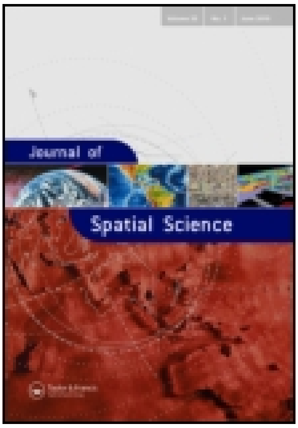

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Mitigation of periodic GPS multipath errors using a normalised least mean square adaptive filter

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Mitigation of Periodic GPS Multipath Errors Using a Normalised Least Mean Square Adaptive Filter

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A field trial over a period of four days has been conducted to investigate the impact of multipath in a highly obstructed environment and the capability of a Least Mean Square (LMS) adaptive filter to model and thereby reduce its effect. The multipath component of the GPS observations has been isolated by computing the double difference carrier phase residuals. Because multipath errors are a function of the satellite-receiver geometry, the resulting daily time series show a repeated pattern of multipath contamination. The numerical analysis presented here demonstrates that adaptive filtering is able to identify and remove these repeated multipath errors. Besides a detailed description of the experimental procedure and the filtering results, a brief summary is also given of the theoretical background for the LMS adaptive filter.

Keywords: adaptive filter, GPS, multipath, structural monitoring

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INTRODUCTION

The use of continuously operating GPS receivers for deformation monitoring of engineering structures has attracted much interest (e.g. DeLoach, 1989; Leach *et al.*, 1992; Ashkenazi *et al.*, 1997; Roberts *et al.*, 2004). The main advantage of a GPS-based monitoring system is that it allows the collection of continuous, real-time, automated, weather independent, high accuracy measurement data in any unobstructed environment. In addition, GPS receivers can provide *absolute* coordinate information, whereas other sensors, such as accelerometers, tilt-meters and strain gauges, which are more commonly used for structural monitoring, yield no information with regard to absolute deflections and deformation trends. GPS technology has therefore emerged as a versatile measurement option for structural deformation monitoring.

In GPS monitoring projects, antennaes often need to be installed in locations that are less than ideal for the reception of clean, high quality satellite data. Components of the structure being monitored as well as surrounding structures can obstruct satellite signals and act as reflectors causing multipath interference. A more extreme case can exist when a GPS antenna is installed to monitor the movement of a point on the face of a high-rise building, bridge tower or pier. In such circumstances, obstructions to the receiver's tracking of the satellites restrict the *completeness* of the recorded data. At the same time, the presence of nearby reflective surfaces which lead to multipath interference, reduce the *quality* of the collected data. Taken together, these two factors pose a significant challenge to the full exploitation of GPS in structural monitoring projects. Incomplete data and/or reduced data quality impact on the reliable detection of structural deformation. It is the latter of these two issues (reduced data quality resulting from multipath interference) that is the particular focus of this paper.

Techniques exist that employ either antenna design (hardware) or digital signal processing techniques (software) to assist in the mitigation of errors caused by GPS signal reflection. The particular approach being investigated here is a software approach that takes advantage of the fact that in a temporally constant environment (which is typically the case in structural monitoring applications) the multipath error will repeat on a daily cycle (sidereal day) due to the daily repetition of the receiver-satellite geometry. The premise therefore is that the multipath error observed on one day can be used to predict and remove the multipath error at the same (sidereal) time on the following day. A method to achieve this objective that is both flexible and relatively easy to implement is the *Least Mean Square (LMS) Adaptive Filter*.

RECEIVER MULTIPATH

As the word implies, multipath at a GPS receiver occurs when a signal from a GPS satellite reaches an antenna via two or more paths. Only one path relates to the direct line of sight between the antenna and the satellite and is the one required for an unbiased measurement of the carrier phase, the other paths are *indirect* and should be rejected. An indirect path results from the signal having been reflected or diffracted from a surface (or surfaces) close to the receiver prior to arriving at the antenna. The result of multipath reflection is an error in the path length and a consequent change in phase of the signal. The direct and indirect signals interfere electronically at the receiver, giving rise to erroneous carrier phase measurements.

The major difficulty with receiver multipath as a source of error in GPS positioning is that it is not spatially correlated. Multipath occurring at one end of a GPS baseline will therefore be independent of that occurring at the other end. Thus the principle of relative GPS positioning, used to eliminate or minimise the influence of spatially correlated errors, does not help in the mitigation of receiver multipath. Research has been carried out into the mathematical modelling of multipath based on the location and nature of reflectors in the vicinity of a GPS receiver (e.g. Georgiadou and Kleusberg, 1988; Arbour and Santerre, 1996). However the environment that typically exists around a GPS antenna being used to monitor structural deformation is complex due to the presence of multiple reflective surfaces with diverse properties. Hence mathematically modelling the influence of multipath in such an environment will be difficult. In the case of C/A-code observations, the errors caused by multipath can typically be in the order of 10-20 metres (Hofmann-Wellenhof *et al.*, 2001). Since the multipath error is a function of wavelength, carrier phase observations are less affected, with the maximum error occurring when the reflected signal has a relative phase offset of one quarter of a wavelength – about 5 cm (Leick, 1995).

There are three strategies commonly employed to minimise the impact of multipath on GPS positioning, these are; site selection, antenna design and digital signal processing. As previously mentioned, in deformation monitoring projects, site selection is often dictated by the physical features of the structure rather than by issues of optimum signal reception. The use of antennas designed specifically to reject reflected signals is common, but costs can sometimes be prohibitive and antenna features, such as choke rings and ground planes, are not always successful in eliminating reflected signals, particularly if the reflection occurs above the plane of the antenna. Perhaps the greatest advances in recent years have occurred in the area of signal processing where the objective is to detect and reject reflected signals at the time of signal reception (e.g. Fenton and Jones, 2005). While improvements continue to be made on this front, the problem of multipath remains a significant challenge for high precision applications of GPS.

It is in light of the limitations of current multipath mitigation techniques that several post-processing strategies have been proposed. For example, Axelrad *et al.* (1996) use the signal to noise ratio of observed signals; Ge (1999) uses adaptive filtering on double differenced GPS phase observations; Bock *et al.* (2000) use a temporal stacking algorithm; Radovanovic (2000) uses day-to-day correlation analysis; Forward *et al.* (2003) use GPS data stacking; Choi *et al.* (2004) use modified sidereal filtering; Satirapod and Rizos (2004) use wavelet analysis; and, Zheng *et al.* (2005) use a Vondrak filter with a cross-validation technique. Among them is the *Least Mean Square Adaptive Filter* investigated in this paper.

The adaptive filter takes advantage of the repetitive nature of multipath errors. More precisely, a multipath template containing observations from the previous day is used as reference input for the filter, enabling the identification of common error components on the following day. The theoretical feasibility of the adaptive filter was investigated by Ge (1999) and since then it has been used by a number of researchers to mitigate multipath in GPS observations (e.g. Ge *et al.,* 2000; Ge *et al.*, 2002; Roberts *et al.*, 2004; Chan *et al.*, 2006).

EXTRACTING MULTIPATH - THE DOUBLE DIFFERENCE RESIDUAL

In order to extract (and subsequently model) the multipath component from GPS carrier phase observations it is convenient to compute the *double difference carrier phase residuals.* This quantity is formed by subtracting the double difference computed using *true (known) ranges* from the double difference derived from the carrier phase observations. In the case of stationary receivers, *true ranges* can be derived from known receiver coordinates and a precise ephemeris. For the purposes of illustrating the computational process, equations relevant to the L1 carrier are developed below. Identical formulae could be derived for the L2 signal.

The observation equation for the raw carrier phase observable (in metres) for receiver *i* and satellite *p* can be written as (Hofmann-Wellenhoff *et al.*, 2001) :

$$\Phi_i^p = c(dt^p - dT_i) + R_i^p - d(ion)_i^p + d(trop)_i^p + d(multi)_i^p - \lambda N_i^p + noise \quad (1)$$

where ;

c	speed of light (m/sec)
dt	satellite clock offset (seconds)
dT	receiver clock offset (seconds)
R	geometric range (metres)
d(ion)	ionospheric delay (metres)
d(trop)	tropospheric delay (metres)
d(multi)	multipath delay (metres)
λ	carrier phase wavelength (metres)
N	integer ambiguity (unit less)
noise	random measurement error (metres)

Formation of the double difference involves combining four such equations combining two receivers (*i* and *j*) and two satellites (*p* and *q*). The double difference process eliminates the receiver and satellite clock errors, giving the double difference observation equation the following form:

$$\Delta\nabla\Phi_{ij}^{pq} = (\Phi_i^p - \Phi_i^q) - (\Phi_j^p - \Phi_j^q)$$

$$\Delta\nabla\Phi_{ij}^{pq} = \Delta\nabla R_{ij}^{pq} - \Delta\nabla d(ion)_{ij}^{pq} + \Delta\nabla d(trop)_{ij}^{pq} + \Delta\nabla d(multi)_{ij}^{pq} - \lambda\Delta\nabla N_{ij}^{pq} + \Delta\nabla noise \quad (2)$$

Fortunately in GPS structural deformation monitoring applications, baselines are normally short. In such cases (e.g. where the distance between receiver *i* and receiver *j* is less than 10 km) the double differenced tropospheric and ionospheric delays will effectively cancel due to the strong spatial correlation of these errors. In such cases, Equation (2) can be further simplified to :

$$\Delta\nabla\Phi_{ij}^{pq} = \Delta\nabla R_{ij}^{pq} + \Delta\nabla d(multi)_{ij}^{pq} - \lambda\Delta\nabla N_{ij}^{pq} + \Delta\nabla noise \quad (3)$$

As previously mentioned, for stationary receivers at *known* locations, the true ranges contained in the double difference range term ($\Delta\nabla P$) can be computed:

$$\Delta\nabla R_{ij}^{pq} = (R_i^p - R_i^q) - (R_j^p - R_j^q) \quad (4)$$

where (for example);

$$R_i^p = \left((X_i - X^p)^2 + (Y_i - Y^p)^2 + (Z_i - Z^p)^2\right)^{1/2} \quad (5)$$

Similarly it is a relatively simple matter to estimate the double difference integer ambiguity ($\Delta\nabla N$). The *double difference residual* ($v_{\Delta\nabla}$) can then be formed as follows:

$$v_{\Delta\nabla} = \Delta\nabla\Phi_{ij}^{pq} - (\Delta\nabla R_{ij}^{pq} - \lambda\Delta\nabla N_{ij}^{pq}) = \Delta\nabla d(multi)_{ij}^{pq} + \Delta\nabla noise \quad (6)$$

As can be seen from Equation (6), the double difference residual consists primarily of the double difference multipath error and combined receiver noise. The multipath component is of course the combination of multipath errors on four observations between two receivers *i* and *j* and two satellites *p* and *q*. This could be seen to be a disadvantage in as much as neither individual site nor individual satellite multipath have not been isolated. However, it should be noted that high accuracy GPS processing is almost always based on the double difference observable and so it is the elimination of the double difference multipath error that is of primary interest.

Multipath errors typically vary relatively slowly with time due to the fact that the satellite-reflector-receiver geometry changes slowly (Ge, 1999). Figure 1 shows a sample time-series of double difference carrier phase residuals computed using Equation (6). The series contains low frequency (long wavelength) behaviour in addition to a high frequency component, which is most likely measurement noise. Based on Equation (6), it is suggested that the low frequency variation shown in Figure 1 is predominantly the result of multipath errors.

SATELLITE CONSTELLATION AND MULTIPATH REPETITION

When viewed from a point on the earth, the direction (azimuth and elevation) of a GPS satellite repeats approximately every sidereal day. In other words the satellite-receiver geometry is the same approximately every 86,164 seconds. In reality, this time shift varies slightly because satellite orbits are subject to minor perturbations (Dodson *et al.*, 2002; Leick, 1995).

The logical consequence of repeated satellite-receiver geometry is that multipath errors also repeat. Repetition of multipath was first used by Genrich and Bock (1992) for multipath mitigation. Consequently many researchers (some of the works are referred to earlier) have developed multipath mitigation techniques based on repetition of multipath. As stated earlier, the Least Mean Square Adaptive Filtering technique is one of these techniques.

In Figure 2 the double difference residuals for satellites 16 and 20 at a single site for two consecutive days have been aligned with a time shift of 86,160 seconds. The result visually confirms the premise that repeated multipath occurs each sidereal day.

Cross-correlation analysis carried out by the authors of double difference residuals from two consecutive days involving six satellite pairs has demonstrated that the actual time shift between the two data sets was closer to 86,154 seconds. This value agrees with that reported by Ge *et al.* (2002) and will be adopted in the subsequent analysis.

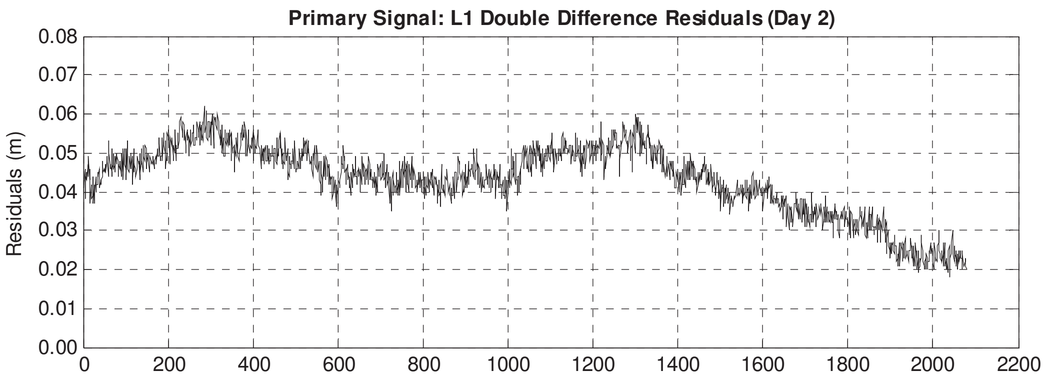

Figure 1. *Double difference residual time series for satellite pair 16-20*

THE LINEAR ADAPTIVE FILTER

The observation of any continuous signal results in a finite number of measured values, known as a *discrete time series*. The process of filtering aims to extract information about the signal at time (*t*) using data up to and including *t* (Haykin, 2002). A filter is said to be *linear* if the output of the filter is a linear combination of the input.

For a given discrete time series $\{x(i), x(i-1), x(i-2),..., x(i-M+1)\}$, where *i* is the current epoch, and *M* is the number of observation epochs to be included in the filter (*filter order*), the output *y(i)* of a linear filter at epoch *i* is given by:

$$y(i) = w_1 x(i) + w_2 x(i-1) + w_3 x(i-2) + \ldots + w_M x(i-M+1) \tag{7}$$

where $\{w_1, w_2,..., w_M\}$ are the filter coefficients, otherwise known as the *tap weights*.

The filter is designed to produce an optimal result in a statistical sense. A common requirement is to *minimise the mean-square error* of the filter output with regard to an expected response given by a reference signal. For a *stationary filter*, the same tap weights are applied for the duration of the time series. However, because of the time-variable nature of the multipath error, a stationary filter is not appropriate. Rather, it becomes necessary to adjust the tap weights on an epoch-by-epoch basis. Refinement of the tap weights over time leads to the concept of an *adaptive filter*. In an adaptive filter, new tap weights are estimated each epoch on the basis of the filter results from the previous epoch. An expression for the linear *adaptive* filter follows from Equation (7) :

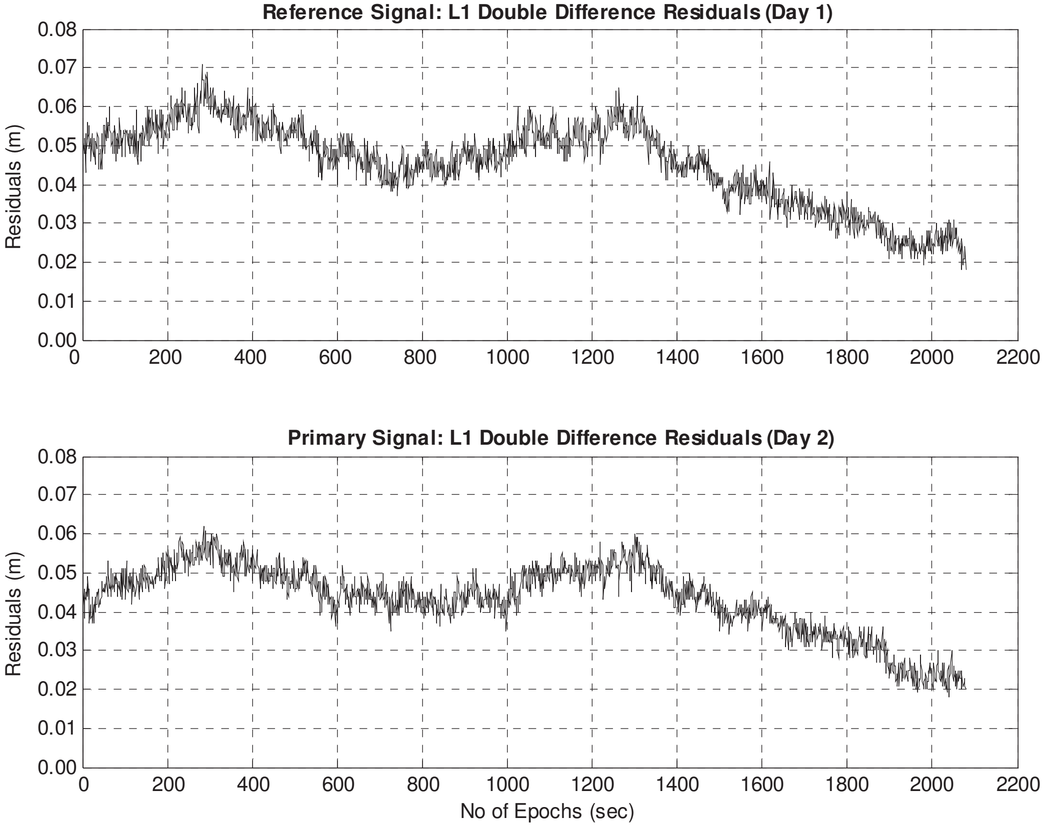

Figure 2. *Day 2 and Day 1 double difference residuals for satellite pair 16-20*

$$y(i) = w_1(i)x(i) + w_2(i)x(i-1) + w_3(i)x(i-2) + \ldots + w_M(i)x(i-M+1)$$

$$y(i) = \mathbf{w}(i)^T \mathbf{x}(i) \quad (8)$$

where;

$$\mathbf{w}(i) = \begin{bmatrix} w_1(i) & w_2(i) & w_3(i) & \ldots\ldots & w_M(i) \end{bmatrix}^T \quad \text{and}$$

$$\mathbf{x}(i) = \begin{bmatrix} x(i) & x(i-1) & x(i-2) & \ldots\ldots & x(i-M+1) \end{bmatrix}^T$$

Correlated and Uncorrelated Input Components

An adaptive filter needs two input signals, the primary signal to be filtered and the reference signal which contains information about the expected behaviour of the primary signal. In the case of adaptive filtering to detect multipath, the primary signal is a time series of double difference carrier phase residuals contaminated by multipath. Because of the expected daily repetition of the multipath errors, the reference signal is the double difference carrier phase residuals for the same satellite-receiver combination at the same site, but recorded on the previous day. The expectation is that the two signals (reference and primary) will be strongly correlated as a result of the repeated multipath error. On this basis, the following relationships will apply.

The primary signal *p(i)* (time series on Day 2) is a combination of multipath errors on Day 2 $m_2(i)$ and residual measurement noise $n_2(i)$:

$$p(i) = m_2(i) + n_2(i) \quad (9)$$

The reference signal *r(i)* (time series on Day 1) is a combination of multipath errors on Day 1 $m_1(i)$ and residual measurement noise $n_1(i)$:

$$r(i) = m_1(i) + n_1(i) \quad (10)$$

Because of repeated satellite-receiver geometry, the multipath components (m_2 and m_1) will be highly correlated whereas the residual measurement noise from day to day will be uncorrelated, as expressed by the following statistical expectations :

$$E[n_1(i)n_2(i)] = 0 \text{ and } E[m_1(i)\, m_2(i)] \neq 0 \quad (11)$$

In order to remove the multipath component from the primary signal, it is necessary to identify that part of the signal which is correlated with the reference signal. The linear adaptive filter is employed for this purpose where the tap weights are continuously *adapted* to give a filter output that is as close as possible to the reference signal. Any remaining difference is taken to be the uncorrelated component (random measurement noise) and is output as the filtering error.

The Least Mean Square Adaptive Filter

An adaptive filter aims to minimise the sum of the mean square errors between the filter output *y(i)* and the expected response *r(i)*.

$$\sum_{i=1}^{M} e^2(i) \rightarrow minimum \quad \text{where} \quad e(i) = r(i) - y(i) \quad (12)$$

In practice, it is not possible to rigorously achieve this objective because the strict minimisation of the sum of the squared errors requires the least squares estimation of multiple tap weights from a single equation. Instead, an iterative adaptation process for updating the tap weights for each new input value is carried out as follows :

Step 1. For epoch *i*, calculate the adaptive filter output (Equation 8)

$$y(i) = \mathbf{w}(i)^T \mathbf{p}(i)$$

Step 2. Compute the filter error for that epoch (Equation 12)

$$e(i) = r(i) - y(i)$$

Step 3. Update the tap weights for use in the next epoch *(i+1)*

$$\mathbf{w}(i+1) = \mathbf{w}(i) + \mu e(i)\mathbf{p}(i) \quad (13)$$

Step 4. Repeat from Step 1

The initial tap weight vector $\mathbf{w}(0)$ is usually set to zero. The update of the tap weight vector is controlled by the *adaptation parameter* (μ.) This parameter determines the rate of convergence of the filter and also influences the final error level. Determination of an optimal adaptation parameter is not trivial and depends on whether fast convergence or a low level of filtering error is preferred. Although formulae exist to calculate an optimal adaptation parameter, they require information about the statistical characteristics of the input data, which is often not readily available. Thus for practical purposes, the adaptation parameter is usually determined empirically.

Normalisation of the LMS Adaptive Filter

Normalisation is employed to ensure the numerical stability of the LMS adaptive filter computations. This is achieved by dividing the second term in the equation for the updated tap weight vector (Equation 13) by the squared Euclidean norm of the primary input signal :

$$\mathbf{w}(i+1) = \mathbf{w}(i) + \frac{\mu}{\mathbf{x}^{\mathrm{T}}(i)\mathbf{x}(i)} e(i)\mathbf{x}(i) \qquad (14)$$

Choice of Filter Order and Adaptation Parameter

Using a sample of double difference carrier phase residuals, experiments using various filter orders revealed that higher order filters do not significantly reduce, and sometimes marginally increase, the standard deviation of the filtering errors. As shown in Table 1, though high order filters (M > 200 epochs) consistently produce good results, optimum filter performance can often be achieved using as few as 20 to 30 epochs. Subsequent analyses were carried out using a filter order of M=40 epochs.

Selecting an appropriate adaptation parameter is crucial to achieving optimal filter performance. As a general rule, a smaller adaptation parameter will slow down the rate of filter convergence (slower adaptation) but will also lead to reduced filtering errors. On the other hand, a larger adaptation parameter will lead to faster filter convergence (faster adaptation) but will also result in larger average filtering errors.

According to Götze (2005), the adaptation parameter for a normalised LMS adaptive filter should be between 0 and 2. Using the same data set as used to derive the results presented in Table 1, various adaptation parameters in the range 0.1 to 1.0 have been used to test the impact of different adaptation parameters. A summary of the filtering results is shown in Table 2. Figure 3 shows the convergence of the filter in each case. For the purposes of subsequent analyses, an adaptation parameter of 0.2 has been used.

MULTIPATH TRIAL DATA AND PROCESSING

From 26 to 30 August 2005, ninety hours of continuous, dual frequency GPS data were recorded using a 1 Hz sampling rate at a point on the roof of the Geomatics Building at the University of Melbourne. A *Leica System 500* GPS receiver equipped with a *Leica AT 502* antenna was used for the data collection. As shown in Figure 4, the antenna was set up immediately adjacent to a high brick wall which blocked approximately 40 percent of the sky to the south of the antenna (Figure 5) and, as subsequent analysis demonstrates, introduced severe multipath errors. The trial was designed to simulate the case of a GPS receiver installed on the vertical wall of a high-rise building gathering information to detect structural deformation.

The coordinates of the antenna position and the location of any obstructions above a 15° cut-off angle were determined using a total station. Mapping the obstructions made it possible to identify observations in the data set to satellites that in fact had no direct line-of-sight to the antenna. These satellites, from

Epochs	2324	2324	2324	2324	2324	2324
Filter order (M)	10	20	40	60	120	300
Adaptation (μ)	0.2	0.2	0.2	0.2	0.2	0.2
Mean filter error	-0.000006	-0.000020	-0.000039	-0.000063	-0.000089	-0.000088
Std dev of error	±0.002394	±0.002356	±0.002349	±0.002359	±0.002370	±0.002349

Table 1. *Adaptive filter statistics for satellite pair 16-20 using different filter orders*

Epochs	2324	2324	2324	2324	2324
Filter order (M)	40	40	40	40	40
Adaptation (m)	0.1	0.2	0.4	0.6	1.0
Mean filter error	-0.000049	-0.000039	-0.000029	-0.000023	-0.000015
Std dev of error	±0.002398	±0.002349	±0.002388	±0.002512	±0.002934

Table 2. *Adaptive filter statistics for satellite pair 16-20 using different adaptation parameters*

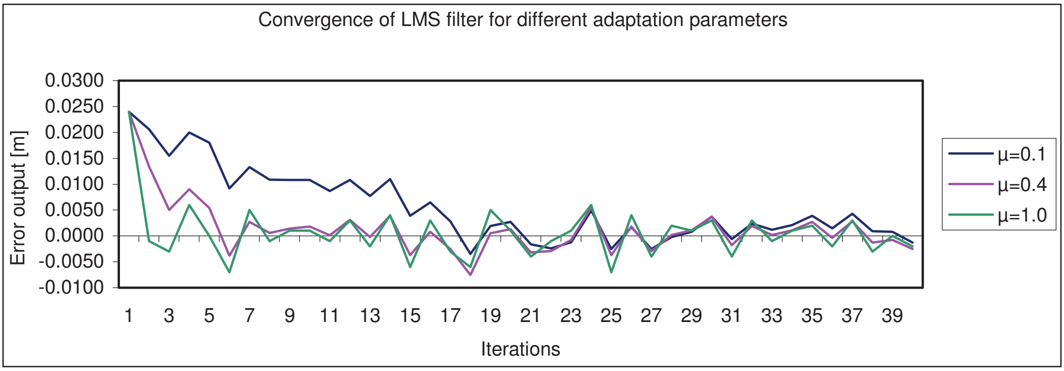

Figure 3. *Convergence rates for satellites16-20 using different adaptation parameters*

which therefore only reflected signals had been received, were subsequently eliminated from the data set. During this process, it was discovered that up to 40 percent of the recorded data had no direct line-of-sight to the satellites, thus confirming the presence of significant multipath at the site.

The cleaned data, along with matching data from a permanent base station located in a low multipath/low obstruction environment less than 1 km to the south, were used to calculate double difference carrier phase residuals via Equation (3). To this end, *true* ranges between the receivers and each tracked satellite were required. These were calculated using known receiver coordinates and satellite coordinates extracted from IGS precise ephemerides (Dow *et al.*, 2005).

FILTERING RESULTS

Double difference carrier phase residuals were computed using purpose written software and *RINEX* classes from the *GPS Toolbox* (Hilla and Adams, 2001). Likewise, software to implement a normalised LMS adaptive filter was developed and used to process these data from two

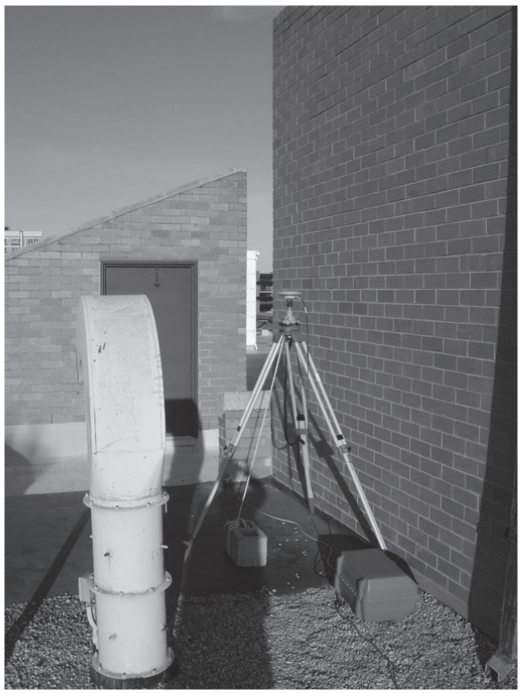

Figure 4. *Data collection site, roof of the Geomatics building, University of Melbourne*

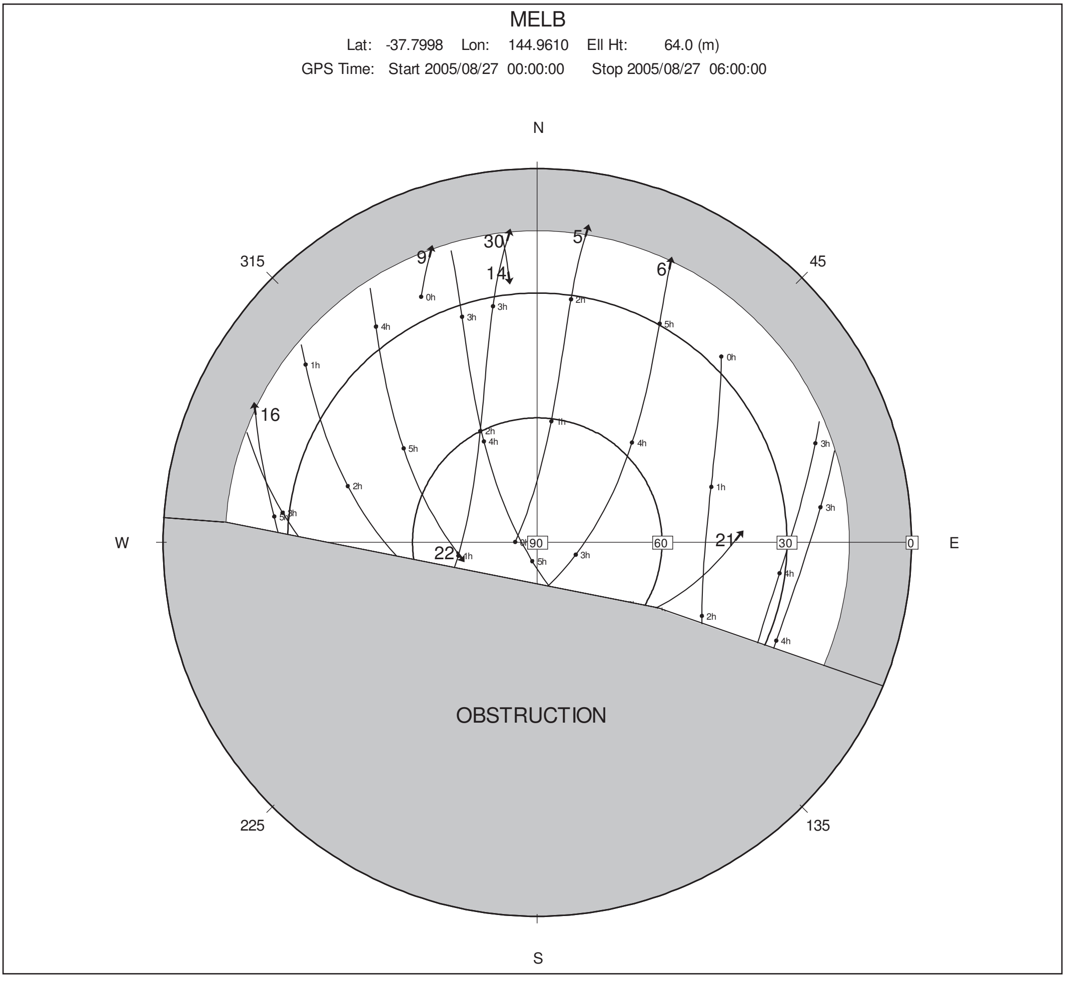

Figure 5. *Obstruction map of the receiver site (created using skyplot.exe, Marshall 2002).*

consecutive days. For the purposes of this paper, three sample datasets (satellite pairs 16-20, 20-1 and 20-24) comprising 2078, 1159 and 1178 epochs of double difference carrier phase residuals respectively, were selected for analysis. These datasets were chosen because they were free of cycle slips and provided good examples of the daily repetition of multipath errors.

As discussed above, the reference signal (from Day 1) and the primary signal (from Day 2) were aligned using a time shift of 86,154 seconds. Note that it is not necessary to exactly align the data with respect to time as the adaptive nature of the filter can readily deal with an offset between the two signals, even if the offset varies. The top two plots in Figures 6, 7 and 8 show the reference and the primary input signals for satellite pairs 16-20, 20-1 and 20-24, respectively.

The filter was run using an order of 40 and an adaptation parameter of 0.2. The statistics from applying the adaptive filter to the data sets shown in Figures 6, 7 and 8 are presented in Table 5.

The filter output and the filtering error for satellite pairs 16-20, 20-1 and 20-24 are shown in the bottom two plots in Figures 6, 7 and 8, respectively. As can be seen from these graphs, the LMS adaptive filter has been very effective in detecting and modelling the repeated multipath error labelled as *adaptive filter output* in the charts. The filtering error, that part of the signal which remains after the time correlated part has been modelled, is small in each case. The error time series have standard deviations of ±2.3 mm, ±3.8 mm and ±2.5 mm respectively, which can be interpreted as combined measurement noise involving two satellites and two receivers. Assuming that each carrier phase measurement has the same standard deviation, the estimates of combined receiver noise equate to individual carrier phase precisions in the range ±1.2 to ±1.9 mm, as would be expected given the wavelength of the carrier phase signals. These results confirm that the adaptive filter has successfully removed the influence of multipath from the double difference carrier phase residuals.

CONCLUSION

GPS data were collected over a four day period using a Leica System 500 receiver installed in a high multipath environment. In order to identify and remove the multipath errors from the observations, double difference carrier phase residuals were formed. An adaptive filter, based on the normalised Least Mean Square (LMS) algorithm, was implemented to remove the repeated multipath errors over two consecutive days from the double difference L1 time series. The test results show that the multipath impact was successfully removed. The remaining double difference noise series had acceptable standard deviations, consistent with expectations, confirming the success of the LMS filter.

Multipath is perhaps the most serious single source of error limiting the usefulness of GPS for structural deformation monitoring using short baselines. The results presented in this paper demonstrate that LMS adaptive filtering, which takes advantage of the daily repetition of the multipath error, can be used to detect and remove the multipath errors from double differenced carrier phase data, thereby allowing *cleaned* data to be used in monitoring structural movement.

A limitation of the filtering method presented here is that it has only been tested using double differenced carrier phase data. The advantage of using the double difference observable is

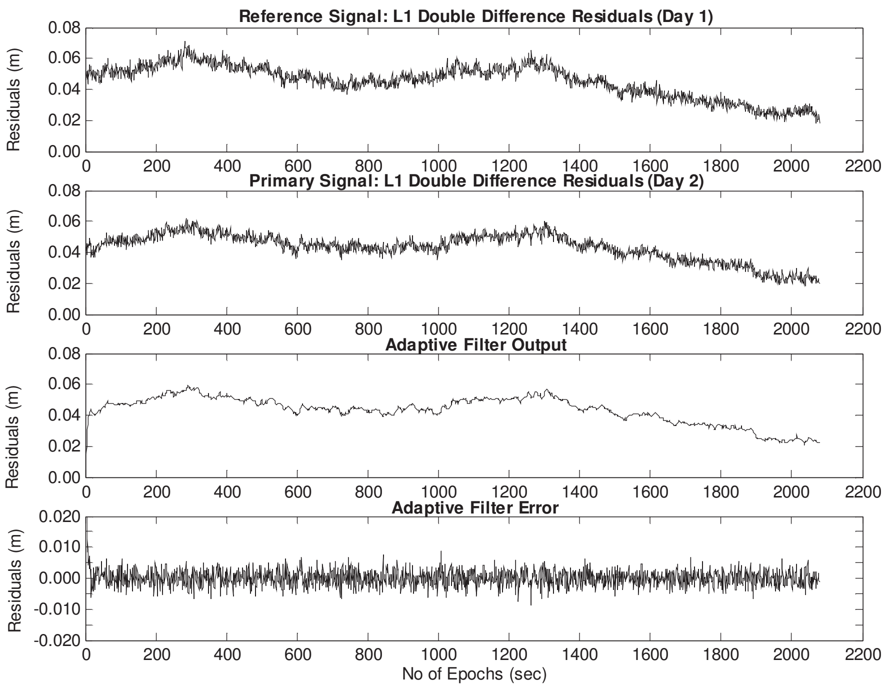

Figure 6. *Adaptive filter applied to L1 double-difference residuals of satellite pair 16-20*

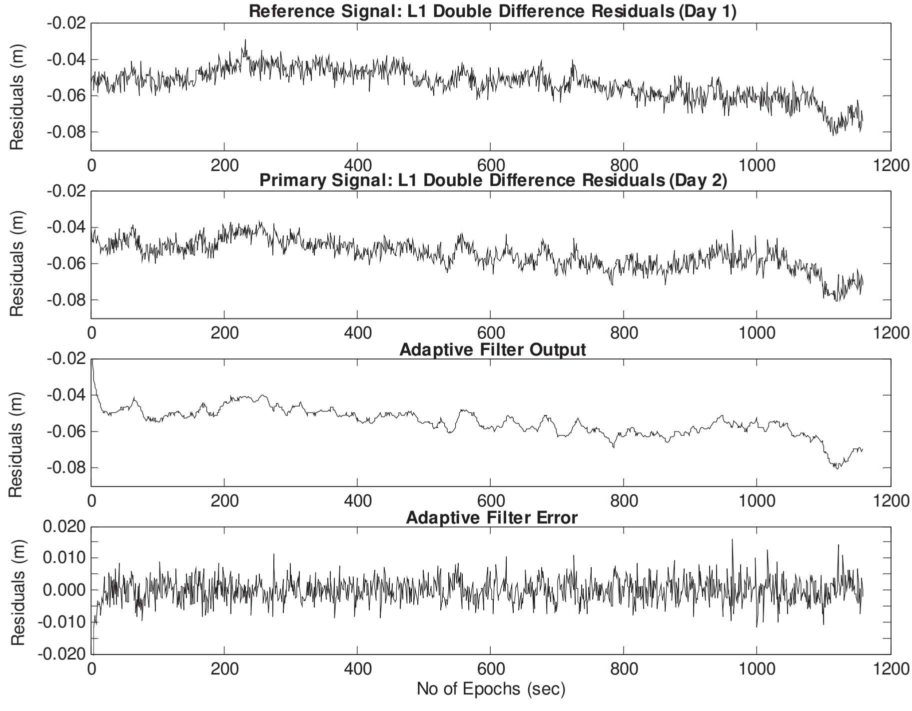

Figure 7. *Adaptive filter applied to L1 double-difference residuals of satellite pair 20-1*

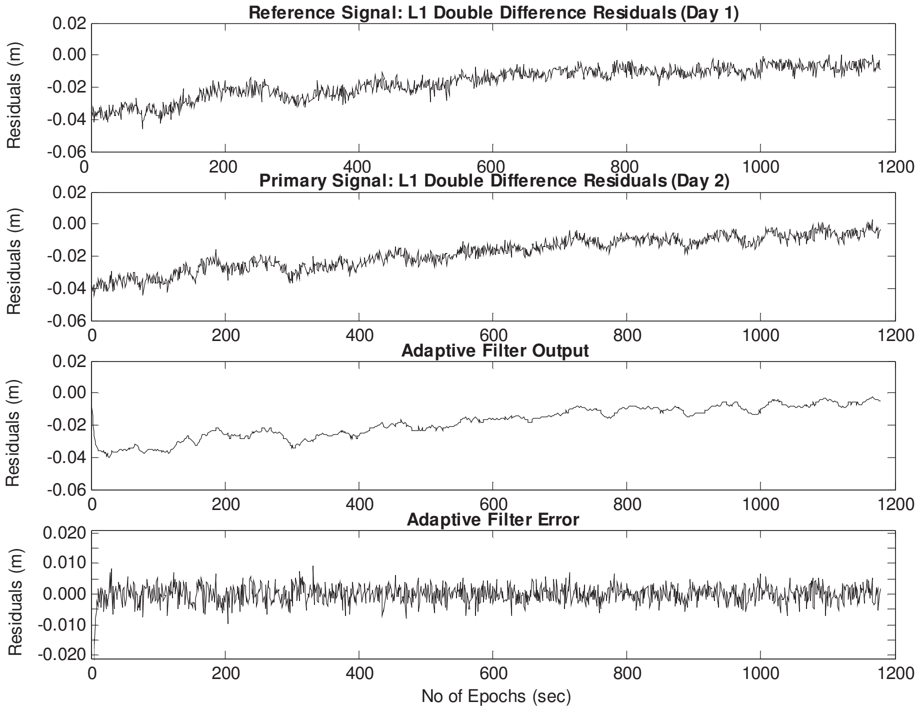

Figure 8. *Adaptive filter applied to L1 double-difference residuals of satellite pair 20-24*

Satellite pair	16-20	20-1	20-24
Epochs	2078	1159	1178
Filter order (M)	40	40	40
Adaptation (μ)	0.2	0.2	0.2
Mean filter error (metres)	-0.000002	0.000096	0.000052
Std dev of error (metres)	±0.002383	±0.003840	±0.002545

Table 5. Statistics of filter input and output

that, over short baselines, it effectively removes all spatially correlated errors affecting the observations. A possible disadvantage is that it does not allow individual site-satellite multipath to be detected.

Another practical problem in this approach is that the presence of obstructions and multipath interference at a site will often lead to cycle slips in the recorded data. Cycle slips will corrupt the continuity of the double difference time series and thus cause the proposed filtering method to fail. Thus cycle slip detection and repair must be carried out before using the adaptive filter to remove the influence of multipath. Of course in the presence of strong multipath, cycle slip fixing can be problematic in its own right and this is an issue that needs further research.

Notwithstanding the limitations identified above, and the need for further research, the authors believe that the LMS adaptive filter holds considerable promise as a technique for mitigating repeated multipath errors in the context of structural deformation monitoring using GPS.

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