PPP-RTK: Results of CORS Network-Based PPP with Integer Ambiguity Resolution *

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ABSTRACT

During the last decade the technique of CORS-based Network RTK has been proven capable of providing cm-level positioning accuracy for rovers receiving GNSS correction data from the CORS network. The technique relies on successful integer carrier-phase ambiguity resolution at both network and user level: at the level of the CORS network, ambiguity resolution is a prerequisite in order to determine very precise atmospheric corrections for (mobile) users, while these users need to resolve their integer ambiguities (with respect to a certain CORS network master reference station) to obtain precise cm-level positioning accuracy. In case of Network RTK a user thus needs corrections from the network, plus the GNSS data of one of the CORS stations. In practice, there exists a variety of implementations of the Network RTK concept, of which VRS, FKP and MAC are best known [1, 2, 3, 4]. In this contribution we discuss a closely related concept, PPP-RTK, and show its performance on two CORS networks.

Keywords: GNSS, Rank defects, PPP-RTK, Ambiguity resolution

I. INTRODUCTION

In this contribution we describe PPP-RTK and show some of its performance. In short, PPP-RTK works very much like standard PPP, but achieves positioning accuracies comparable to Network-RTK. This high accuracy is due to the RTK ambiguity resolution capability at the user site.

Our CORS-based Network PPP-RTK approach can be described as follows. The un-differenced GNSS observations from the CORS network are processed based on the reparameterized observation equations as developed in Delft in the 1990s, see [5, 6, 7]. The estimated network parameters include single-differenced (biased) receiver clocks, (biased) satellite clocks, (biased) phase and code instrumental delays, double-differenced ambiguities, single-differenced zenith tropospheric delays and ionospheric model parameters. The biases of partially aforementioned estimable parameters are mainly due to a reparameterization process adopted for the rank-deficiency elimination. After CORS network ambiguity resolution, the very precise ambiguity-fixed network estimates are stored in a database, which is made available to RTK users. By forming certain combinations of these network parameters for correcting their single-receiver GNSS phase and code data, users can perform integer ambiguity resolution and realize cm-level positioning. In the paper the performance of this CORS-based Network PPP-RTK concept will be demonstrated by means of two tests. The first test is based on the Hong Kong CORS Network, which is of relatively small size, with inter-station distances below 30 km. The second test is based on the GPS Network Perth having inter-station distances up to 70 km.

II. PPP-RTK THEORY

In this section we give a brief outline of the theory

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of PPP-RTK. The theory is applicable to any GNSS and for an arbitrary number of frequencies.

2.1 Un-differenced Observation Equations

We start from the set of un-differenced carrier phase and pseudo range (or code) observation equations. For a receiver-satellite combination *r-s* at frequency *j*, we have [8]:

$$\begin{aligned} E(\varphi_{r,j}^s) &= l_r^s + \tau_r^s + \delta t_{r,j} - \delta t_{,j}^s - \mu_j I_r^s + \lambda_j M_{r,j}^s \\ E(p_{r,j}^s) &= l_r^s + \tau_r^s + dt_{r,j} - dt_{,j}^s + \mu_j I_r^s \end{aligned} \tag{1}$$

where $\varphi_{r,j}^s$ and $p_{r,j}^s$ denote the phase and code observable, l_r^s the receiver-satellite range, τ_r^s the slant tropospheric delay, $\delta t_{r,j}$ and $\delta t_{,j}^s$ the frequency dependent, receiver and satellite phase clock errors, $dt_{r,j}$ and $dt_{,j}^s$ the frequency dependent, receiver and satellite pseudo range clock errors, I_r^s the (first-order) slant ionospheric delays on the first frequency $(\mu_j = \lambda_j^2 / \lambda_1^2)$ and $M_{r,j}^s$ the (non-integer) ambiguity, with λ_j the wavelength of frequency j. Note that all clock errors are in units of range.

If un-differenced observation equations are used, the design matrix of the network will show a rank defect. This rank defect can be eliminated through an appropriate reduction and redefinition of the unknown parameters. Here we will follow the method of reparametrization as developed in Delft in the 1990s. It is based on the theory of S-transformations [5, 6, 7, 9].

Just for the purpose of *illustrating* the theory, we make some simplifying assumptions concerning the network. These assumptions do not reduce the general applicability of our method, but for the present discussion simplify the derivations somewhat.

The network is assumed to consist of *n* receivers (*r*=1,...,*n*), tracking the same *m* satellites (*s*=1,…,*m*) on the same *f* frequencies (*j*=1,…*f*). The position of the receivers and the position of the satellites are assumed known. Thus the geometric range l_r^s is assumed known. We assume further that the network is sufficiently small, so that $\tau_1^s = \tau_2^s = \cdots = \tau_n^s \equiv \tau^s$ and $I_1^s = I_2^s = \cdots = I_n^s \equiv I^s$. Based on these assumptions, the network observation equations become

$$\begin{aligned} E(\varphi_{r,j}^s - l_r^s) &= \delta t_{r,j} - \delta t'^s_{,j} - \mu_j I^s + \lambda_j M_{r,j}^s \\ E(p_{r,j}^s - l_r^s) &= dt_{r,j} - dt'^s_{,j} + \mu_j I^s \end{aligned} \tag{2}$$

where $\delta t'^s_{,j} = \delta t_{,j}^s - \tau^s$, $dt'^s_{,j} = dt_{,j}^s - \tau^s$, i.e. the tropospheric delay has been lumped with the phase and pseudo range satellite clock errors. As will be shown, the rank defect of this system of network equations is

$$\text{rank defect} = f + nf + mf + m$$

Since the redundancy is defined as the number of observations (2*mnf*) minus the number of parameters (2*nf*+2*mf* clocks, *m* ionospheric delays, *mnf* ambiguities), plus the rank defect (*f*+*nf*+*mf*+*m*), the single-epoch redundancy of our network model is

$$\text{redundancy} = f(m-1)(n-1)$$

2.2 Null-space Identification

Before we can eliminate the rank defects we need to identify the null-space of the network's design matrix. The null-space can be identified by showing which parameter changes leave the observations invariant.

The clocks

The phase and pseudo range observations remain invariant if we add an arbitrary constant, per frequency, to the phase clocks and to the pseudo range clocks. Hence, the observations are invariant for the following two transformations,

$$\begin{bmatrix} \delta_{r,j} \\ \delta'^s_{,j} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} x \text{ and } \begin{bmatrix} d_{r,j} \\ d'^s_{,j} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} x \tag{3}$$

The rank defect contribution is therefore 2*f*. These rank defects can be eliminated in many different ways. A simple choice is to fix the (phase and pseudo range) receiver clocks of the first network station: $\delta t_{1,j}$ and $dt_{1,j}$.

The redefined clocks become therefore,

$$\begin{aligned} \delta\bar{t}_{r,j} &= \delta t_{r,j} - \delta t_{1,j}\ ,\ \ \delta\bar{t}_{,j}^s = \delta t'^s_{,j} - \delta t_{1,j} \\ d\tilde{t}_{r,j} &= dt_{r,j} - dt_{1,j}\ ,\ \ d\bar{t}_{,j}^s = dt'^s_{,j} - dt_{1,j} \end{aligned} \tag{4}$$

The ionosphere

Since every perturbation in the ionospheric delay can be nullified by the satellite clocks, the observations also remain invariant under the transformation,

$$\begin{bmatrix} \delta\bar{t}_{,j}^s \\ d\bar{t}_{,j}^s \\ I^s \end{bmatrix} + \begin{bmatrix} -\mu_j \\ +\mu_j \\ 1 \end{bmatrix} x \tag{5}$$

This gives an additional rank defect of *m*, which can be eliminated by fixing I^s. This results in the redefined satellite clocks

$$\delta\bar{\bar{t}}_{,j}^s = \delta t'^s_{,j} - \delta t_{1,j} + \mu_j I^s\ ,\ \ d\tilde{t}_{,j}^s = dt'^s_{,j} - dt_{1,j} - \mu_j I^s \tag{6}$$

Ambiguities and satellite clocks

A constant shift of all the ambiguities of satellite *s* on frequency *j* can be nullified by an appropriate change in the *j*-frequency satellite clock. Hence, the observations remain invariant under the transformation,

$$\begin{bmatrix} \delta \overset{=}{t}{}^{s}_{,j} \\ M^{s}_{1,j} \\ \vdots \\ M^{s}_{n,j} \end{bmatrix} + \begin{bmatrix} \lambda_j \\ 1 \\ \vdots \\ 1 \end{bmatrix} x \quad (7)$$

This gives an additional rank defect of *mf*. They can be eliminated by fixing the ambiguity of the first network station: $M^{s}_{1,j}$. This results in redefined satellite clocks and redefined ambiguities,

$$\delta \tilde{\tilde{t}}{}^{s}_{,j} = \delta t'^{s}_{,j} - \delta t_{1,j} + \mu_j I^s - \lambda_j M^{s}_{1,j}\ ,\ M^{s}_{1r,j} = M^{s}_{r,j} - M^{s}_{1,j} \quad (8)$$

Note that the ambiguity is now a between receiver, single-differenced ambiguity.

Ambiguities and receiver clocks

Like above, a constant shift of all the ambiguities of receiver *r* on frequency *j* can be nullified by an appropriate change in the *j*-frequency receiver clock,

$$\begin{bmatrix} \delta \bar{t}_{r,j} \\ M^{1}_{1r,j} \\ \vdots \\ M^{m}_{1r,j} \end{bmatrix} + \begin{bmatrix} -\lambda_j \\ 1 \\ \vdots \\ 1 \end{bmatrix} x \quad (9)$$

This gives an additional rank defect of *(n-1)f*, and not of *nf*, since the parameter vector is identically zero for *r=1*. These rank defects can be eliminated by fixing the ambiguity of the first satellite: $M^{1}_{1r,j}$. This results in redefined receiver clocks and redefined ambiguities,

$$\delta \tilde{t}_{r,j} = \delta t_{r,j} - \delta t_{1,j} + \lambda_j M^{1}_{1r,j}\ ,\ M^{1s}_{1r,j} = M^{s}_{1r,j} - M^{1}_{1r,j} \quad (10)$$

Note that the ambiguity is now a double-differenced ambiguity and therefore integer.

2.3 The Full-rank Network System

We are now in the position to formulate and interpret the full-rank, un-differenced network observation equations. The *S-basis* or *minimum constraint set* [5], [9], that we used to eliminate the rank deficiency, is given by the set:

$$\begin{cases} \delta t_{1,j} \text{ and } dt_{1,j} \text{ for } j = 1,\dots,f \\ I^s \text{ for } s = 1,\dots,m \\ M^{s}_{1,j} \text{ for } j = 1,\dots,f;\ s = 1,\dots,m \\ M^{1}_{1r,j} \text{ for } j = 1,\dots,f;\ r = 1,\dots,(n-1) \end{cases} \quad (11)$$

With these minimum constraints, we obtain a full-rank system of observation equations

$$\begin{aligned} E(\varphi^{s}_{r,j} - l^{s}_{r}) &= \delta \tilde{t}_{r,j} - \delta \tilde{\tilde{t}}{}^{s}_{,j} + \lambda_j M^{1s}_{1r,j} \\ E(p^{s}_{r,j} - l^{s}_{r}) &= d\tilde{t}_{r,j} - d\tilde{\tilde{t}}{}^{s}_{,j} \end{aligned} \quad (12)$$

in which the reparametrized phase and pseudo range clocks and integer ambiguities are given as

$$\begin{cases} \delta \tilde{t}_{r,j} = \delta t_{r,j} - \delta t_{1,j} + \lambda_j M^{1}_{1r,j}\ , \\ \delta \tilde{\tilde{t}}{}^{s}_{,j} = \delta t'^{s}_{,j} - \delta t_{1,j} + \mu_j I^s - \lambda_j M^{s}_{1,j}\ , \\ d\tilde{\tilde{t}}{}^{s}_{,j} = dt'^{s}_{,j} - dt_{1,j} - \mu_j I^s\ , \\ d\tilde{t}_{r,j} = dt_{r,j} - dt_{1,j}\ , \\ M^{1s}_{1r,j} = M^{s}_{r,j} - M^{s}_{1,j} - M^{1}_{r,j} + M^{1}_{1,j} \end{cases} \quad (13)$$

The full-rank network system can now be solved using the integer constraints of the ambiguities. This gives ambiguity resolved estimates for the satellite clocks, $\delta \tilde{\tilde{t}}{}^{s}_{,j}$ and $d\tilde{\tilde{t}}{}^{s}_{,j}$. When these precise estimates are passed on to the user, the above given definition of these clocks ensures that the ambiguities of the user are also integer and that therefore ambiguity resolution becomes able at the user side as well. This is the principle of PPP-RTK.

2.4 The Full-rank User System

To demonstrate this principle, note that for the roving user '*u*', a similar un-differenced, full-rank system of observation equations can be formulated. But in this case, the satellite clocks, $\delta \tilde{\tilde{t}}{}^{s}_{,j}$ and $d\tilde{\tilde{t}}{}^{s}_{,j}$, are known (as provided by the network) and the range l^{s}_{u} is unknown,

$$\begin{aligned} E(\varphi^{s}_{u,j} + \delta \tilde{\tilde{t}}{}^{s}_{,j}) &= l^{s}_{u} + \delta \tilde{t}_{u,j} + \lambda_j M^{1s}_{1u,j} \\ E(p^{s}_{r,j} + d\tilde{\tilde{t}}{}^{s}_{,j}) &= l^{s}_{u} + d\tilde{t}_{u,j} \end{aligned} \quad (14)$$

The user can now resolve his integer ambiguities $M^{1s}_{1u,j}$ and compute his ambiguity resolved position fast and accurately.

Note that other minimum constraints can be used, than the one we used, to eliminate the rank defect in the network's design matrix. Each minimum constrained solution can be transformed to another minimum constrained solution by means of an *S-Transformation* [9]. Importantly, the user's ability to solve for his unknown parameters is not affected by the choice of minimum constraints.

In the next two sections we will apply the PPP-RTK principle to two CORS networks, one in Hong Kong (China) and one in Perth (Australia). Instead of assuming $\tau^{s}_{1} = \tau^{s}_{2} = \cdots = \tau^{s}_{n} \equiv \tau^{s}$ and $I^{s}_{1} = I^{s}_{2} = \cdots = I^{s}_{n} \equiv I^{s}$, as was done in the previous section, we now assume the atmospheric delays τ^{s}_{r}, I^{s}_{r} to vary from station to station.

Although this will result in a different, and higher dimensioned, null space of the network's design matrix, our same method to overcome the various rank defects has been applied, i.e. null space identification and rank defect elimination through reparametrization.

The function of the network is to provide the user with satellite clocks and interpolated ionospheric delays. For the orbits, the precise IGS orbits are used. For the network processing, a Kalman filter was used, assuming the ambiguities time-invariant, while for the user an epoch-by-epoch least-squares processing was used, thus providing truly instantaneous single-epoch solutions. The integer ambiguity resolution of both network and user was based on the LAMBDA method [10], with the Fixed Failure Ratio Test [11]. The critical values of this test were computed for a failure rate of 0.001. For the data quality control we applied the recursive DIA procedure for the detection, identification and adaptation of outliers and cycle slips [12].

III. HONG KONG PPP-RTK

The used Hong Kong network is a small CORS network of 4 stations with interstation distances ranging from 9 to 27 km, see Fig 1. The dual-frequency (L1-L2-C1-P2) Leica GPS data has been collected over 24hrs on 5th March 2001 with 30 sec sampling rate.

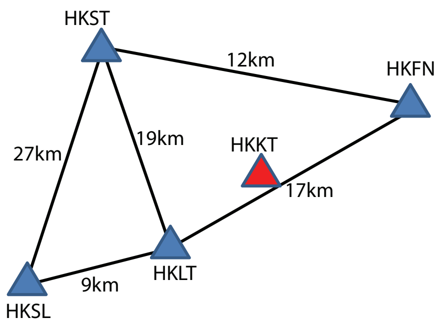

Figure 1 Hong Kong network consisting of 4 stations (blue triangle) plus user station HKKT (red triangle)

3.1 Hong Kong Network Processing

The undifferenced standard deviations of dual-frequency phase and pseudo range (in local zenith) were set at 2mm and 20cm, respectively; in addition all observations are weighted according to their elevation, with a cut-off elevation of 10 degrees. The ionosphere-weighted model [6] was used, with an a-priori ionospheric standard deviation of 50cm. For the troposphere, we applied Saastamoinen tropospheric corrections and relative zenith tropospheric delay (ZTD) estimation;

Our network Kalman filter processing is characterized as: Filter initialization based on first epoch; Dynamic models for ZTDs (time-constant), DD ambiguities (time-constant), ionospheric delays (random walk with standard deviation of 1 cm).

Full LAMBDA ambiguity resolution was done after an ambiguity initialization time of 10 epochs (5 min). After initialization LAMBDA+FFRatio test were executed every epoch. If a new satellite rises, its float ambiguities are solved immediately, however their integers are only resolved after 60 epochs (30 min). Fig 2 shows the results of the Hong Kong network ambiguity resolution. All integer ambiguity solutions were accepted.

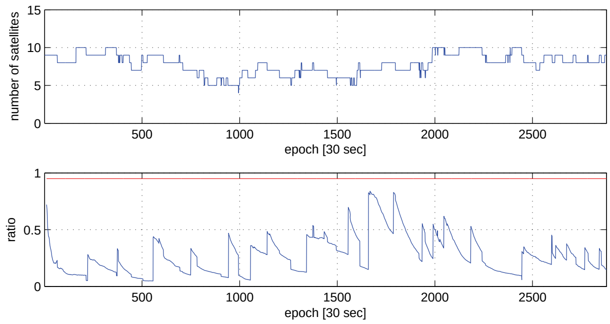

Figure 2 Hong Kong network: # tracked satellites (top) and ratio vs. threshold value of the FFRatio test (bottom; ratio depicted in blue and threshold in red).

3.2 Hong Kong User Processing

The corrections received by the user are the satellite clocks and the interpolated ionospheric delays. Satellite clocks for each epoch were added as user pseudo-observations, with appropriate variance matrix. Interpolation of the ambiguity-fixed network ionospheric delays to the location of rover HKKT was based on Kriging.

Figure 3 shows the double-differenced (DD) ionospheric delays based on fixed integer ambiguities for the user station HKKT with respect to network station HKSL. As can be seen at the beginning and end of the day the DD ionospheric delays are relatively small, but increase to large values of about 50 cm for this 16-km baseline during the middle of the day. This is due to the fact that the Hong Kong network is in a tropical region, in which the ionospheric conditions are usually more severe than at mid-latitudes. In addition, the GPS data are from 2001, close to the most recent maximum of the solar cycle.

The user processing settings differs in the following way from that of the network: no estimation of zenith tropospheric delay was applied and the undifferenced ionospheric standard deviation of the ionosphere-weighted model was set at 5mm. Moreover, a truly epoch-by-epoch processing was applied. This resulted in a user empirical ambiguity success rate of 2864/2869 = 99.8%, i.e. for almost all epochs the (correct) integer ambiguities passed the FFRatio test, see Figure 4.

Figure 5 shows the float and fixed epoch-by-epoch positioning results for PPP-RTK user HKKT. Horizontal (East-North) and vertical (Up) position errors are depicted with respect to known ground-truth coordinates.

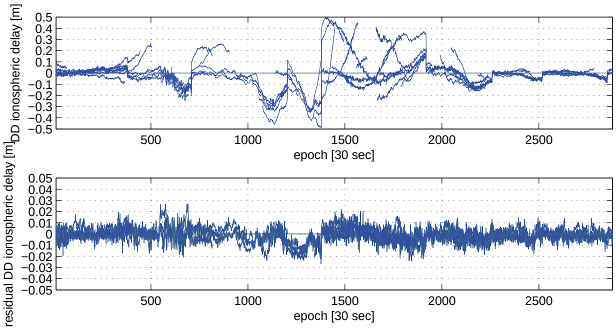

Figure 3 Ambiguity-fixed DD ionospheric delays for baseline HKSL-HKKT (16 km) before applying interpolated ionospheric network corrections (top) vs. after correction (bottom)

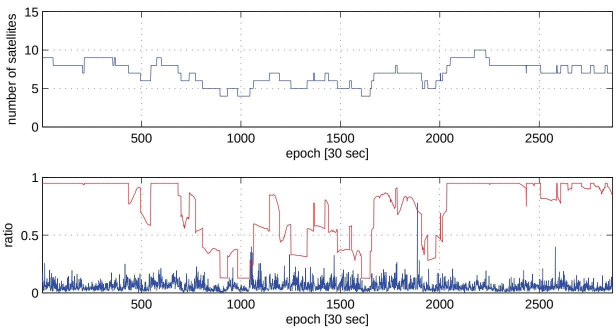

Figure 4 PPP-RTK positioning for Hong Kong user station HKKT: Number of satellites (top) vs. ratio (bottom; blue) and threshold value of FFRatio test (bottom; red).

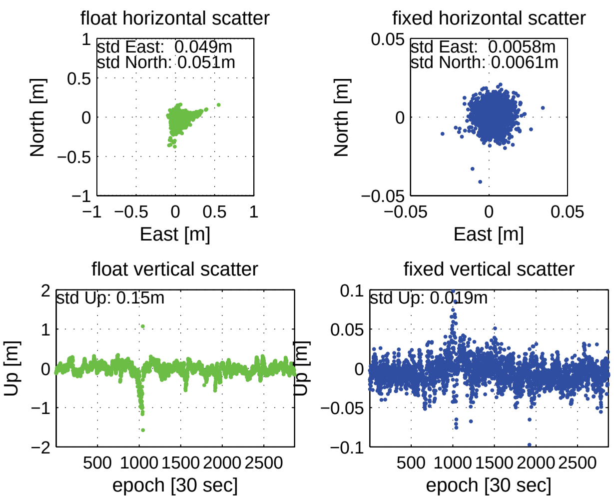

Figure 5 PPP-RTK epoch-by-epoch positioning for Hong Kong user station HKKT: float horizontal scatter (top left) and float vertical time series (bottom left) vs. fixed horizontal scatter (top right) and fixed vertical time series (bottom right). Computed standard deviations are included in the graphs.

IV. PERTH PPP-RTK

The used Perth network is a CORS network of 4 stations with interstation distances of about 60 km, see Fig 6. The dual-frequency (L1-L2-C1-P2) Trimble NetR5 GPS data has been collected over 16hrs on 31th July 2010 with 10 sec sampling rate.

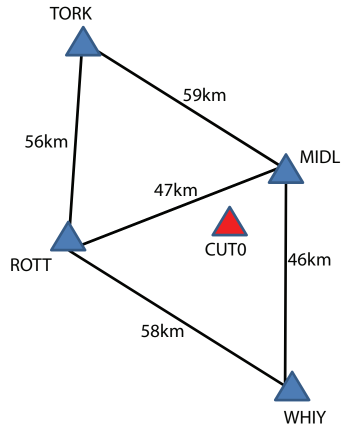

Figure 6 Perth network consisting of 4 stations (blue triangles) plus user station CUT0 (red triangle)

4.1 Perth Network Processing

The same procedure and almost the same settings as in the Hong Kong network were used. The difference being that full LAMBDA ambiguity resolutiion was now done after an initialization time of 60 epochs (10 min) and if a new satellite rises, their integers are only resolved after 210 epochs (35 min). Fig 7 shows the results of the Perth network ambiguity resolution. All integer ambiguity solutions were accepted.

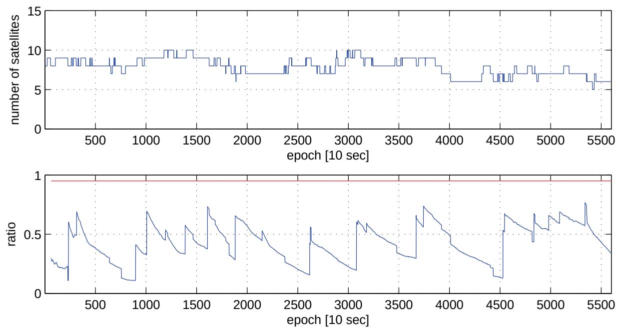

Figure 7 Perth network: # tracked satellites (top) and ratio vs. threshold value of the FFRatio test (bottom; ratio depicted in blue and threshold in red).

4.2 Perth User Processing

The corrections received by the user are the satellite clocks and the interpolated ionospheric delays. Figure 8 shows the performance of the ionospheric delay interpolation. It can be seen that the residual double-differenced ionospheric delays are within ±5 cm. At the user site the same procedure and settings were used as for the Hong Kong user, except that the undifferenced ionospheric standard deviation was now set at 1cm. The user single-epoch success rate turned out to be 91.8%, which is somewhat lower than for the Hong Kong case, due to the slightly larger ionospheric residuals. However, if we replace the epoch-by-epoch processing, by a batch processing of small windows of only 6 epochs (= 1min), a user success-rate of 100% is achieved, see Fig 9. This means that if the user would be using a Kalman filter, a complete loss-of-lock could be successfully re-initialized after only 6 epochs. The corresponding positioning results of the 6 epoch batch processing are shown in Fig 10.

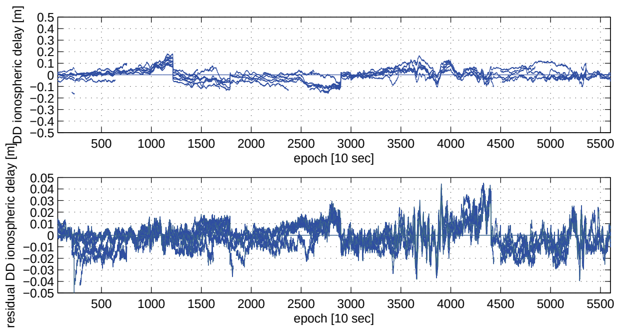

Figure 8 Ambiguity-fixed DD ionospheric delays for baseline TORK-CUT0 (65 km) before applying interpolated ionospheric network corrections (top) vs. after correction (bottom)

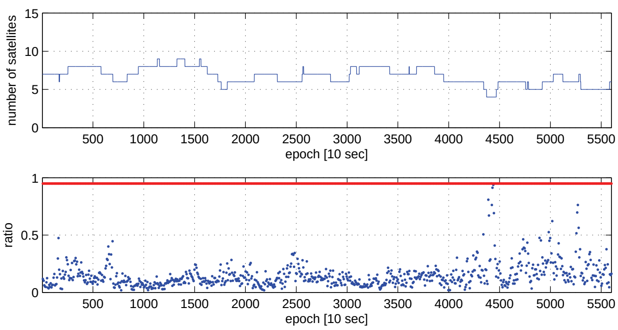

Figure 9 Perth network: # tracked satellites (top) and ratio vs. threshold value of the FFRatio test (bottom; ratio depicted in blue and threshold in red) for batches of 6 epochs.

V. CONCLUSIONS

In this contribution we described the PPP-RTK concept by means of the method of reparametrizing the undifferenced GNSS observation equations so as to eliminate the various rank defects. It was shown that PPP-RTK works very much like standard PPP, but has the quality of Network-RTK due to its ambiguity resolution ability. Its excellent performance was demonstrated by means of two test networks, one in Hong Kong and one in Perth.

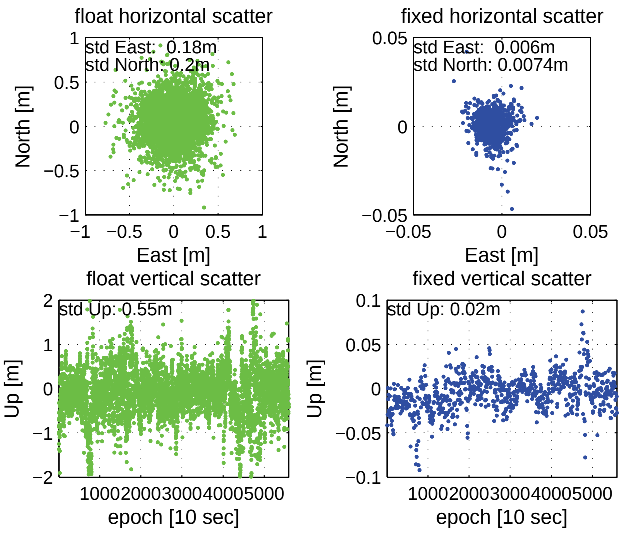

Figure 10 PPP-RTK positioning for Perth user station CUT0: float horizontal scatter (top left) and float vertical time series (bottom left) vs. fixed horizontal scatter (top right) and fixed vertical time series (bottom right). Computed standard deviations are included in the graphs.

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