 0.0036 & -0.0003 \\ 0.0036 & 0.1160 & 0.0011 \\ -0.0003 & 0.0011 & 0.0036 \end{bmatrix}$$

The above two examples illustrate the ease with which decorrelated ambiguities can be obtained with the LAMBDA-method. The process is automatic and it makes use of all available information, like type and structure of the observation equations and the level of observational precision. Changes in any of these will generally result in a different set of transformed ambiguities, just like the above two sets differ because of the two different models used, one with the ionospheric delay excluded and one with the delay included.

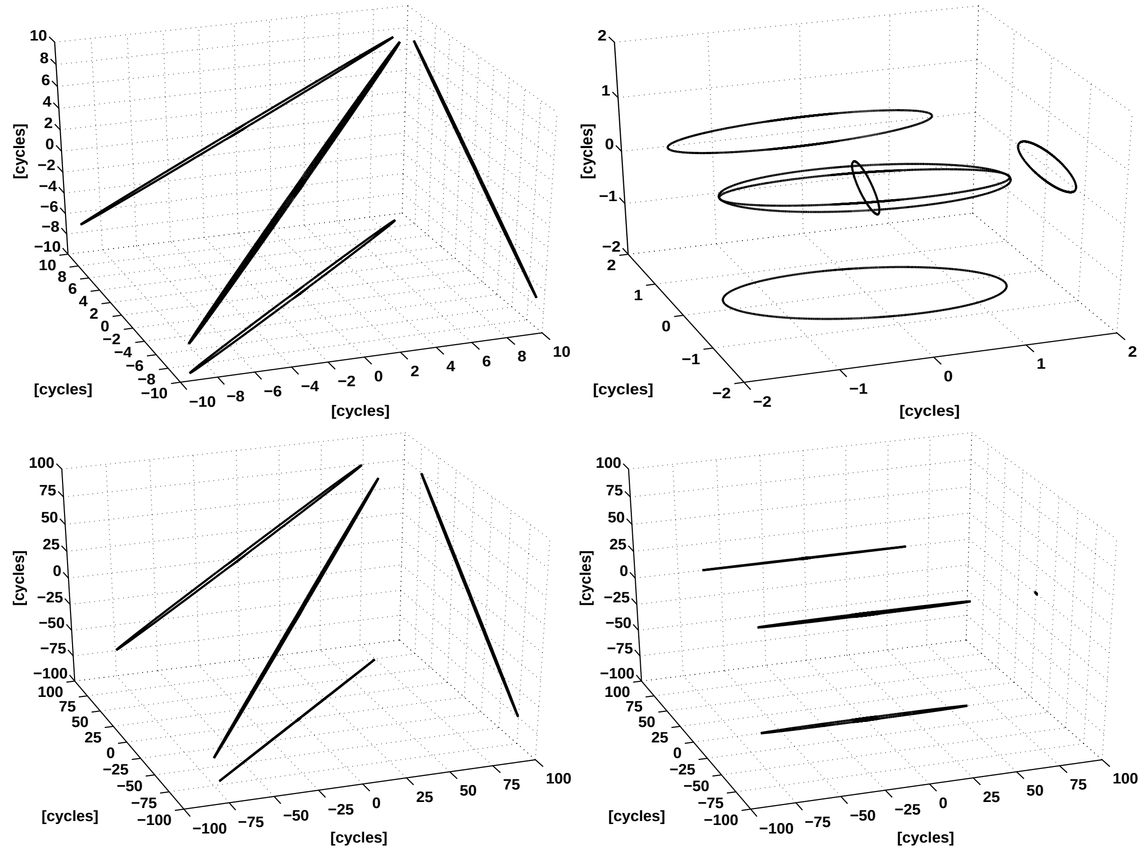

Figure 1: Shape of the 3-dimensional search space before (left) and after (right) performing decorrelation using the Z-transformation. Note the alignment of the principal axes of the ellipses. In the decorrelated case they are (almost) aligned with the three coordinate axes, indicating decorrelation to a large extent. The top row is based on the ionosphere-fixed model, the bottom row is based on the ionosphere-float model.

At this point it can also be made clear in what way the LAMBDA-method differs from the classical ‘wide-laning’ techniques. In this comparison one has to discriminate between two aspects, namely in the way transformed ambiguities are constructed and in the choice of integer estimation principle used. The classical ‘wide-laning’ techniques aim to resolve the ambiguities as integers by rounding the ambiguities of between-frequency phase differences to their nearest integer values. These, or similar techniques are also described and utilized for the three-frequency GNSS-models, see e.g. [1], [2] [3] and [10]. One difference between the classical ‘wide-laning’ techniques and the LAMBDA-method lies thus in the choice of integer estimator. The LAMBDA-method uses the optimal integer least-squares estimator, whereas the ‘wide-laning’ techniques do not. A second difference lies in the way transformed ambiguities are constructed. In the LAMBDA-method the ambiguity transformation is multivariate and it is constructed on the basis of all the relevant information as captured in the ambiguity vc-matrix. This is not the case with the ‘wide-laning’ techniques. They only consider one linear combination at a time, whereby their first aim is to achieve long wavelengths. These techniques do therefore not make use of all the relevant information. They are, for instance, not capable of making use of the relative receiver-satellite geometry. In the LAMBDA-method this information is automatically incorporated.

When the relative receiver-satellite geometry is absent, such as is the case with the simple geometry-free model of above, it may happen that some of the LAMBDA-transformed ambiguities can be recognized as ‘wide-lane’ ambiguities. This is the case for instance with both ambiguity transformations of above. They both contain the ‘super-widelane’, being the difference between the two closest frequencies 1589.742MHz and 1561.098MHz. No other obvious ‘wide-lane’ combinations are present however. Instead, the two other ambiguities are combinations made up of all three original DD ambiguities, and these combinations differ for the two models.

LAMBDA success-rates

As proven in [9] the integer least squares estimator has a superior performance in the sense that it maximizes the probability $P(\breve{a} = a)$ of estimating the ambiguities at their *correct* integer values. This probability, being the ambiguity success-rate of the LAMBDA-method, is given as the integral:

$$P(a = a) = \int_{S_a} p_{\hat{a}}(x)dx$$

with $p_{\hat{a}}(x)$ the probability density function of the float ambiguities and S_a the ambiguity pull-in region. It is stressed that the success-rate should be used as measure for predicting the success of ambiguity resolution and not the standard deviations of the ambiguities. When using standard deviations as measure of success very misleading results can be obtained. First, the correlations are neglected when only the standard deviations are considered. Second, the standard deviations can be manipulated at will by means of ambiguity transformations. The above success-rate on the other hand is invariant for any ambiguity transformation and it gives by means of a single number a complete picture of how successful one can expect the ambiguity resolution to be.

As an illustration of the LAMBDA success-rates, they are computed for a number of cases, namely for the proposed European GNSS-2 system, using the frequencies 1589.742MHz, 1561.098MHz and 1256.244MHz, and for the current and future American GPS system, with the frequencies 1575.420MHz, 1227.600MHz and 1176.450MHz. All computations are based on the single-epoch, geometry-free model using a 3 mm standard deviation of the undifferenced phases.

The results shown in figure 2 are based on the geometry-free, ionosphere-fixed model, whereas the results of figure 3 are based on the geometry-free, ionosphere-float model.

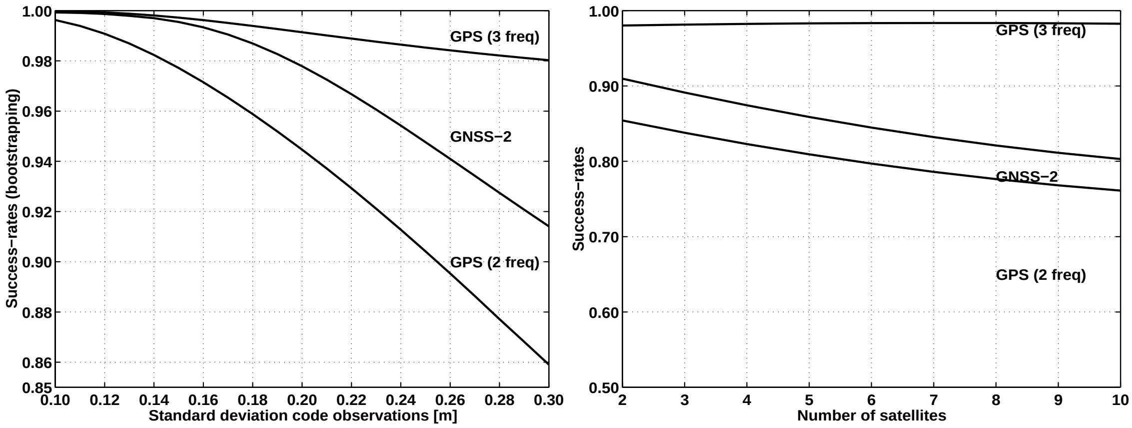

Figure 2: LAMBDA success-rates, on the left as a function of the standard deviation of the code measurements (2 satellites), on the right as a function of the number of satellites (σ_{code}=30[cm]).

From these graphs we learn that using a third frequency gives an improvement in all cases. We also see that 3-frequency GPS outperforms 3-frequency GNSS-2. This is due to the larger frequency spacing used with GPS. Note that this is in contrast with popular belief that two frequencies should be chosen close to each other, thus allowing for ‘wide-lane’ ambiguities to be resolved. Although this often works if one is interested in resolving only a single ambiguity or a subset of all ambiguities, it will not work if one aims at resolving the complete set of ambiguities. This has been shown in [5] where the success-rates were studied in their dependence on the third frequency.

From the left graph of figure 3 we learn that in case of long baselines, when no a-priori information on the ionospheric delays is available, instantaneous ambiguity resolution becomes impossible. Although the addition of a third frequency helps, it is by far not enough to make instantaneous ambiguity resolution possible. The graph on the right of figure 3 shows for example that, with a pseudorange standard deviation of 30 cm, the necessary number of epochs to achieve a success-rate of 99% decreases from 6500 for 2-freq. GPS, via around 4000 for GNSS-2 to around 750 for 3-freq. GPS.

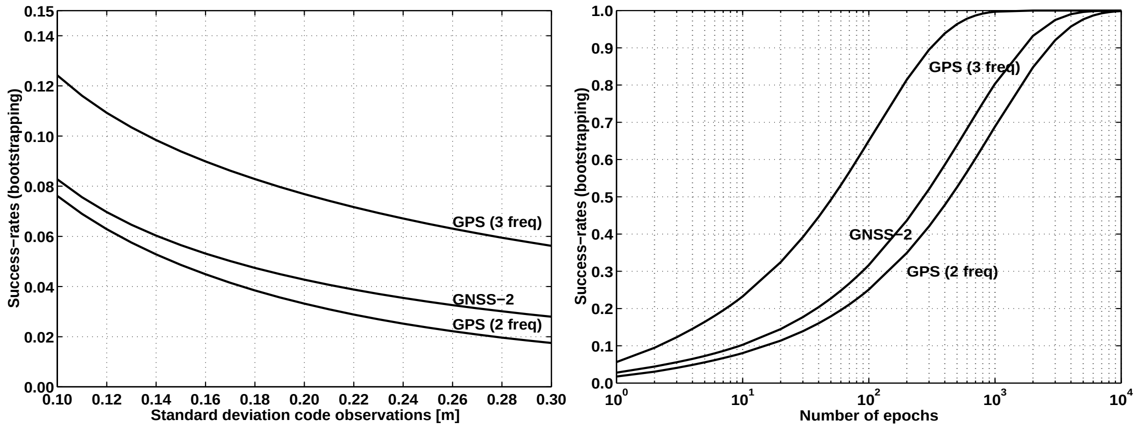

Figure 3: LAMBDA success-rates for the ionosphere-float model, left as a function of the standard deviation of the code observations (instantaneous), right as function of the number of epochs (σ_{code}=30[cm]).

Conclusions

In this contribution it has been stressed that the LAMBDA-method is applicable for any model in which unknown integer parameters appear. The method produces an optimal integer ambiguity solution by means of efficiently solving the integer least-squares problem through the use of a decorrelation process. At present the method is widely used for various single- and dual-frequency GPS processing applications. The method is however also applicable and suitable for a three carrier system (modernized GPS, Galileo), without any modification to the method at all. This has been exemplified by showing to what extent the ambiguity success-rates improve when use is made of an additional third frequency.

References

[1] Ericson, S. A study of linear phase combinations in considering future civil GPS frequencies. In: Proceedings ION GPS NTM-99 (1999), pp. 677-686.

[2] Forsell, B., Martin-Neira, M. and Harris, R., Carrier phase ambiguity resolution in GNSS-2. In proceedings ION GPS-97 (1997), pp. 1727-1736.

[3] Hatch, R., The promise of a third frequency. GPS World (May 1996).

[4] Jonge, P.J. de and Tiberius, C.C.J.M., The LAMBDA-method for integer ambiguity estimation: implementation aspects. LGR series No. 12, Delft Geodetic Computing Centre, Delft University of Technology, The Netherlands, 1996.

[5] Jonkman, N.F., Teunissen, P.J.G., Joosten, P. and Odijk, D.,GNSS long baseline ambiguity resolution: Impact of a third frequency, IUGG 1999, Birmingham, UK, July 1999.

[6] Teunissen, P.J.G., Least-squares estimation of the integer GPS ambiguities. IAG General Meeting, Beijing, China, 1993.

[7] Teunissen, P.J.G., On the integer normal distribution of the GPS ambiguities, Artificial Satellites 33-2, 1998, pp. 49-64.

[8] Teunissen, P.J.G., Success-probability of integer GPS ambiguity rounding and bootstrapping, Journal of Geodesy 72, 1998, pp. 606-612.

[9] Teunissen, P.J.G., A theorem on miximizing the probability of correct integer estimation, Artificial Satellites 34-1, 1999, pp. 3-9.

[10] Vollath, U., Birnbach, S. and Landau, H., Analysis of three-carrier ambiguity resolution (TCAR) technique for precise relative positioning in GNSS-2. Proceedings ION GPS-98, 1999, pp. 417-426