Reliable Multi-Carrier Ambiguity Resolution in the Presence of Multipath

Sandra Verhagen and Dennis Odijk, *Delft Institute of Earth Observation and Space systems, Delft University of Technology, The Netherlands*
Frank Boon, *Septentrio Satellite Navigation, Belgium*
José María López Almansa, *GMV, Spain*

BIOGRAPHY

Sandra Verhagen obtained her PhD from and is now an assistant professor at the Delft Institute of Earth Observation and Space Systems of Delft University of Technology. Her research interests are ambiguity resolution and quality control for real-time kinematic GNSS applications.

Dennis Odijk holds a PhD in Geodetic Engineering of Delft University of Technology. Currently he works as a researcher with the Delft institute for Earth Observation and Space systems (DEOS). He is involved in several GNSS-related projects, focusing on ambiguity resolution, ionosphere modeling for precise positioning and indoor GPS.

Frank Boon holds a MSc. in Aerospace Engineering from Delft University of technology. He is head of the research group at Septentrio. His research interests include error modeling of GNSS measurements and optimization of positioning algorithms for highly dynamic applications. He previously worked on OTF ambiguity fixing methods and GNSS/IMU integration.

José María López-Almansa holds a MSc. degree in Telecommunications Engineering from the Polytechnic University of Madrid. He joined GMV at the end of 1999, where his activities include GNSS signal processing (multipath and interference analysis, receiver performance), simulation tools and navigation and integrity algorithms.

ABSTRACT

In this contribution it will be shown that a correct stochastic model will dramatically improve the degraded performance of carrier-phase ambiguity resolution in the presence of biases. The focus in this paper is on code biases caused by multipath, as it is known to be a dominant error source in high-accuracy GNSS applications.

A bias in the observations will propagate in the parameter estimates, which may cause severe positioning errors. Therefore, it is essential to use the a-priori knowledge about errors in the model if possible. With the A-Posteriori Multipath Estimation (APME) method this is done by applying corrections to the observations prior to the parameter estimation. With the APME technique the multipath error affecting the code tracking is estimated, and this estimated error is subtracted from the code-phase measurements. APME also provides the standard deviation of the multipath estimates, so that the stochastic model can be adapted.

In this contribution we will specifically investigate whether APME and the improved stochastic model may improve the reliability of ambiguity resolution, and/or may serve as an indication that reliable ambiguity resolution is not feasible.

In conclusion, it will be shown that a good error mitigation method together with a correct stochastic model will significantly improve the reliability of ambiguity resolution, and if Ratio Test Integer Aperture estimation is applied APME automatically provides an indication when reliable ambiguity resolution is not possible.

INTRODUCTION

Carrier phase integer ambiguity resolution is a key aspect to (near) real-time high-accuracy and reliable GNSS positioning. The LAMBDA method is a very well-known mechanization of integer least-squares ambiguity estimation, which means that the highest probability of correct integer estimation is obtained. With LAMBDA the position solution is obtained in three steps. First, a standard least-squares adjustment is applied to obtain real-valued estimates for all unknown parameters. This solution is referred to as the float solution. In a second step, LAMBDA is applied to obtain the integer ambiguity estimates. Finally, the float position estimates are adjusted according to their correlation with the ambiguities.

A bias in the observations will propagate in the parameter estimates. Firstly, the float solution will be affected. As a result, the biased float ambiguities may result in an incorrect fixed ambiguity solution, even though the formal precision and thus the success rate (based on the model) are very high. Therefore, it is important to consider the impact of biases on ambiguity resolution. Here, we will specifically look at biases caused by multipath.

Because of the risk of using the wrong integer solution, which could cause a significant error in the fixed position solution, ambiguity resolution does not only involve integer estimation, but also validation. Currently, often the ratio test is used for this purpose. It is known that the weak point of this test is the choice of the threshold value which determines whether or not the fixed solution will be accepted. Mostly a fixed value is used, not depending on the model under consideration. This approach is often too conservative, meaning that the fixed solution is rejected while the probability of being wrong is actually very low. Therefore, a new approach has been proposed referred to as Ratio Test Integer Aperture estimation, where a model-driven threshold value is used. This threshold value is then determined by the condition that the failure rate may not exceed a certain user-defined value.

The A-Posteriori Multipath Estimation (APME) method developed by Septentrio aims at multipath mitigation. With APME the multipath error affecting the code tracking is estimated, and this estimated error is subtracted from the code-phase measurements. APME also provides the standard deviation of the multipath estimates, so that the stochastic model can be adapted.

In this contribution we will specifically investigate whether APME and the improved stochastic model may improve the reliability of ambiguity resolution, and/or may serve as an indication that reliable ambiguity resolution is not feasible. With Ratio Test Integer Aperture estimation the decision whether or not to fix the ambiguities is based on the condition that the failure rate may not exceed a user-defined value. Since with APME both the functional and stochastic model are adapted, the ratio test serves as a good decision tool whether reliable ambiguity resolution is possible given the presence of multipath errors.

Simulated data are used to investigate the ambiguity resolution and positioning performance with Galileo signals in the presence of severe multipath, with and without APME. Epoch-by-epoch processing was used in order to investigate the feasibility of instantaneous ambiguity resolution. Both the triple-frequency (L1+E6+E5) and dual-frequency (L1+E5) scenario are considered.

It will be shown here that a good error mitigation method together with a correct stochastic model will significantly improve the reliability of ambiguity resolution, and if Ratio Test Integer Aperture estimation is applied APME automatically provides an indication when reliable ambiguity resolution is not possible.

The outline of this paper is as follows. First a review of the integer ambiguity resolution problem is given. The impact of multipath on ambiguity resolution is the topic of the next section. In the following sections the experimental set-up and results are presented, followed by the conclusions.

INTEGER AMBIGUITY RESOLUTION

The next generation Global Navigation Satellite Systems will offer positioning information on more frequencies as compared to the currently available dual-frequency GPS. It is well-known that rapid and accurate positioning requires using the very precise carrier phase observations, which are ambiguous by an unknown number of cycles. This section describes the integer ambiguity resolution (IAR) problem.

Integer estimation

Let the linearized GNSS model be given as follows:

$$E\{\boldsymbol{y}\} = \begin{pmatrix} \boldsymbol{B} & \boldsymbol{A} \end{pmatrix} \begin{pmatrix} \boldsymbol{b} \\ \boldsymbol{a} \end{pmatrix}; \quad D\{\boldsymbol{y}\} = \boldsymbol{Q}_y \tag{1}$$

Where ***E**{.}* and ***D**{.}* are the mathematical expectation and dispersion operators. The double difference (DD) code and carrier phase observations are collected in is the (*m*-1)-vector $\boldsymbol{y}$. The *p*-vector $\boldsymbol{b}$ contains all real-valued unknown parameter increments, including the baseline unknowns and for example atmospheric delays. In the sequel it will be referred to as the vector of baseline unknowns. The unknown integer ambiguities are denoted with the vector $\boldsymbol{a}$, which is of order *n*. The functional relationship between the observations and the unknown parameters is captured by the matrices $\boldsymbol{B}$ and $\boldsymbol{A}$. Finally, the precision of the observations is described by the variance matrix $\boldsymbol{Q}_y$.

The first step in GNSS data processing is to determine the so-called float solution of the unknown parameters, using a standard least-squares adjustment. After this step, real-valued estimates of the parameters are available together with their associated variance matrix. Let this float solution be given as:

$$\begin{pmatrix} \hat{\boldsymbol{b}} \\ \hat{\boldsymbol{a}} \end{pmatrix}; \quad \begin{pmatrix} \boldsymbol{Q}_{\hat{b}} & \boldsymbol{Q}_{\hat{b}\hat{a}} \\ \boldsymbol{Q}_{\hat{a}\hat{b}} & \boldsymbol{Q}_{\hat{a}} \end{pmatrix} \tag{2}$$

The next step is then to resolve the integer ambiguities, meaning that the float ambiguities are all mapped to integer values.

The choice of an integer estimator should be based on probability of correct integer estimation, since using the wrong integer ambiguity may severely deteriorate the baseline solution, whereas the goal is of course to improve its precision. The probability of correct integer estimation is referred to as the success rate and can be computed without the need for actual observations, since the GNSS model can be set up once the satellite positions with respect to the user location are known. For that purpose, almanac files can be used.

The integer estimator which maximizes the success rate is the integer least-squares estimator, which is therefore called the optimal integer estimator. LAMBDA is an efficient implementation of this estimator, e.g. [Teunissen, 1993; Teunissen, 1998].

Based on the integer ambiguity solution, finally the float baseline solution is adjusted for the difference between the float and the integer ambiguities.

This will result in a more precise solution, the so-called fixed solution. The fixed baseline solution is obtained as follows:

$$\breve{\boldsymbol{b}} = \hat{\boldsymbol{b}} - \boldsymbol{Q}_{\hat{b}\hat{a}} \boldsymbol{Q}_{\hat{a}}^{-1} (\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}) \tag{3}$$

The goal of integer ambiguity estimation is of course to improve the precision of the baseline estimates. This will certainly be the case if it can be assumed that the fixed ambiguities are non-stochastic, since then application of the propagation law of variances to Eq.(3) will result in:

$$\boldsymbol{Q}_{\breve{b}} = \boldsymbol{Q}_{\hat{b}} - \boldsymbol{Q}_{\hat{b}\hat{a}} \boldsymbol{Q}_{\hat{a}}^{-1} \boldsymbol{Q}_{\hat{a}\hat{b}} << \boldsymbol{Q}_{\hat{b}} \tag{4}$$

In reality, the fixed ambiguities are stochastic and their distribution function is a probability mass function. For an example see Figure 1. Only if the probability mass located at the correct integer is very close to one, the fixed ambiguities may indeed be considered non-stochastic. This probability is the same as the success rate mentioned at the beginning of this section, see e.g. [Teunissen, 1997; 1998].

Integer validation: ratio test

The success rate is an important measure for the reliability of integer ambiguity resolution. However, the success rate is only model-driven; the actual integer ambiguity solution is not considered. Therefore, also integer validation is required, meaning that it must be tested whether or not a specific integer ambiguity solution is sufficiently more likely than any other integer candidate.

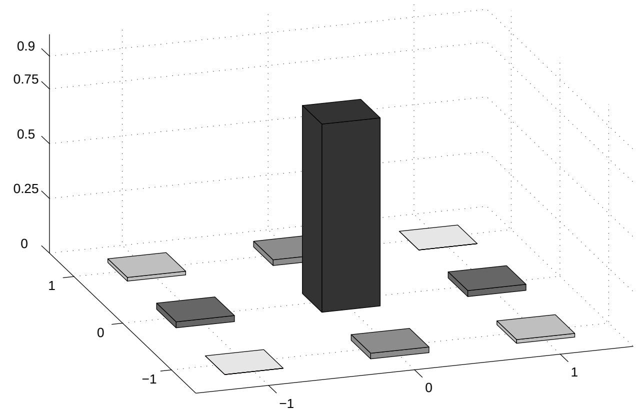

Figure 1. Probability mass function. The success rate equals the probability mass located at the correct integer, in this example (0,0).

Even though the success rate may be very close to one, the float ambiguity estimate may be close to the boundary of the integer least-squares pull-in region. In that case, the integer corresponding to the adjacent pull-in region will be almost as likely to be correct. We say then that the integer solution cannot be distinguished with enough confidence from the second-best integer solution.

The best and second-best integer solution in the integer least-squares sense are the ones for which the following holds:

$$\left\|\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}\right\|^2_{\boldsymbol{Q}_{\hat{a}}} \leq \left\|\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_2\right\|^2_{\boldsymbol{Q}_{\hat{a}}} \leq \left\|\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_{\boldsymbol{i}}\right\|^2_{\boldsymbol{Q}_{\hat{a}}}, \quad \forall \breve{\boldsymbol{a}}_{\boldsymbol{i}} \in \mathbb{Z}^{\boldsymbol{n}} \setminus \{\breve{\boldsymbol{a}}\} \tag{5}$$

where $\breve{\boldsymbol{a}}_2$ the second-best integer ambiguity estimate given by LAMBDA.

A well-known integer validation test which considers the best and second-best integer solution is the so-called ratio test, [Euler and Schaffrin, 1991; Wei and Schwarz, 1995; Han and Rizos, 1996]. The reciprocal of this test reads:

Accept the integer least-squares ambiguity solution $\breve{\boldsymbol{a}}$ if and only if:

$$\frac{\left\|\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}\right\|^2_{\boldsymbol{Q}_{\hat{a}}}}{\left\|\hat{\boldsymbol{a}} - \breve{\boldsymbol{a}}_2\right\|^2_{\boldsymbol{Q}_{\hat{a}}}} \leq \boldsymbol{\mu} \tag{6}$$

The reciprocal of the ratio test is used here, because then the threshold value $\boldsymbol{\mu}$ should always be between 0 and 1, as follows from Eq.(5).

The rationale behind this test is that if the floating ambiguity solution is sufficiently closer to the integer ambiguity solution than to any other integer candidate, where the distance is measured in the metric of the variance matrix $Q_{\hat{a}}$, then $\breve{a}$ can be considered sufficiently more likely than any other integer candidate. Hence, the ratio test is a discrimination test; it is not suitable for detecting biases, since a bias which affects the floating ambiguities can be such that the floating ambiguities are close to an integer, but the wrong one. Hence, it is not a validation test in the sense that it tells you whether the fixed solution is correct or incorrect.

The problem with the ratio test is the choice of the threshold (or: critical) value μ. In practice a fixed value is generally used, e.g. μ=1/2 [Wei and Schwarz, 1995] or μ=1/3 [Leick, 2003] . However, this value may sometimes be too conservative such that the integer solution is unnecessarily rejected, in other cases the opposite may be true such that the integer ambiguity solution is wrongly accepted

TU Delft has developed a method, based on the theory of integer aperture estimation [Teunissen, 2003], which allows one to choose the threshold value based on the measurement model under consideration such that the failure rate (probability of accepting the wrong integer solution) is guaranteed to be below a certain user-defined value. Hence, the threshold value will be different depending on the satellite geometry, the parameterization of the model and the measurement noise, see e.g. [Teunissen and Verhagen, 2004; Verhagen and Teunissen, 2006; Verhagen, 2006]. In case of a very strong model, the failure rate will be (very close to) zero, and this would imply that the threshold value becomes equal to 1 (integer solution is always accepted). In order to build in some additional protection, one could however choose to assign a maximum value to the threshold value.

AMBIGUITY RESOLUTION IN THE PRESENCE OF MULTIPATH

A bias in the observations will propagate in the parameter estimates. Firstly, the float solution will be affected. As a result, the biased float ambiguities may results in an incorrect fixed ambiguity solution, even though the precision and thus the theoretical success rate is very high. Therefore, it is important to study the impact of biases on IAR. The focus will be on multipath errors. It is investigated whether multipath estimation may either improve the reliability of IAR, or may serve as an indication that reliable IAR is not feasible.

Bias-affected success rates

A mis-specification in the GNSS model will generally lead to a biased least-squares estimator and thus the float ambiguity and baseline solutions will be biased in that case. Such biases could be generated by outliers in the code data, cycle slips in the carrier phase data, multipath, the presence of unaccounted atmospheric delays, or any other unmodeled error source. Most of these problems can be addressed by adapting the mathematical model accordingly. Usually this results in a somewhat weaker model, due to the inclusion of extra unknown parameters in the model, but it also results in an unbiased solution.

Multipath forms a specific problem, as it is often not clear how the mathematical model should be adapted. Consequently, of all probable causes of bias, multipath is currently regarded as one of the major problems in GNSS applications where speed and accuracy are important.

In the presence of a bias, the correct functional model reads:

(7) $$E\{y\} = \begin{pmatrix} B & A \end{pmatrix} \begin{pmatrix} b \\ a \end{pmatrix} + \nabla y$$

instead of model (1). The bias is thus given by ∇y . This bias propagates in the float solution, which is denoted as:

(8) $$\begin{pmatrix} \hat{b}_{\nabla} \\ \hat{a}_{\nabla} \end{pmatrix} = \begin{pmatrix} \hat{b} + \nabla \hat{b} \\ \hat{a} + \nabla \hat{a} \end{pmatrix}$$

where $\hat{b}$ and $\hat{a}$ are the float solutions corresponding to the unbiased model (1), and $\nabla \hat{b}$ and $\nabla \hat{a}$ are the biases caused by ∇y .

The success rate was already introduced as the probability of correct integer estimation. A bias in the float ambiguity solution implies a higher probability that the wrong integer solution is obtained, i.e. in a lower success rate, which is called the *bias-affected* success rate.

The effect of multipath

Multipath is known to be a dominant error source in high-accuracy GNSS applications. Therefore, multipath mitigation has received considerable attention in the past years.

Typical multipath mitigation methods used in high-end receivers attempt to compensate multipath errors by sensing the deviations of the shape of the correlation peak from its ideal shape.

When no multipath is present, a GNSS receiver only receives the line-of-sight signal from the satellite. It computes the correlation function of this signal with the local replica of the PRN code and tracks the main peak of this correlation function. In ideal conditions (infinite signal bandwidth and no multipath), the correlation function is a triangle in the case of GPS signals.

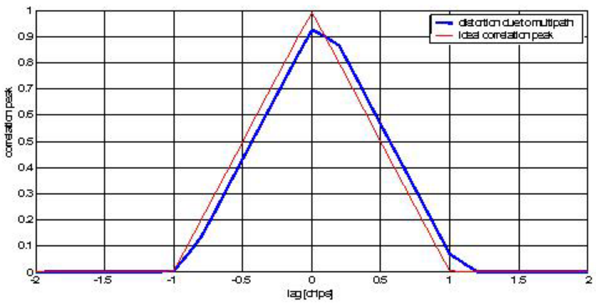

Figure 2. Distortion in the correlation peak due to multipath.

When multipath is present, the incoming signal is a superposition of several copies of the same signal with different delays, amplitudes and phases. All these copies contribute to the correlation process in such a way, that the composite correlation peak deviates from its ideal triangular shape. Such distortion is illustrated in Figure 2.

Septentrio has developed an empirical approach to mitigating the multipath effect by taking advantage of the amount of the distortion of the correlation peak. This technique, called A-Posteriori Multipath Estimation technique (APME), corrects the measurements by estimating the multipath delay according to the following formula [Sleewaegen and Boon, 2001]:

(9) $$\boldsymbol{MP} = -0.42 \cdot \left(1 - \frac{\boldsymbol{I}_{+2}}{\boldsymbol{I}_0}\frac{1}{1-\boldsymbol{d}}\right)$$

where:

$\boldsymbol{I_0}$ is the punctual correlation value;

$\boldsymbol{I_{+2}}$ is the correlation value measured at a delay of d chips with respect to the punctual code.

One of the underlying assumption in this formula is that the slope of the correlation peak for multipath-free signal is constant for all satellites.

Hence, with APME the multipath error affecting the code tracking is estimated a-posteriori, and this estimated error is subtracted from the code-phase measurement. APME also provides the standard deviation of the multipath estimates.

As a starting point, we will now consider the model:

(10) $$\boldsymbol{E}\{\boldsymbol{y}\} = \begin{pmatrix}\boldsymbol{B} & \boldsymbol{A}\end{pmatrix}\begin{pmatrix}\boldsymbol{b}\\ \boldsymbol{a}\end{pmatrix} + \boldsymbol{Mm}\,; \quad \boldsymbol{D}\{\boldsymbol{y}\} = \boldsymbol{Q_y}$$

Where $\boldsymbol{m}$ is a vector with the multipath errors on the code measurements of one or more satellites, and $\boldsymbol{M}$ specifies which measurements are affected. If the float solution would be computed without taking into account this term $\boldsymbol{Mm}$, we would obtain a biased solution, with $\nabla \boldsymbol{y} = \boldsymbol{Mm}$.

With APME, we do not estimate $\boldsymbol{m}$ together with the other unknown parameters, but before computing the float solution. As such, an estimate $\hat{\boldsymbol{m}}$ and associated variance matrix $\boldsymbol{Q}_{\hat{\boldsymbol{m}}}$ are obtained. This estimate is then used to correct the observations:

(11) $$\boldsymbol{y^c} = \boldsymbol{y} - \boldsymbol{M\hat{m}}; \qquad \boldsymbol{Q}_{\boldsymbol{y^c}} = \boldsymbol{Q_y} + \boldsymbol{M}\boldsymbol{Q}_{\hat{\boldsymbol{m}}}\boldsymbol{M^T}$$

It follows that the precision of $\boldsymbol{y^c}$ is lower than that of the uncorrected observations, since we have to take into account the uncertainty in the multipath estimates. Recall, however, that without the correction, we would obtain a biased solution. The decreased precision implies that the success rate of IAR will also decrease as compared to the multipath-free case. It will be investigated here whether this success rate is still higher than the bias-affected success rate, i.e. the success rate obtained if APME would not be applied.

EXPERIMENTAL SET-UP

Simulated data will be used to study the performance of IAR in the presence of multipath, with and without APME. The performance will be compared for IAR

- without applying a ratio test,
- with the traditional ratio test using a fixed threshold value of 0.5,
- with the ratio test using the model-driven threshold value.

Although the ratio test is not intended for bias-detection, it is interesting to study the bias-robustness of IAR with ratio test.

Simulated data

This analysis is based on simulated Galileo and multipath data for a time interval of 600 seconds (1 Hz). The baseline is very short (approximately 584 meters), so that atmospheric errors can be assumed to be completely cancelled out after double differencing.

Observations on the following Galileo frequencies are available:

- L1 (1575.42 MHz), BOC(1,1)
- E6 (1278 MHz), BOC(10,5)
- E5 (1191.795 MHz), AltBOC(15,10)

For the simulations, the broadcasted GPS ephemeris file for day 205/06 was used (hence GPS constellation, but Galileo signals were considered).

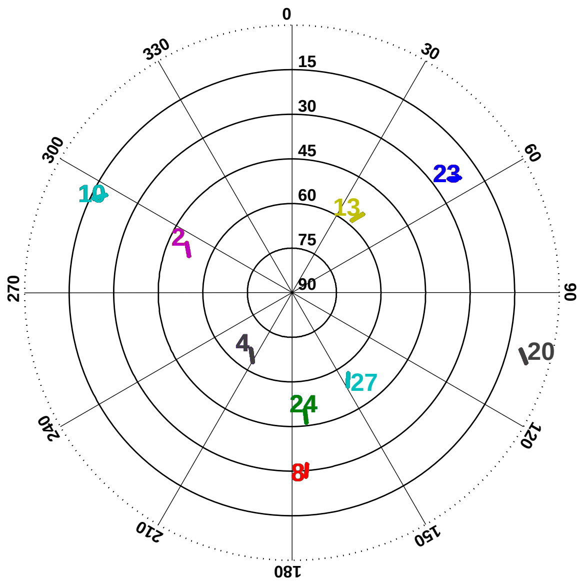

Figure 3. Skyplot of visible satellites.

The corresponding skyplot for the time interval under consideration is shown in Figure 3. It can be seen that 9 satellites are in view.

The noise on the observations is shown in Figure 4.

The following scenarios will be considered:

- all satellites in view, all frequencies
- cut off elevation of 20º, all frequencies
- cut off elevation of 20º, L1 and E5 frequency

With a cut off elevation of 20º PRN 10 and 20 are not tracked. Both without and with the elevation mask the PDOP value is well below 1. Furthermore, the theoretical integer least-squares success rate is equal to 1 in both cases if no multipath would be present. However, with 9 satellites in view the bias-affected success rate is approximately 0.6, whereas with an elevation mask of 20º the it is only approximately 0.3. Hence, in the presence of biases, the number of satellites is a very important factor, as will be clear from the results presented here as well.

For each scenario the following options are considered:

- No MP : data without multipath
- MP : data with multipath
- APME : APME-corrected data.

Epoch-by-epoch processing is applied in order to investigate the feasibility of instantaneous ambiguity resolution.

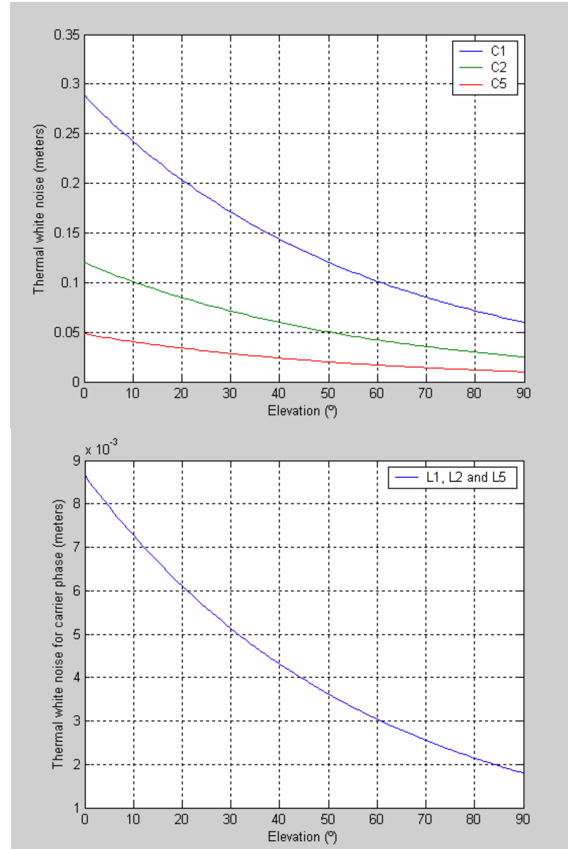

Figure 4. Code noise (top) and carrier noise (bottom). C2 and L2 correspond to code and phase on E6 respectively.

This would be a realistic processing strategy if dynamics are high or even unknown, although in practice one might keep the ambiguities fixed as soon as reliable IAR is possible. But here the IAR performance is analyzed, and therefore IAR is applied every epoch a-new.

Note that the success rates corresponding to the MP option are the *bias-affected* success rates.

Results of position components will be given as the difference with respect to the multipath-free data.

APME corrections

APME corrections were only applied to the L1 code and E6 code observations. Hence, $\hat{\boldsymbol{m}}$ does not contain estimates for the multipath errors on E5 code observations and neither on all phase observations. The true multipath errors on the code observations and corresponding APME corrections are shown in Figure 5 - Figure 7, and Figure 12 - Figure 17 (at the end of this paper).

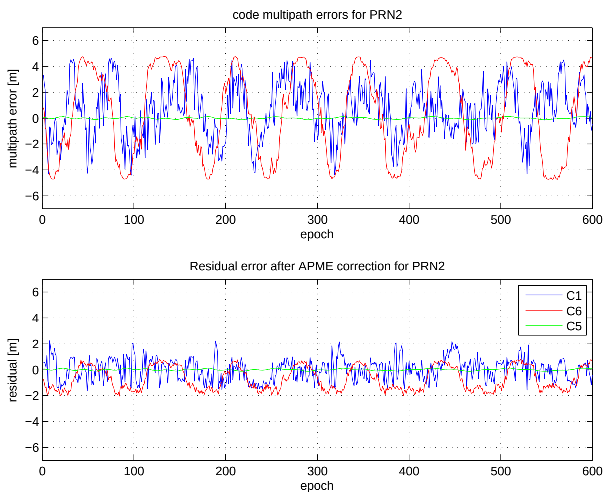

Figure 5. Multipath errors for PRN 2.

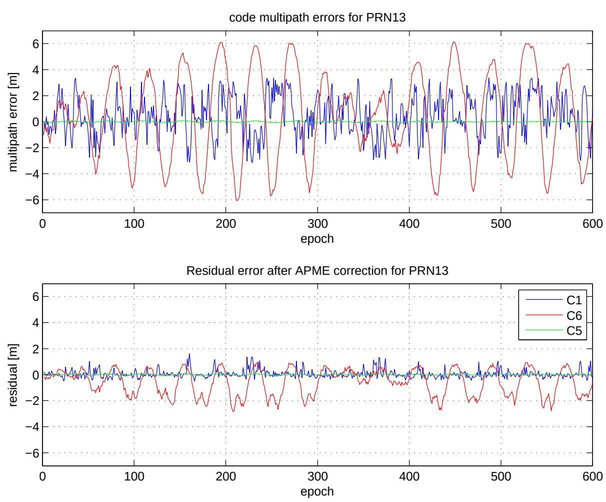

Figure 6. Multipath errors for PRN 13.

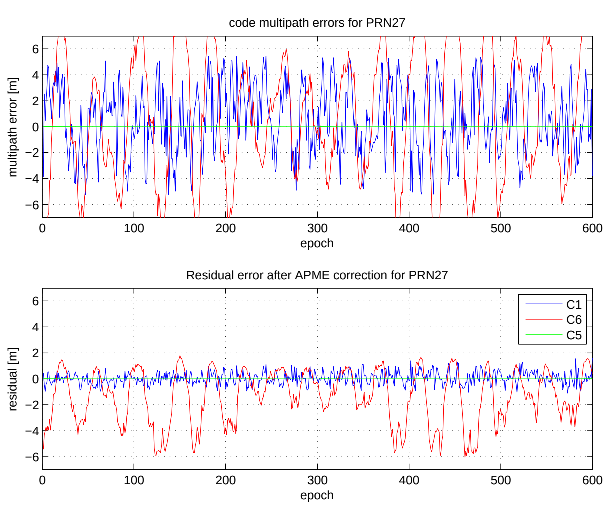

Figure 7. Multipath errors for PRN 27.

It can be seen that the remaining errors on E6 code observations are still considerable in most cases. The reason is that the effectiveness of APME depends very much on the correlation curves of the signals, which is very different for the different GPS and Galileo signals. Unfortunately, the correlation curves of the Galileo E6 signal were not known yet at the time of the simulation. In the future, APME will therefore be improved and give better results.

The undifferenced code standard deviations (L1 and E6) are adjusted for the uncertainty of the APME corrections [Sleewaegen and Boon, 2001] if applicable:

(12)
$$\boldsymbol{\sigma}_{c1,\boldsymbol{APME}}^2 = \boldsymbol{\sigma}_{c1}^2 + 0.085^2\,\boldsymbol{m}$$
$$\boldsymbol{\sigma}_{c6,\boldsymbol{APME}}^2 = \boldsymbol{\sigma}_{c6}^2 + 0.085^2\,\boldsymbol{m}$$

This means a vertical shift upwards in Figure 4 for the code standard deviations on L1 and E6. The uncertainty of the APME corrections is not elevation dependent. It will be shown that this adjustment is essential for improved IAR performance.

Ratio test

The higher standard deviations as a result of the APME corrections will result in higher integer least-squares failure rates, and thus to lower critical value for the ratio test (i.e. test will be more conservative). Hence, the APME corrections will improve the float solution since part of the multipath bias is removed, and at the same time the ratio test provides additional protection against incorrect ambiguity fixing.

The maximum value of the threshold value $\boldsymbol{\mu}$ was set to 0.9.

EXPERIMENTAL RESULTS

Table 1 presents the following results:

- Probability that correct decision is made ($P_{correct}$),
- Probability that ambiguities are fixed correctly (P_s),
- Probability that ambiguities are fixed incorrectly (P_f),
- Probability that the fixed solution is unnecessarily rejected (P_{false}),
- Empirical standard deviations.

Figure 8 and Figure 9 show as an example the position solutions for the 3-frequency with 7 visible satellites.

Below the results are analyzed for the 3- and 2-frequency scenarios, either with or without APME.

Table 1. Empirical results for different scenarios.

frequency	scenario	decision	ratio test results				standard deviations [m]		
			$P_{correct}$	P_s	P_f	P_{false}	σ_E	σ_N	σ_U
9 satellites									
L1,E6,E5	no MP		1	1	0	0	0.002	0.002	0.004
	MP	P_f=0.001	0.883	0.537	0.057	0.060	0.378	0.325	0.536
		μ = 0.5	0.547	0.143	0	0.453	0.429	0.367	0.631
		μ = 1	0.597	0.597	0.403	0	0.348	0.304	0.557
	APME	P_f=0.001	1	1	0	0	0.002	0.002	0.005
		μ = 0.5	1	1	0	0	0.002	0.002	0.005
		μ = 1	1	1	0	0	0.002	0.002	0.005
7 satellites									
L1,E6,E5	no MP		1	1	0	0	0.003	0.002	0.007
	MP	P_f=0.001	0.608	0.227	0.337	0.055	0.677	0.392	1.226
		μ = 0.5	0.795	0.077	0	0.205	0.693	0.404	1.275
		μ = 1	0.282	0.282	0.718	0	0.655	0.381	1.201
	APME	P_f=0.001	1	1	0	0	0.003	0.002	0.009
		μ = 0.5	0.888	0.888	0	0.112	0.036	0.057	0.299
		μ = 1	1	1	0	0	0.003	0.002	0.009
7 satellites									
L1,E5	no MP		1	1	0	0	0.004	0.003	0.010
	MP	P_f=0.001	0.990	0.987	0.002	0.008	0.012	0.021	0.119
		μ = 0.5	0.813	0.808	0	0.187	0.038	0.069	0.385
		μ = 1	0.995	0.995	0.005	0	0.009	0.013	0.091
	APME	P_f=0.001	0.997	0.997	0	0.003	0.005	0.009	0.074
		μ = 0.5	0.868	0.868	0	0.132	0.029	0.062	0.324
		μ = 1	1	1	0	0	0.005	0.003	0.013

3-frequency MP scenario

The IAR performance with model-driven ratio test is much better than without the ratio test, while the standard deviations are of the same order.

Note that with 9 satellites in view, the maximum position error without ratio test is 2.26 meters, whereas if the ratio test (either with model-driven or fixed threshold value) is applied the maximum error is 1.98 meters. The reason is that incorrectly fixed ambiguities may cause large biases. On the other hand, incorrect fixing may also lead to an improved position.

Hence, empirical standard deviations of the fixed position components are of the same order with and without applying the ratio test, but the maximum error is smaller thanks to the smaller probability of incorrect fixing if the ratio test is applied.

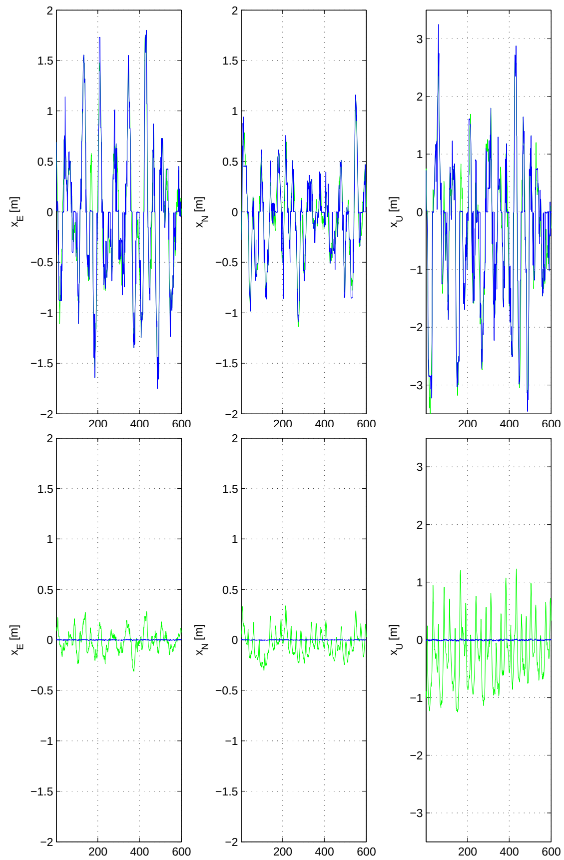

Figure 8. Float (green) and fixed (blue) position with model-driven ratio test. 7 satellites, 3 frequencies.
Top: MP; Bottom: APME.

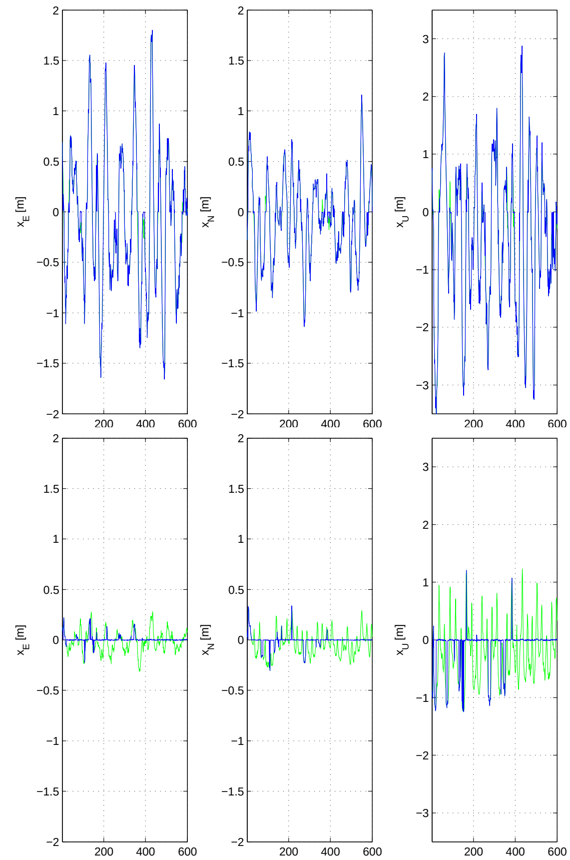

Figure 9. Float (green) and fixed (blue) position with traditional ratio test. 7 satellites, 3 frequencies.
Top: MP; Bottom: APME.

With the traditional ratio test, which is much more conservative, the failure rate is equal to zero, but the false alarm rate is much higher. As a result the empirical standard deviations are higher.

In summary:

- on average a higher failure rate (i.e. less-conservative ratio test) is acceptable in the presence of multipath in terms of precision, but
- a high failure rate means less reliability and may lead to larger position errors.

3-frequency APME scenario

Even though the APME corrections for E6 code observations are not so good, the results after APME is applied are much better. IAR performs very well, and the standard deviations of the fixed position components are at the sub-centimeter level.

When only 7 satellites are tracked, the traditional ratio test is far too conservative, which degrades the fixed positions in many epochs.

2-frequency MP scenario

The 2-frequency scenario is only considered here when 7 satellites are visible. With 9 satellites the success rates would always be equal to or very close to 1.

It can be seen that the more conservative traditional ratio test results in higher empirical standard deviations due to the higher false alarm rates. The E5 signal contributes so much to the precision, that the multipath errors on the L1 observations do not harm the solution very much.

In this case the IAR performance is actually a little bit better if no ratio test is applied at all.

2-frequency APME scenario

With APME applied the empirical standard deviations of the position are very good, only one or two millimeters worse than those obtained if there would be no multipath at all.

Note that the results with the 2-frequency scenarios are better than with the 3-frequency ones, with and without applying APME. The reason is that the (remaining) multipath errors on the E6 code observations have a severe impact.

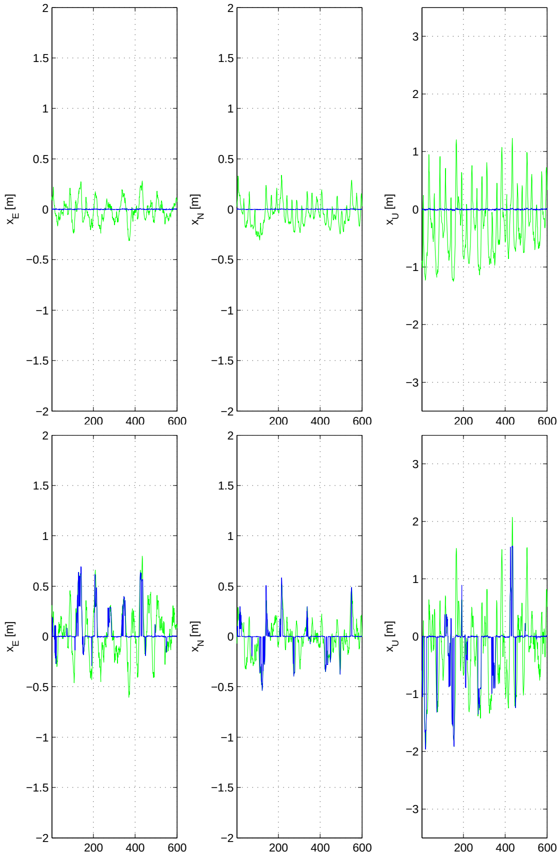

Figure 10. Float (green) and fixed (blue) position with model-driven ratio test. 7 satellites, 3 frequencies.
Top: σ_{APME} = 0.085 m; Bottom: σ_{APME} = 0 m.

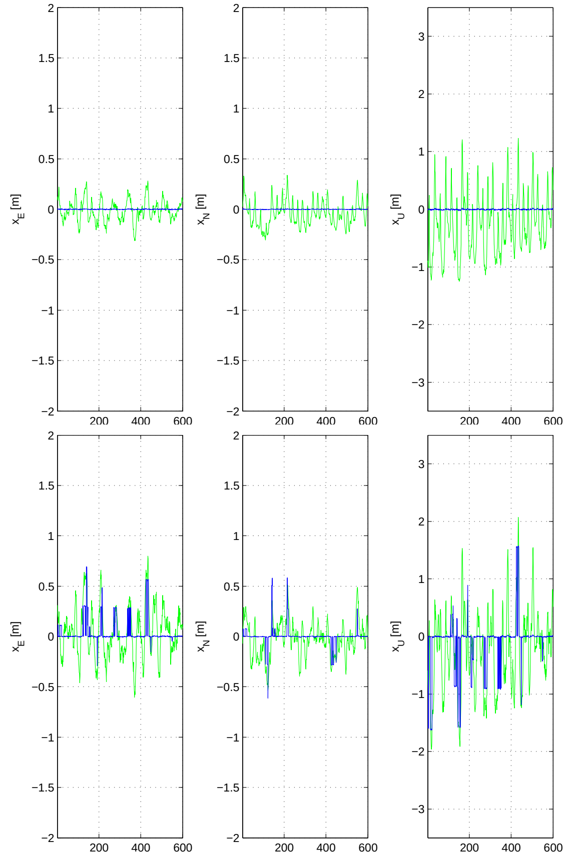

Figure 11. Float (green) and fixed (blue) position without ratio test. 7 satellites, 3 frequencies.
Top: σ_{APME} = 0.085 m; Bottom: σ_{APME} = 0 m.

The E6 observations are not considered in the 2-frequency scenarios, and there is almost no multipath on the E5 observations, which at the same time are much more precise compared to the L1 observations, and hence get a larger weight.

Stochastic model

The stochastic model was adjusted here by adding a constant to the undifferenced code standard deviations of all satellites in order to reflect the uncertainty of the APME corrections, see Eq.(12). This constant should of course represent this uncertainty correctly. If it would be ignored, the IAR performance could be seriously worse.

As an example of the impact, Figure 10 and Figure 11 show the float and fixed positioning errors for the 3-frequency APME scenario with 7 visible satellites, with and without adding the constant to the code standard deviations. Obviously, the IAR success rate with or without the ratio test is much lower if the uncertainty of the APME corrections is not properly taken into account. As a result, the precision of the fixed solutions is much worse.

CONCLUDING REMARKS

The application of multipath mitigation and an adjusted stochastic model has a very beneficial effect on the reliability of IAR and consequently the precision of the position solution is very much improved.

The ratio test with model-driven threshold value offers some protection against biases and results in relatively small standard deviations. The probability that the correct decision is made based on the ratio test is nearly always larger than if the fixed solution would be unconditionally accepted (***μ***=1); and in general it is also larger than if the traditional ratio test is applied (***μ***=0.5), except if the bias is very large. But in that case, fixing the wrong ambiguities does not have a significant effect on the precision of the fixed position. So, in the presence of (severe) multipath, a more conservative ratio test is beneficial in term of IAR performance, but NOT per se in terms of positioning accuracy. In this case it is important of course, that one should not continue to use the wrongly fixed ambiguities in the next epochs, even though the incorrect fixing is not problematic in the current epoch. It is therefore recommended to apply ambiguity resolution on an epoch-by-epoch basis. So, even if the processing scheme is based on a Kalman filter or recursive estimation, ambiguity resolution can be applied every epoch a-new, at least until one is sufficiently sure that they are correct (when they did not change for a long time). This strategy is especially desirable in case one has some indication that (multipath) biases may be present. This will not harm the ambiguity resolution performance. Note that the float ambiguities from the current epoch can be used in the time-update of the Kalman filter.

In some cases, IAR performance and/or positioning accuracy without applying a ratio test seems to be better. See the results in table 1 for the APME scenario with L1 and E5. In that case, this is caused by setting the maximum threshold value at 0.9. Setting the maximum threshold to a larger value, notably to 1, would however cause worse performance of the ratio test with model-driven threshold in other cases. Hence, in some situations the ratio test slightly degrades the performance because of a higher false alarm rate. This is the price one has to pay for improved reliability, i.e. a better protection against large position biases due to incorrect fixing. In other words, the ratio test brings higher reliability, but not always improved precision.

Remarks

- Results with respect to multipath mitigation are rather pessimistic since APME, which is an empirical method designed with GPS signals, has not yet been tuned to the correlation curves of Galileo;
- A severe multipath environment was simulated here. Results would improve if not all satellites were affected by multipath.
- Combined Galileo/GPS will give even better results; in that case scenarios with less than 10 satellites are only expected in environments with large obstructions (due to signal blocking).

ACKNOWLEDGMENTS

The research underlying this paper was carried out in the frame of the Swirls project, aiming at developing a professional Galileo/GPS receiver. The Swirls project is coordinated by Septentrio and is partially funded by the European GNSS Supervisory Authority. It is part of the projects started under the 2nd call of the 6th Framework Programme for Galileo Research and Development. Swirls consists of a consortium of nine European partners. See http://www.swirls-gju.com

REFERENCES

Euler, H. J. and Schaffrin, B. (1991). *On a measure for the discernibility between different ambiguity solutions in the static-kinematic GPS mode*. IAG Symposia no.107, Kinematic Systems in Geodesy, Surveying, and Remote Sensing, Springer-Verlag, New York, pages 285–295.

Han, S. and Rizos, C. (1996). *Integrated methods for instantaneous ambiguity resolution using new-generation GPS receivers*. Proc. of IEEE PLANS'96, Atlanta GA, pages 254–261.

Leick, A. (2003). *GPS Satellite Surveying*. John Wiley and Sons, New York, 3rd edition.

Sleewaegen JM and Boon F (2001). *Mitigating short-delay multipath: a promising new technique*. Proc. of ION GPS-2001, Salt Lake City UT..

Teunissen, P. J. G. (1993). *Least squares estimation of the integer GPS ambiguities*. Invited lecture, Section IV Theory and Methodology, IAG General Meeting, Beijing.

Teunissen PJG (1997). *A theorem on maximizing the probability of correct integer estimation*. Artificial Satellites, 34(1):3-9.

Teunissen, P. J. G. (1998). *GPS carrier phase ambiguity fixing concepts*. In: PJG Teunissen and Kleusberg A, GPS for Geodesy, Springer-Verlag, Berlin.

Teunissen, P. J. G. (1999). *An optimality property of the integer least-squares estimator*. Journal of Geodesy, 73(11):587–593.

Teunissen, P. J. G. (2003). *Integer aperture GNSS ambiguity resolution*. Artificial Satellites, 38(3):79–88.

Teunissen, P. J. G. and Verhagen, S. (2004). *On the foundation of the popular ratio test for GNSS ambiguity resolution*. Proc. of ION GNSS-2004, Long Beach CA, pages 2529–2540.

Verhagen, S. and P. J. G. Teunissen (2006). *New global navigation satellite system ambiguity resolution method compared to existing approaches*. Journal of Guidance, Control, and Dynamics, vol. 29, no. 4, pp. 981–991.

Verhagen, S. (2006). *Improved performance of Multi-Carrier Ambiguity Resolution based on the LAMBDA method*. In Proc. of Navitec 2006, ESA-ESTEC, Noordwijk NL, 8 pages.

Wei, M. and Schwarz, K. P. (1995). *Fast ambiguity resolution using an integer nonlinear programming method*. Proc. of ION GPS-1995, Palm Springs CA, pages 1101–1110.

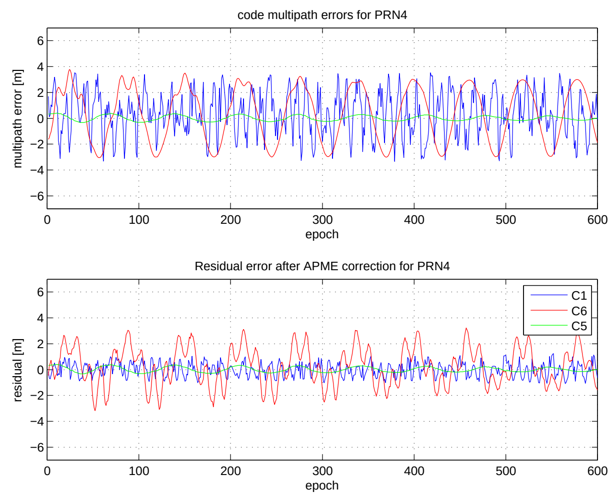

Figure 12. Multipath errors for PRN 4.

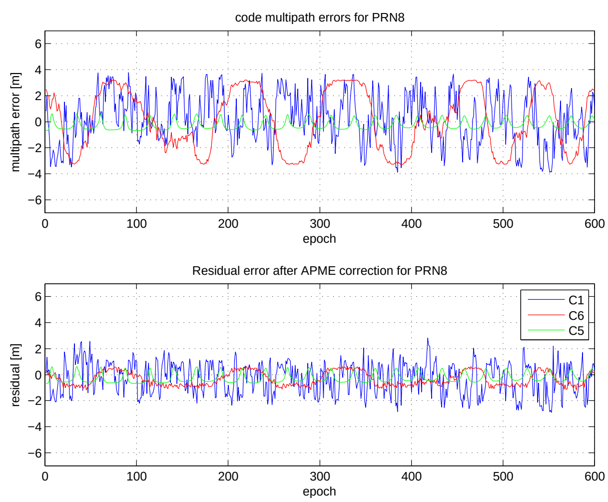

Figure 13. Multipath errors for PRN 8.

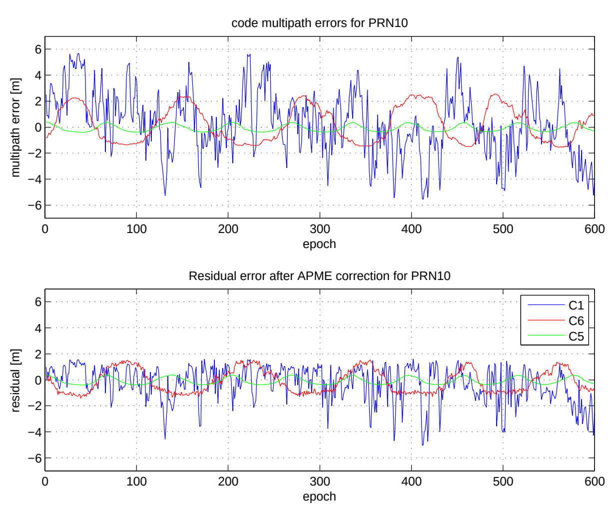

Figure 14. Multipath errors for PRN 10.

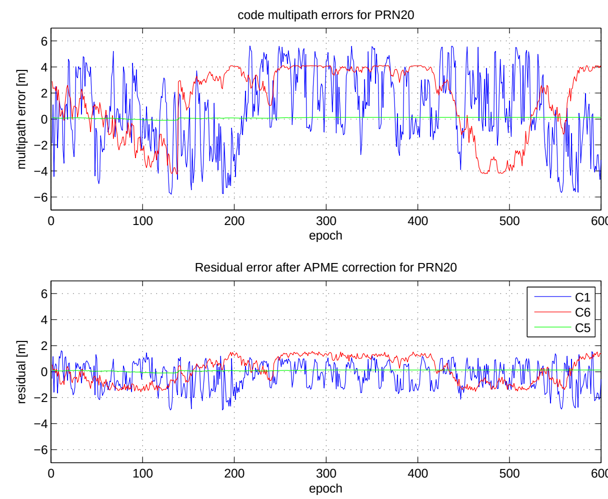

Figure 15. Multipath errors for PRN 20.

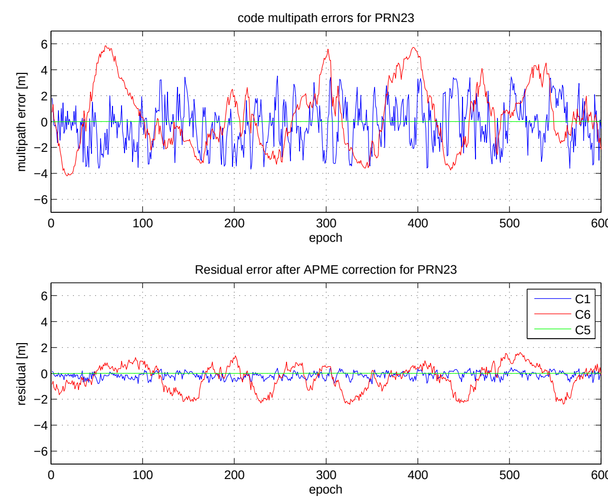

Figure 16. Multipath errors for PRN 23.

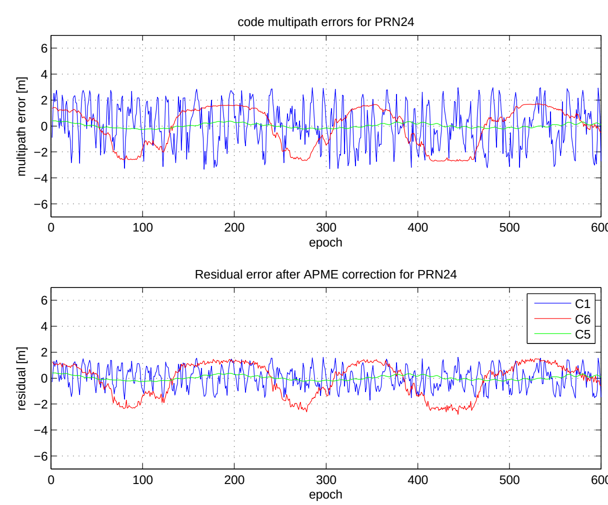

Figure 17. Multipath errors for PRN 24.