A Linearized Approach for Ambiguity Resolution Applied to the GNSS Compass Problem: Experimental Results

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Abstract: The nonlinearity of the baseline length constrained GNSS compass model prohibits the use of standard methods of ambiguity resolution. In this paper we therefore test to what extent a linearized version of the constrained GNSS compass model is applicable. We present experimental results for the linearized model and compare these results with results obtained for the unconstrained and nonlinearly constrained models. The results of the unconstrained and linearized constrained models are obtained with the standard LAMBDA method, while those of the nonlinear model are obtained with the constrained LAMBDA method. Our focus is on the most challenging case, being single-frequency, single-epoch ambiguity resolution.

Keywords: GNSS Compass - Nonlinear constraints - Ambiguity Resolution - LAMBDA.

1 INTRODUCTION

The GNSS Compass problem consists in estimating the orientation of a baseline given a set of GNSS code and phase observations. In order to achieve high degrees of angular accuracy, it is necessary to solve for the integer-valued ambiguities that are inherent to the carrier phase observables. Once the ambiguities are fixed, the precise data can be used for a wide range of demanding applications, ranging from terrestrial to maritime, air and space. Different approaches have been developed for resolving the GNSS Compass ambiguity resolution problem, see e.g. [1]-[6]. We use the popular LAMBDA method [7]-[8], which is an efficient implementation of the ILS (Integer Least-Squares) theory. In the Compass problem, two antennae are assumed to be kept at a known and fixed distance: the information on the baseline length can then be exploited for strengthening the observation model. A rigorous least-squares solution of the GNSS Compass problem, taking into account both the integerness of the ambiguities and the geometrical constraint on the baseline, was given in [9]-[11], and experimental results were reported in [12]-[16]: the new method was coined the Constrained (C-) LAMBDA method. The demonstrated improvement in the capacity of resolving the correct integer ambiguity vector for the constrained method comes at the cost of a more complicated search strategy. As discussed in [10], under certain circumstances it is possible to make use of a linearized approximation of the constrained method, for which the standard search routines implemented in the LAMBDA method can still be used. This approach works well as long as the baseline length is longer than a certain threshold, which depends on the quality of the observation data. In this contribution we analyze the linearized approach, providing some experimental results.

This contribution is structured as follows. In Section 2 we introduce the unconstrained and the constrained GNSS model. In Section 3 the unconstrained solution is given, while in Section 4 we present the constrained solution. The solution corresponding to the linearized model is discussed in Section 5. Finally, in Section 6 we present our experimental results that are obtained from actual data of a static ground experiment.

2 THE GNSS OBSERVABLES

If we track $m+1$ satellites from 2 antennae, a set of $2m$ Double Difference (DD) single-frequency, single-epoch phase and code observations is available. These observables can be linked to the vectors of unknowns by means of the linear(ized) observation equations

$$\begin{aligned} E(y) &= Aa + Bb \quad a \in \mathbb{Z}^n; b \in \mathbb{R}^p \\ D(y) &= Q_{yy} \end{aligned} \tag{1}$$

where $E(\cdot)$ is the expectation operator, y is the vector of DD code and carrier phase observables (order $2m$), a contains the n integer-valued ambiguities and b is the vector of remaining p real-valued unknowns. It is assumed that the antennae are separated a short distance (within hundreds of meters). We can therefore neglect the atmospheric delays and assume that the three baseline coordinates are the only real-valued unknowns ($p = 3$). A is the matrix of carrier wavelengths, while B is the matrix of line-of-sight vectors.
$D(\cdot)$ is the dispersion operator: the vector of observables is assumed to be Gaussian-distributed, characterized by the variance-covariance (v-c) matrix Q_{yy}.
For the GNSS-compass problem, one can strengthen the above unconstrained model by exploiting the knowledge of the baseline length $||b|| = l$. As a result we obtain the constrained model

$$\begin{aligned} E(y) &= Aa + Bb \quad a \in \mathbb{Z}^n; b \in \mathbb{R}^p; \|b\| = l \\ D(y) &= Q_{yy} \end{aligned} \tag{2}$$

Due to the nonlinear constraint $||b|| = l$, the search for the integer ambiguities needs to be modified, as illustrated in Section 4.

3 THE UNCONSTRAINED MODEL SOLUTION

The set of linear equations (1) is solved by applying the well-known ILS principle, the solution of which can be obtained from the following three consecutive steps:

1) First the so-called float solution is obtained, i.e. the least-squares solution of (1) disregarding the integer nature of the ambiguities:

$$\begin{aligned} \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} &= N^{-1} \begin{bmatrix} A^T Q_{yy}^{-1} y \\ B^T Q_{yy}^{-1} y \end{bmatrix} \\ N &= \begin{bmatrix} A^T Q_{yy}^{-1} A & A^T Q_{yy}^{-1} B \\ B^T Q_{yy}^{-1} A & B^T Q_{yy}^{-1} B \end{bmatrix} \end{aligned} \tag{3}$$

The variance-covariance (v-c) matrix of the float solutions is obtained by the inversion of the normal matrix N.

2) Then the integer ambiguity vector is estimated as

$$\check{a}_{\Lambda} = \arg \min_{a \in \mathbb{Z}^n} ||\hat{a} - a||^2_{Q_{\hat{a}\hat{a}}} \tag{4}$$

where $(.)^T Q_{\hat{a}\hat{a}}^{-1} (.) = ||.||^2_{Q_{\hat{a}\hat{a}}}$. The search for $\check{a}_{\Lambda}$, which minimizes the distance with respect to the float solution $\hat{a}$ in the metric of $Q_{\hat{a}\hat{a}}$, is performed by the LAMBDA method.

3) Finally, the fixed baseline solution is obtained as

$$\check{b}_{\Lambda} = \hat{b}(\check{a}_{\Lambda}) = (B^T Q_{yy}^{-1} B)^{-1} B^T Q_{yy}^{-1} (y - A\check{a}_{\Lambda}) \tag{5}$$

4 THE CONSTRAINED MODEL SOLUTION

The solution of (2) follows the same steps, although the search for the integer vector is modified to incorporate the constraint on the baseline length:

1) The float solutions and their v-c matrices are found as in (3).

2) The sum-of-squares expression that has to be minimized in the constrained case reads

$$\begin{aligned} &\min_{a \in \mathbb{Z}^n, b \in \mathbb{R}^3, \|b\| = l} \|y - Aa - Bb\|^2_{Q_{yy}} = ||\hat{e}||^2_{Q_{yy}} + \\ &+ \min_{a \in \mathbb{Z}^n} \left(\|\hat{a} - a\|^2_{Q_{\hat{a}\hat{a}}} + \min_{b \in \mathbb{R}^3, \|b\| = l} \|\hat{b}(a) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}} \right) \end{aligned} \tag{6}$$

where $Q_{\hat{b}(a)\hat{b}(a)}$ is the v-c matrix of $\hat{b}(a)$. Hence, the integer ambiguities are now estimated as the solution of the minimization problem

$$\check{a}_{C\Lambda} = \arg \min_{a \in \mathbb{Z}^n} \left(\|\hat{a} - a\|^2_{Q_{\hat{a}\hat{a}}} + \|\hat{b}(a) - \check{b}'(a)\|^2_{Q_{\hat{b}(a)\hat{b}(a)}} \right) \tag{7}$$

with

$$\check{b}'(a) = \arg \min_{b \in \mathbb{R}^3, \|b\| = l} \|\hat{b}(a) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}}$$

The search for the integer minimizer (7) is more complex than in the unconstrained case, because of two reasons: firstly, the search space is no longer ellipsoidal; secondly, the evaluation of the objective function implies the solution of a nonlinear least squares problem to extract the vector $\check{b}'(a)$, and this has to be done for each integer candidate.
It is shown in [13]-[15] how to perform such a search in an efficient way, by employing one of two search algorithms of the C-LAMBDA method, namely the *Search and Shrink* approach and the *Search and Expansion* approach.

3) When the integer minimizer $\check{a}_{C\Lambda}$ is found, the constrained fixed baseline solution is obtained as

$$\check{b}_{C\Lambda} = \check{b}'(\check{a}_{C\Lambda}) = \arg \min_{b \in \mathbb{R}^3, \|b\|=l} \|\hat{b}(\check{a}_{C\Lambda}) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}} \tag{8}$$

The constrained method achieves higher performance due to the rigorous inclusion into the integer estimation process of the nonlinear constraint, as given by the known distance between the GNSS antennae. The better performance in terms of higher success rates are reported in [12]-[16].
The C-LAMBDA method is however more complex than the standard LAMBDA method. It is therefore of interest to analyze a linearized version for which the standard search routines of the LAMBDA method can still be applied. This is discussed in the next section.

5 A QUADRATIC APPROXIMATION

If one uses linearization one obtains a quadratic approximation to the objective function. The quadratic approximation of the implicit ambiguity objective function was derived in [10]. The solution of the linearized model follows the following three steps procedure:

1) Instead of the unconstrained float solution, the constrained float solution is now used. From the unconstrained float baseline solution $\hat{b}$, one first obtains the constrained float baseline solution $\bar{b}$, from which one can obtain the constrained float ambiguity solution $\bar{a}$:

$$\begin{aligned} \bar{b} &= \arg \min_{||b||=l} ||\hat{b} - b||^2_{Q_{\hat{b}\hat{b}}} \\ \bar{a} &= (A^T Q_{yy}^{-1} A)^{-1} A^T Q_{yy}^{-1} (y - B\bar{b}) \end{aligned} \tag{9}$$

2) The integer ambiguity solution is obtained as [10]:

$$\check{a}_{L\Lambda} = \arg \min_{a \in \mathbb{Z}^n} ||\bar{a} - a||^2_{(\partial^2_{aa} F(\bar{a}))^{-1}} \tag{10}$$

where $\partial^2_{aa} F(\bar{a})$ is the Hessian matrix of the ambiguity objective function (7) evaluated at the point $\bar{a}$. This expression is a quadratic minimization problem for which the routines of the standard LAMBDA method can be applied again.
The higher order terms that are neglected in the quadratic approximation are bounded by a function inversely proportional to the baseline length: the longer the baseline, the smaller the neglected terms in the approximation, and the better the performance of the method. This has a straightforward geometrical interpretation: the non linearity of the constrained method is due to the curved manifold upon which the baseline solution is projected (the sphere of radius l), and the longer the baseline, the smaller is the curvature of the sphere, and thus the smaller the local nonlinearity.

3) When the minimizer $\check{a}_{L\Lambda}$ is found, the baseline solution is derived as

$$\check{b}_{L\Lambda} = \arg \min_{b \in \mathbb{R}^3, \|b\|=l} \|\hat{b}(\check{a}_{L\Lambda}) - b\|^2_{Q_{\hat{b}(a)\hat{b}(a)}} \tag{11}$$

Therefore, this approach only differ from the standard one by the shape of the weight matrix in the ambiguity search process and the derivation of the float solution. Purpose of this paper is to find how well this linearized version works compared to the standard LAMBDA and to the Constrained LAMBDA methods: this is investigated in the following section.

6 EXPERIMENTAL RESULTS

The linearized method described in the previous section has been tested with data collected during a field test and compared against the LAMBDA and C-LAMBDA methods. The performance indicator which has been looked into is the single-frequency, single-epoch success rate, i.e. the ratio of correctly fixed ambiguity vectors with respect

Fig. 1. Placement of the receivers on field

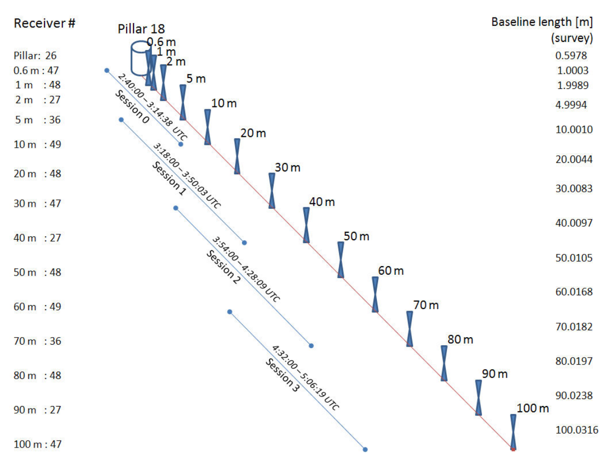
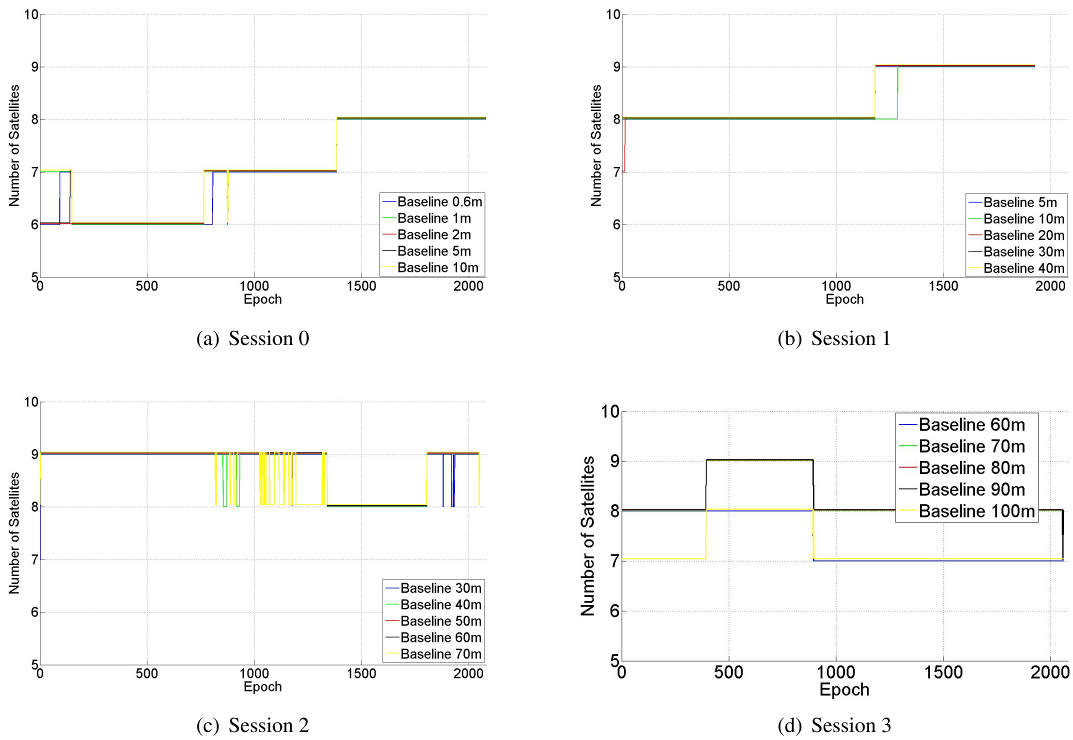


Fig. 2. Scheme of the experiment



(a) Session 0 (b) Session 1

(c) Session 2 (d) Session 3

Fig. 3. Number of satellites tracked at the different baselines

to the total number of epochs processed when only a single epoch of data collected tracking a single frequency is considered: this represents the most challenging scenario to evaluate.

6.1 The experiment set-up

In order to test the linearized method, a field experiment has been carried at CUT (Curtin University of Technology) on 31 August 2009. Since the performance of the tested linearized method strongly depend on the baseline length, fourteen different baselines have been set up with a common reference antennae. The antennae have been placed at distances of 0.6-1-2-5-10-20-30-40-50-60-70-80-90-100 meter from the reference one, lined up as shown in Figure 1. Six receivers (Sokkia GSR 2700 ISX) were used for the test, therefore 4 different measurements sessions have been necessary to cover the fourteen different cases, testing at each session 5 different baseline lengths and keeping three (the reference and the furthest two) receivers on place when changing session, as to allow two overlaps between consecutive sessions.
Figure 2 illustrates the placement of the receivers as function of the session (0 to 3); also the precise baseline lengths measured on field are reported. The ground truth for the experiment has been surveyed with a Sokkia Set1X total station (2 mm distance measurement acuracy). The number of satellites tracked varied between 6 and 9, as shown in figure 3, with PDOP values ranging between 2.2 and 3 for most of the time, with a peak of 16 for the first 800 epoch of the first session, due to a bad distribution of the six tracked satellites in the sky.
Each session lasted about 40 minutes, and each of the datasets collected was processed with the LAMBDA, the Constrained LAMBDA (C-LAMBDA) and the Linearized (LC-LAMBDA) methods.

6.2 Success rate performance

Table 1 reports the single-frequency, single-epoch success rates of the three ambiguity estimators for all the sessions examined. Figure 4 gives a graphical representation of the results, illustrating the success rates as function of the baseline lengths. Clearly visible in the graph are the different overlaps between consecutive sessions: for the receivers placed at 5m and 10m, 30m and 40m, 60 and 70m, two different experimental success rates are plotted for each method.
According to the theory, the performance of the linearized method increases with the baseline length, with a curve which is relatively insensitive to the number of satellites tracked. The results relative to the constrained (C-LAMBDA) and standard (LAMBDA) methods are flatter, although the latter shows a certain variability, due to the change of number of tracked satellites between and within the ses-

Table 1. Single-frequency, single-epoch success rates for the LAMBDA , constrained LAMBDA, and Linearized methods.

	Epochs	Baseline length [m]	LAMBDA	C-LAMBDA	LC-LAMBDA
Session 0	2079	0.5977	76.4791	98.5570	13.2275
		1.0003	76.8158	99.6633	10.7263
		1.9989	74.1703	97.7393	12.8908
		4.9994	75.5171	99.0861	27.5132
		10.0010	77.8259	99.0380	34.7763
Session 1	1924	4.9994	90.5925	99.8960	28.7440
		10.0010	90.4886	99.5842	41.3721
		20.0044	96.6216	99.8441	58.2121
		30.0083	98.0769	99.9480	71.4137
		40.0097	98.6486	99.9480	82.6403
Session 2	2050	30.0083	86.2439	99.4634	69.6098
		40.0097	93.6098	99.9024	81.1220
		50.0105	78.0488	99.7073	84.0488
		60.0168	89.0244	99.9024	87.1707
		70.0182	90.1463	96.0976	85.4634
Session 3	2060	60.0168	64.6602	98.8835	82.7184
		70.0182	85.5825	99.8058	91.4563
		80.0197	87.3786	99.8544	94.1748
		90.0238	75.9223	99.1262	90.6311
		100.0316	61.5904	99.8441	84.6117

sions. The C-LAMBDA method achieves almost a 100% of sucess rate on all the datasets processed, showing a large robustness. The LAMBDA method gives good results but it is less robust to the variation of the number of satellites tracked than the other methods: this can be easily observed comparing the sessions 0 and 1, where a higher number of satellites tracked in session 1 effectively increases the number of correctly fixed ambiguities. The Linearized approach works well for baseline longer than few tens of meters, and it achieves the same or higher performance than the standard LAMBDA method for baseline lengths in the range of 40 to 60m.

The anomalities in the last part of the graph for the LAMBDA and the Linearized method are due to the signal (not) received from a setting satellite (PRN 14), which is totally blocked by a building as seen from the receiver placed at 100m (see 3d) and it suffers (although not blocked) a large degradation for the receiver placed at 90m, as the

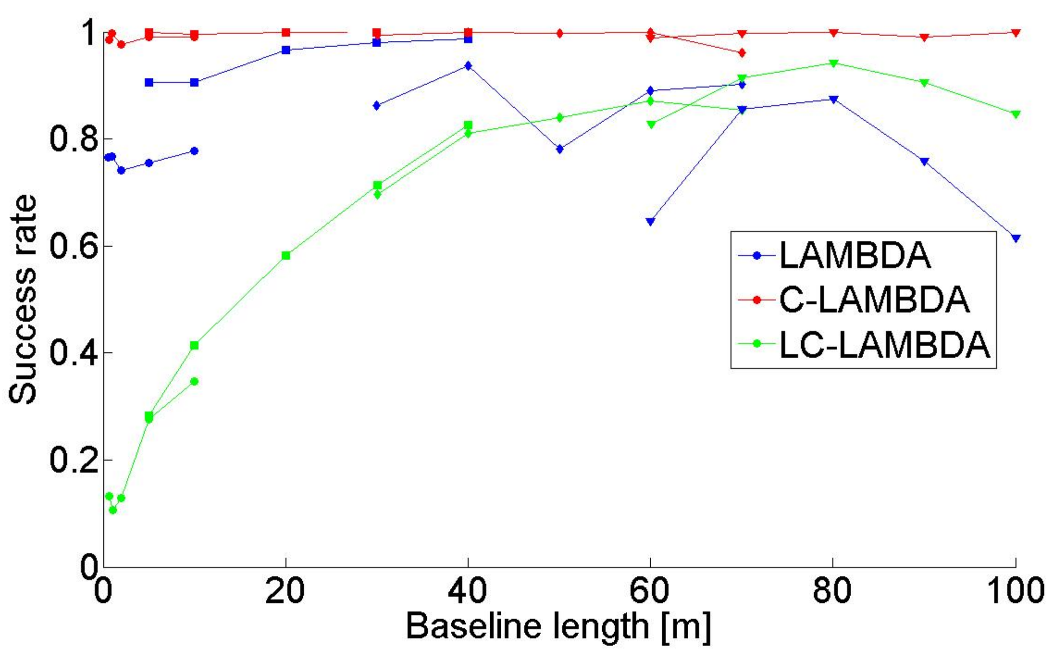


Fig. 4. Experimental single-epoch,single-frequency success rates for the three methods examined

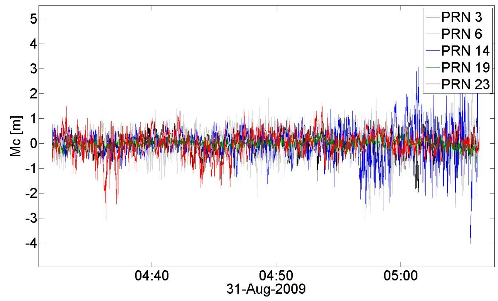


Fig. 5. Multipath combination (Mc=C1-4.092 L1 + 3.092 L2), Session 3, Receiver 27 placed at 90m from the reference

multipath combination reported in figure 5 reveals. The drop in success rate for the receiver placed at 60m is due to a sudden drop of the number of satellites tracked from the receiver placed at that distance, as reported in figure 3d.

7 CONCLUSIONS

It is studied in this contribution the performance achievable with a quadratic approximation of the Constrained Integer Least Squares. The linearization described has the advantage of being of a reduced complexity and the search for the integer minimizer can still be performed in an ellipsoidal search space with the efficient routines of the LAMBDA method. The method has an inherent dependency on the baseline length, and a field test has been performed in order to explore which are the conditions which make the quadratic approximation of the constrained method preferable to the standard approach.

An experimental test focused on testing the method against the LAMBDA and the Constrained LAMBDA for different baseline lengths: it is shown that the linearized method has equal or better performance than the unconstrained method for baselines larger than few tens of meters, and it theoretically tends to equal the performance of the Constrained method for longer baselines.

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REFERENCES

[1] I Y Bar-Itzhack, P Montgomery, and J Garrick. Algorithm for Attitude Determination using Global Positioning System. *Journal Of Guidance, Control and Dynamics*, 21(6), 1998.

[2] C E Cohen. Attitude Determination Using GPS. *Ph.D. Thesis*, 1992.

[3] G Lu. Development of a GPS Multi-antenna System for Attitude Determination. *UCGE Reports 20073, Dept. of Geomatics Eng., University of Calgary*, 1995.

[4] C Park, I Kim, J G Lee, and G I Jee. Efficient Ambiguity Resolution Using Constraint Equation. *Proceedings of IEEE: Position, Location and Navigation Symposium PLANS96, Atlanta, Georgia, USA*, 1996.

[5] C Park, I Kim, J G Lee, and G I Jee. Efficient Technique to Fix GPS Carrier Phase Integer Ambiguity On-the-fly. *Proceedings of IEEE: Radar, Sonar and Navigation, Vol. 144, No. 3*, 1997.

[6] M Ziebart and P Cross. LEO GPS Attitude Determination Algorithm for a Macro-satellite Using Boom-arm Deployed Antennas. *GPS Solutions, 6:242-256*, 2003.

[7] P J G Teunissen. The Least-Squares Ambiguity Decorrelation Adjustment: a Method for Fast GPS Integer Ambiguity Estimation. *Journal of Geodesy*, 70:65–82, 1995.

[8] P J G Teunissen. Least Squares Estimation of the Integer GPS Ambiguities. *Invited lecture, Section IV Theory and Methodology, IAG General Meeting, Beijing. Also in: LGR series No.6, Delft Geodetic Computing Center, Delft University of Technology*, 1993.

[9] P J G Teunissen. The LAMBDA Method for the GNSS Compass. *Artificial Satellites, Vol.41,N.3*, 2006.

[10] P J G Teunissen. GNSS Ambiguity Resolution for Attitude Determination: Theory and Method. *Proceedings of the International Symposium on GPS/GNSS, Tokyo, Japan*, 2008.

[11] P J G Teunissen. Integer Least Squares Theory for the GNSS Compass. *Journal of Geodesy, Springer, submitted*, 2009.

[12] C Park and P J G Teunissen. A New Carrier Phase Ambiguity Estimation for GNSS Attitude Determination Systems. *Proceedings of International GPS/GNSS Symposium, Tokyo*, 2003.

[13] P J Buist. The Baseline Constrained LAMBDA Method for Single Epoch, Single Frequency Attitude Determination Applications. *Proceedings of ION GPS*, 2007.

[14] C Park and P J G Teunissen. A Baseline Constrained LAMBDA Method for Integer Ambiguity

Resolution of GNSS Attitude Determination Systems. *Journal of Control, Robotics and Systems (in Korean), 14(6), 587-594*, 2008.

[15] G Giorgi, P J G Teunissen, and P J Buist. A Search and Shrink Approach for the Baseline Constrained LAMBDA: Experimental Results. *Proceedings of the International Symposium on GPS/GNSS, Tokyo, Japan*, 2008.

[16] G Giorgi and P J Buist. Single-epoch, Single Frequency, Standalone Full Attitude Determination: Experimental Results. *4th ESA Workshop on Satellite Navigation User Equipment Technologies, NAVITEC*, 2008.