THE LAMBDA-METHOD FOR FAST GPS SURVEYING

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Abstract:

Fast and high precision relative GPS positioning based on short observation time span data is possible, when reliable estimates of the integer double difference ambiguities can be determined in an efficient manner. In the present contribution it will be shown how this can be achieved by means of the Least-squares AMBiguity Decorrelation Adjustment (LAMBDA). The method was introduced in [Teunissen, 1993] and consists of two steps: (a) a decorrelation of the double difference ambiguities; and (b) a sequential conditional least-squares based search. The decorrelation is based on an integer approximation of the conditional least-squares transformation. Through the decorrelation, the spectrum of sequential conditional ambiguity variances is flattened and lowered, and a dramatic improvement in ambiguity precision is reached. Both the theoretical and practical intricacies of the method will be reviewed.

1. Introduction

The GPS double difference carrier phase measurements are ambiguous by an unknown integer number of cycles. The a priori knowledge of the integerness of the ambiguities can be used to strengthen the baseline solution. This is in particular of relevance for applications where use is made of short observation time spans. The principle of least-squares is used to determine the most likely integer estimates of the ambiguities. Due to the integer nature of the ambiguities, no direct method exists for the computation of these most likely integer estimates. Hence, use has to be made of a discrete search process. Although one can construct rather straightforward search procedures for the correct determination of the integer ambiguity estimates, difficulties will be encountered when one requires that the search procedures are to be efficient, in terms of computational speed, as well. This is particularly true when short observation time spans are used.

In the literature, many important contributions have been made in the area of GPS integer ambiguity estimation. Starting from rather simple but time consuming rounding schemes, the methods have evolved into complex and efficient search algorithms. Examples are the methods and refinements, which have been proposed in [Blewitt, 1989], [Euler and Landau, 1992], [Frei, 1991], [Hatch, 1991] and [Wübbena, 1991]. Nevertheless, it has been our experience that still some room of improvement, in terms of efficiency and general applicability, is possible. It is our believe, that these requirements are satisfied by the least-squares ambiguity decorrelation adjustment. The basic principles of the method, together with some typical results, will therefore be presented in this contribution.

2. Integer least-squares estimation

The nonlinear observation equation for the difference between the simultaneous phase measurements of a receiver *j* of the signals transmitted by two different satellites, *k* and *l*, and the simultaneous measurements of a second receiver *i* of the same signals, reads

$$\Phi_{ij}^{kl}(t) \;=\; \rho_{ij}^{kl} - I_{ij}^{kl} + T_{ij}^{kl} + \delta m_{ij}^{kl} + \lambda N_{ij}^{kl} \;+\; \varepsilon_{ij}^{kl} \tag{1}$$

The unknown parameters in this equation are: (a) ρ_{ij}^{kl}, the linear combination of the four geometric distances between

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the two receivers, i and j, and the two satellites, k and l; it depends in a nonlinear way on the unknown position of receiver j with respect to receiver i; (b) the two linear combinations, I_{ij}^{kl} and T_{ij}^{kl}, of the four ionospheric and tropospheric delay terms; (c) the combined multipath term δm_{ij}^{kl}; and (d) the integer double difference ambiguity N_{ij}^{kl}. The wavelength is denoted by λ and ε_{ij}^{kl} denotes the noise term.

With the above double difference (DD) carrier phases, we can form a system of observation equations, which after linearization with respect to the unknown parameters, gives the linear system of equations

$$y = Aa + Bb + e \,, \tag{2}$$

where y is the vector of observed minus computed DD carrier phases, a is the unknown integer vector of DD ambiguities, b is the unknown vector containing all real-valued parameters, A and B are the corresponding design matrices, and e is the vector that contains the measurement noise. In the following, we will assume for reasons of simplicity, that multipath is absent, that the troposphere has been accounted for by means of an a priori model and that the length of the baseline is sufficiently short, so that we may neglect the ionospheric delay. Hence, the entries of the vector b consist of the three unknown baseline coordinates. It is remarked however, that this simplification of the observation equations is not a prerequisite for the theory that will be dealt with in this paper.

The least-squares criterion for solving the above linear system of observation equations reads

$$\min_{a,\,b} \; (y - Aa - Bb)^* Q_y^{-1} (y - Aa - Bb), \text{ with } a \in Z^m, \; b \in R^3 \tag{3}$$

Due to the integer constraint $a \in Z^m$, this minimization problem has been referred to as an *integer least-squares* problem in [Teunissen, 1993]. The solution of the integer least-squares problem will be denoted as $\check{a}$ and $\check{b}$, and the solution of the corresponding unconstrained least-squares problem will be denoted as $\hat{a}$ and $\hat{b}$. The estimates $\check{a}$ and $\check{b}$ will be referred to as the *fixed* solution and the estimates $\hat{a}$ and $\hat{b}$ will be referred to as the *float* solution.

It has been shown by Teunissen (1993) that the above minimization problem can be solved in the following steps. The first step consists of solving the unconstrained least-squares problem. This implies solving (3), with $a \in Z^m$ replaced by $a \in R^m$. As a result, real-valued estimates for both the ambiguities and baseline components are obtained, together with their corresponding variance-covariance matrices:

$$\begin{pmatrix} \hat{a} \\ \hat{b} \end{pmatrix}, \begin{pmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{pmatrix} \tag{4}$$

This result forms the input of the second step. In the second step one solves for the vector of *integer* least-squares estimates of the ambiguities, $\check{a}$. It follows from solving

$$\min_{a} \; (\hat{a} - a)^* Q_{\hat{a}}^{-1} (\hat{a} - a), \text{ with } a \in Z^m \tag{5}$$

Once the solution $\check{a}$ has been obtained, the residual $(\hat{a} - \check{a})$ is used to adjust the *float* solution $\hat{b}$ of the first step, to get the *fixed* solution $\check{b}$. It reads

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \check{a}) \tag{6}$$

3. On the baseline precision

The relevance of integer least-squares estimation, in case of GPS surveying, becomes clear if we consider the precision of the baseline. Application of the error propagation law to (6), assuming that the integer ambiguities are nonstochastic, gives

$$Q_{\check{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}} \tag{7}$$

The precision of the float baseline, as described by the variance-covariance matrix $Q_{\hat{b}}$, is usually very poor in case of short observational time spans. This is due to the very high altitude orbits of the GPS satellites. The relative positions of the satellites with respect to the receivers change slowly, which implies that in case of short observational time spans, the ambiguities - when treated as being real-valued - become very poorly separable from the baseline coordinates. As a result, the precision with which the baseline can be estimated will be poor. This situation changes drastically however, when one explicitly aims at resolving for the integer values of the ambiguities. If one considers the ambiguities to be known and nonstochastic, the high precision carrier phase observables will start to act as if they were high precision pseudorange observables. As a result, the baseline coordinates become estimable with a comparable high precision. Hence, we have $Q_{\check{b}} << Q_{\hat{b}}$.

Figure 1 shows, as an example, the standard deviations of both the float and fixed baseline coordinates. The baseline coordinates are expressed in a local North, East, Up system. The figure is based on an example of a 2.2 km baseline, using dual-frequency phase data to seven satellites. The standard deviations in the figure, are given - on a logarithmic scale - as function of the observation time span, which ranges from 1 second to 50 minutes. Each time, only two epochs of data were used; at the start and at the end of the time span. The a priori standard deviation of the undifferenced phase was set to 3 mm. The figure clearly shows the large gain in precision, which is experienced due to the inclusion of the integer constraint $a \in Z^m$.

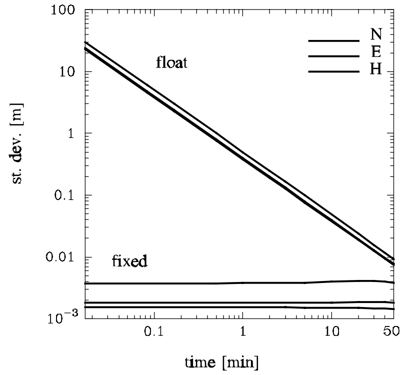

Figure 1: Baseline precision (float and fixed).

Figure 1 shows the *formal* precision of the baseline. In line with theory however, we should see a similar effect if we consider the *empirical* precision of the baseline. As an example we constructed an experiment that was repeated 100 times. Each experiment resulted in a computed float and fixed baseline. For each experiment, the observation time used was set to 5 seconds, using only two epochs of data. The results of the experiments are shown in the 3-dimensional scatter plots of figure 2. Figure 2a shows the scatter plot for the float baseline and figure 2b shows the corresponding scatter plot for the fixed baseline. Again we note the poor precision of the float solution; the spread amounts to several metres. The spread in the fixed solution is however drastically smaller; it amounts to about 1 cm, even for this short observation time span.

(a) (b)

Figure 2: Baseline scatter plot (a: float; b: fixed)

4. On the precision of the ambiguities

In the previous section, it was shown that the sole purpose of ambiguity-fixing, is , to be able, via the inclusion of the integer constraint $a \in Z^m$, to obtain a drastic improvement in the precision of the baseline solution. However, in order to be able to compute $\check{b}$, we first need to compute $\check{a}$ by solving (5). As it turns out, this is a rather computationally demanding task, in particular if use is made of a short observation time span. As it will be shown, the reason for this can be found in the statistics of the real-valued least-squares ambiguities $\hat{a}$.

It will be clear that the mimimization problem (5) can be solved rather easily when all ambiguities are decorrelated. In that case, the variance-covariance matrix Q_a is diagonal and (5) amounts to the minimization of a sum of independent squares

$$\min_{a_i} \; (\hat{a}_i - a_1)^2/\sigma_1^2 + \ldots + (\hat{a}_m - a_m)^2/\sigma_m^2, \;\text{with}\; a_i \in Z \tag{8}$$

In this case, the integer least-squares estimates of the ambiguities are found by means of a simple rounding of the least-squares estimates $\hat{a}_i$ to their nearest integer. This simple rounding scheme does not produce the required integer least-squares estimates however, in case the matrix $Q_{\hat{a}}$ is nondiagonal. As it was shown in [Teunissen, 1993], the objective function of (5) can be cast into a form that resembles the sum-of-squares structure of (8), if we make use of a *sequential conditional least-squares adjustment* on the ambiguities,

$$\min_{a_i} \; (\hat{a}_1 - a_1)^2/\sigma_1^2 + (\hat{a}_{2|1} - a_2)^2/\sigma_{2|1}^2 + \ldots + (\hat{a}_{m|M} - a_m)^2/\sigma_{m|M}^2 \;,\; a_i \in Z \tag{9}$$

The estimate $a_{i|I}$ is the least-squares estimate of a_i conditioned on a_j, $j = 1,\ldots,i-1$. In order to solve for (9), a search for the integer least-squares ambiguities is performed, which is based on the following sets of bounds

$$(\hat{a}_{i|I} - a_i)^2 \le l_i \sigma_{i|I}^2 \chi^2, \; i = 1,\ldots,m \tag{10}$$

where

$$l_i = (1 - \chi_{i-1}^2/\chi^2) \;\text{and}\; \chi_{i-1}^2 = \sum_{j=1}^{i-1} (\hat{a}_{j|J} - a_j)^2/\sigma_{j|J}^2$$

The search based on (10) is discussed in detail in [Teunissen, 1993, 1994c]. In the bounds of (10), we recognize the *sequential* conditional variances of the ambiguities,

$$\sigma_1^2, \; \sigma_{2|1}^2, \; \sigma_{3|2,1}^2 \;, \ldots, \sigma_{m|(m-1),\ldots,1}^2 \tag{11}$$

In figure 3, a typical example is given of the spectrum of sequential conditional standard deviations of the DD ambiguities. This example is based on a single baseline, using dual-frequency carrier phase data only, observing 7 satellites with a time span of 1 second (2 epochs). The spectrum shows a large discontinuity when passing from the third to the fourth ambiguity. This discontinuity is very typical for GPS surveying. Its cause has been discussed in detail in [Teunissen, de Jonge and Tiberius, 1994]. Due to the large discontinuity, the first three conditional standard deviations are very large, while the remaining nine conditional standard deviations are extremely small. The implication of this discontinuity for the search of the integer least-squares DD ambiguities is as follows. Since the first three conditional variances are rather large, the first three bounds of (10) will be rather loose. Hence, quite a number of integer triples will satisfy these first three bounds. The remaining conditional variances however, are very small. The corresponding bounds in (10) will therefore be very tight indeed. This implies, that the potential for search halting is very significant. As a consequence a large number of trials are required, before one is able to find an integer *m*-tuple that satisfies all bounds. This is therefore the reason, why in case of short observation time spans with carrier phase data only, the search for the integer least-squares DD ambiguities performs so poorly.

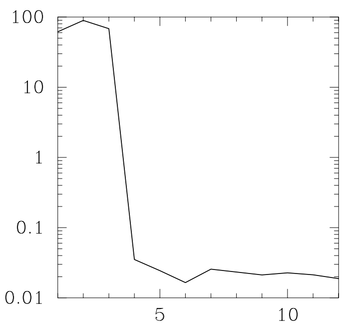

Figure 3: Spectrum of sequential conditional standard deviations

5. The LAMBDA-method

In the previous section, we have seen that the search for the integer least-squares solution suffers from the fact that the spectrum of conditional variances of the DD ambiguities shows a large discontinuity. In this section we will review the principles of the LAMBDA-method and show how the method enables one to compute the required integer ambiguities in a highly efficient manner. The two main features of the method are:

(*i*) the decorrelation of the least-squares ambiguities

(*ii*) the sequential conditional least-squares based search

On the decorrelation of the least-squares ambiguities

The first idea is to reparametrize the minimization problem (5). Let Z be an $m\times m$ matrix of full rank, that reparametrizes the ambiguities as

$$z = Z^{*}a,\ \hat{z} = Z^{*}\hat{a},\ Q_{\hat{z}} = Z^{*}Q_{\hat{a}}Z \tag{12}$$

Then

$$(\hat{a}-a)^{*}Q_{\hat{a}}^{-1}(\hat{a}-a) = (\hat{z}-z)^{*}Q_{\hat{z}}^{-1}(\hat{z}-z) \tag{13}$$

Ideally, we would like the new variance-covariance matrix $Q_{\hat{z}}$ to be diagonal. In that case namely, the new ambiguities become completely decorrelated and the integer least-squares problem can be solved by using simply the scheme of rounding to the nearest integer. Standard approaches of diagonalization are based on using for instance the LDU-decomposition or the SVD-decomposition. Unfortunately however, these standard approaches do not work for our present problem. The reason being, that the transformation matrices of the LDU-decomposition and SVD-decomposition do not preserve the integer-nature of the ambiguities. This dilemma points out that only a restricted class of transformations qualifies for reparametrizing the ambiguities. As it was shown in [Teunissen, 1994b], it is necessary and sufficient for transformations to qualify as an *ambiguity transformation*, if they are volume preserving and have entries which all are integer.

Within the class of ambiguity transformations, we now would like to construct a transformation that decorrelates the ambiguities. The concept will briefly be explained for the two-dimensional case. For a more detailed treatment the reader is referred to [Teunissen, 1993, 1994c]. Let the ambiguities and their variance-covariance matrix be given as

$$\hat{a} = \begin{pmatrix}\hat{a}_1\\ \hat{a}_2\end{pmatrix} \text{ and } Q_{\hat{a}} = \begin{pmatrix}\sigma_1^2 & \sigma_{12}\\ \sigma_{21} & \sigma_2^2\end{pmatrix} \tag{14}$$

We know that a *conditional* least-squares adjustment results in a full decorrelation. Hence, the transformation

$$\begin{pmatrix}1 & -\sigma_{12}\sigma_2^{-2}\\ 0 & 1\end{pmatrix} \tag{15}$$

diagonalizes $Q_{\hat{a}}$. This transformation however, is not admissible since $\sigma_{12}\sigma_2^{-2}$ is real-valued in general. The idea is now to *approximate* (15), such that the resulting transformation becomes admissible. This is done by replacing $\sigma_{12}\sigma_2^{-2}$ by its nearest integer $[\sigma_{12}\sigma_2^{-2}]$. The resulting transformation

$$\begin{pmatrix}1 & -[\sigma_{12}\sigma_2^{-2}]\\ 0 & 1\end{pmatrix} \tag{16}$$

is now admissible, since it is both area-preserving and has entries which are integer. As it was shown in [Teunissen, 1993, 1994a], the above transformation reduces the correlation and improves the precision of the first ambiguity. With (16), we have of course only dealt with one of the two ambiguities, namely the first one. We therefore continue applying transformations of the type of (16), everytime followed by an interchange of the role of the two ambiguities. In this way we construct a concatenated form of transformations, in which each individual transformation reduces the variance of one of the ambiguities. The string of transformations is completed once the individual transformation reduces to the trivial identity transformation. The concatenated form of transformations is then finally our sought for ambiguity transformation. And it can be shown, that after the transformation has been applied, the transformed ambiguities have a correlation coefficient that is less than or equal to a half in absolute value. Geometrically, our decorrelating ambiguity transformation can be given the interpretation as shown in figure 4. Figure 4 shows two steps of the sequence of steps that form the decorrelating ambiguity transformation. In the first step, the vertical tangents of the confidence ellipse are pushed inwards to a level that preserves the integerness and in the second step the horizontal tangents are pushed inwards to a level that preserves the integerness. Note, since the area of the confidence ellipse is kept constant at all times, whereas the area of the box that encloses the ellipse is reduced in each step, that the confidence ellipse is forced to become more sphere-like and that the transformed ambiguities are of a better precision than the original DD ambiguities. For a proof, the reader is referred to [Teunissen, 1994a].

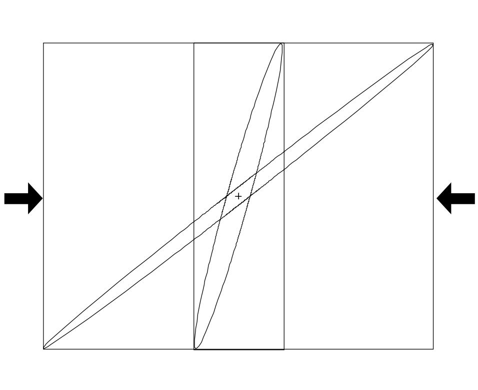

Figure 4: Ambiguity decorrelation by pushing tangents.

To indicate to which extent our decorrelating ambiguity transformation is capable of reducing the correlation among the ambiguities, figure 5 shows as an example, the two histograms of absolute values of the correlation coefficients of the original DD ambiguities a_i and of the transformed ambiguities $\hat{z}_i$. This figure shows that the original ambiguities are generally highly correlated indeed, whereas the transformed ambiguities have correlation coefficients that all are pushed towards zero in absolute value.

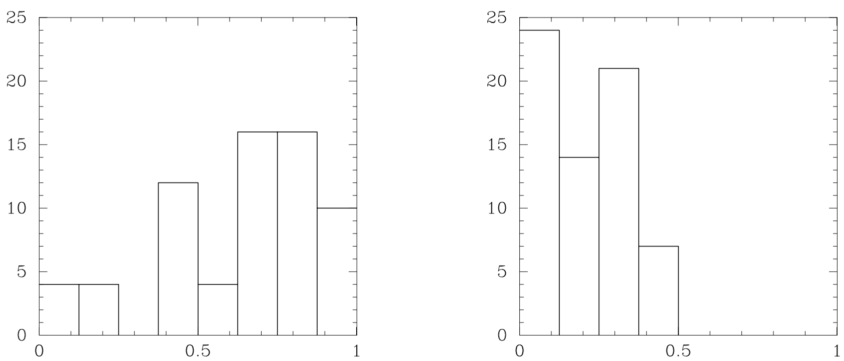

Figure 5: Histograms of absolute ambiguity correlation coefficients.

On the sequential conditional least-squares based search

Once the decorrelating ambiguity transformation Z^* has been constructed, our original minimization problem (5) can be replaced by the equivalent minimization problem

$$\min_z \; (\hat{z}-z)^* Q_{\hat{z}}^{-1} (\hat{z}-z), \text{ with } z \in Z^m \tag{17}$$

And analogous to (10), the integer minimizer of (17) is computed using the bounds that follow from a sequential conditional least-squares adjustment of the ambiguities. Now however, the search is performed in the space of the transformed ambiguities. The bounds are given as

$$(\hat{z}_{i|I} - z_i)^2 \le l_i \sigma^2_{z_{i|I}} \chi^2, \; i = 1,\ldots,m \tag{18}$$

where

$$l_i = (1 - \chi^2_{i-1}/\chi^2) \text{ and } \chi^2_{i-1} = \sum_{j=1}^{i-1} (\hat{z}_{j|J} - z_j)^2 / \sigma^2_{z_{j|J}}$$

To show that the search for the integer least-squares ambiguities can now be performed in a highly efficient manner, we consider the sequential conditional variances of the transformed ambiguities. As an example, figure 6 shows both the original as well as the transformed spectrum of sequential conditional standard deviations. This example is based on the same data as was used for figure 3. We clearly observe a dramatic improvement in the spectrum. The discontinuity has been removed and the transformed conditional standard deviations are now all of about the same order. The flattening and lowering of the spectrum implies, that our search based on (18) already commences with very tight bounds, thus assuring with a high likelihood that the first chosen integer candidates are indeed the values of the sought for integer least-squares ambiguity vector $\check{z}$. As a consequence, the integer minimizer $\check{z}$ can be computed very efficiently. With $\check{z}$, the fixed baseline can be computed as

$$\check{b} = \hat{b} - Q_{\hat{b}\hat{z}} Q_{\hat{z}}^{-1} (\hat{z} - \check{z})$$

Alternatively, one can recover the integer minimizer of (5) from invoking $\check{a} = Z^{*-1}\check{z}$ and use (6).

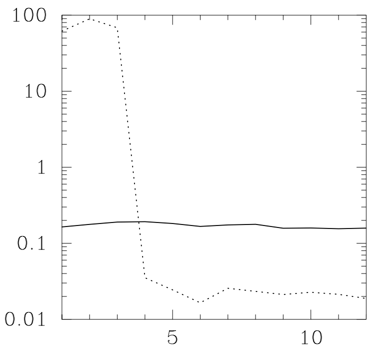

Figure 6: The original and transformed spectrum of sequential conditional standard deviations

6. Concluding remarks

In this contribution, a brief overview was given of the principles of the LAMBDA-method and of some typical results that can be obtained with the method. As opposed to some methods that have been proposed in the literature, the LAMBDA-method is generally applicable in the sense that it is independent of the form and structure of the observation equations used. In the present contribution we have followed the customary practice of working with the double difference version of the carrier phase observation equations. This however, is not really necessary. One could as well work with the undifferenced versions, as long as the original set of undifferenced noninteger ambiguities is reparametrized into a set of integer ambiguities plus a remaining set of noninteger parameters.

On the observational side, the method is also independent in principle, on whether code data are used or not, and on whether dual-frequency data are used or not. And when dual-frequency data are used, one could also apply the method to other types of ambiguities, such as the wide-lane ambiguities. Examples of transformed spectra of these different cases are given in [Teunissen, de Jonge and Tiberius, 1994]. The method is also not restricted to the case where the unknowns in the model are merely ambiguities and baseline components. If needed and when estimable, additional parameters, such as differential delays, can be included without affecting the principle of the approach. Also, the multi baseline case can be treated in quite the same way as the single baseline case was treated.

The LAMBDA-method can also be applied to the case where code measurements are used directly together with phase measurements to estimate the integer ambiguities [Hatch, 1982], [Euler and Goad, 1990]. In that case, the relative receiver-satellite geometry is dispensed with and the data are treated on a time serie by time serie basis. An analytically based study of this case, with and without the inclusion of ionospheric delays, is presented in [Teunissen, 1995b].

As it was pointed out, the LAMBDA-method is particularly suited for applications where very fast and high precision positioning results are required. This has also been confirmed by studies reported in [Tiberius and de Jonge, 1995], [Jonge, de and Tiberius, 1994, 1995] and [Goad, C. and M. Yang, 1994]. Based on a comparative study of six different methods for ambiguity resolution, Han (1995) reports on the high efficiency and reliability of the LAMBDA-method.

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