Regularized Solution to Fast GPS Ambiguity Resolution

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Abstract: In rapid global positioning systems (GPS) positioning one of the key problems is to quickly determine the ambiguities of GPS carrier phase observables. Since carrier phase observations are generally collected only for a few minutes in the mode of rapid GPS positioning, the least squares floating solution of the ambiguities will be highly correlated and the decorrelation approach has often been used in order to reduce the search space of integer ambiguities. In this paper we propose a regularized algorithm as an alternative approach to decorrelation, and compute the regularization parameter by minimizing the trace of mean squared errors. Since regularization has been essential to solve inverse ill-posed problems and shown to be very significant in reducing the condition number of normal matrices, we will explore possible applications of regularization for improving the high correlation of the estimated float ambiguities. Numerical experiments with 50 epochs of single frequency observations show that the condition number after regularization reduces to half of that of the floating solution if the ambiguities could be known to 2–3 cycles. If better knowledge about the ambiguities could be obtained to within 1 cycle, further improvement can be achieved. The results indicate that regularization could be used for fast GPS ambiguity resolution. Our experiments also demonstrate that a scale factor of about 8 is needed to multiply the estimated variance of unit weight for obtaining a reasonable estimator for the accuracy of float ambiguities.

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Introduction

Fast ambiguity resolution plays a very important role in global positioning system (GPS) rapid precise positioning. In relative positioning for the baseline of a few kilometers, we need to find the proper algorithm to reliably determine the carrier phase integer ambiguities with only a few epochs of observations (see, e.g., Frei and Beutler 1990) or even on the fly. In this case, the covariance of the solved float ambiguity parameters is normally highly correlated. Teunissen proposed a method called "decorrelation" and some authors developed different decorrelation algorithms in order that the integer ambiguities could be rapidly fixed (see, e.g., Teunissen 1994, 1995, 1996; Teunissen et al. 1997; Xu 1998a, 2006; Liu et al. 1999; Grafarend 2000). Numerical investigations by Xu (2001) and Lou and Grafarend (2002) have shown that these algorithms can considerably reduce the search space of integer ambiguities, except that the dimension of ambiguities is not too large.

As is well known, inverse problem theory is a powerful tool to solve ill-posed problems, which has been widely applied to solve inverse ill-posed geodetic problems, in particular, in satellite gravimetry (see, e.g., Xu 1992, 1998b; Xu and Rummel 1994a). Since GPS decorrelation problems are known to be highly unstable, inverse problem theory should find a natural application to the problems of this kind. As a matter of fact, ill-posed problems were first identified and defined by Hadamard (1932). Tikhonov proposed the regularization method to ill-posed models (Tikhonov 1963a,b). Hoerl and Kennard proposed the ridge regression method to estimate the ridge/regularization parameter by minimizing the mean squared error (Hoerl and Kennard 1970b, 1970a). Xu and Rummel (1994b) advanced a multiple parameter regularization method (see also Xu et al. 2006). For some more details on inverse problem theory, the reader may refer to Tarantola (2005).

This paper will investigate potential applications of inverse problem theory to GPS, in particular, as an alternative method to GPS decorrelation. "Regularized Solution as Alternative Method to Decorrelation" will formulate the regularized solution to GPS decorrelation by minimizing the mean squared error of the integer ambiguities. "Estimation of Regularization Parameter" will discuss the method to estimate regularization parameter based on the criterion of minimizing the mean square error. Finally, numerical cases are demonstrated to verify the regularization model advanced in this paper.

Regularized Solution as Alternative Method to Decorrelation

Precise positioning depends on proper handling of systematic errors, which generally include GPS receiver and satellite's clock errors, ephemeredes error, ionospheric, and tropospheric effects. In order to significantly reduce and eliminate these systematic errors, we always use the double difference operator to raw phase or code observations, which has been demonstrated to be very

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effective for baselines of a few kilometers. For GPS phase observables at epoch i, the linearized double difference observation equations can be simply written as follows

$$\mathbf{v}_i = \mathbf{A}_i\mathbf{x} + \lambda\mathbf{z} - \mathbf{l}_i \tag{1}$$

where $\mathbf{l}_i$=vector of double difference phase observables; $\mathbf{v}_i$=vector of the residuals; $\mathbf{x}$=vector of three-dimensional coordinate parameters; z=integer vector of ambiguity parameters, $\mathbf{A}_i$=design matrix which consists of three direction cosines from receiver to satellite; and λ=wavelength of carrier phase. If we track $(n+1)$ GPS satellites with two receivers, we have n observations and n ambiguity parameters at each epoch.

If we have m epochs of observables and collect them together, then we have the following system of observation equations

$$\mathbf{v} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{l} \tag{2}$$

where $\mathbf{l}=(\mathbf{l}_1^T\mathbf{l}_2^T\cdots\mathbf{l}_m^T)^T$, $\mathbf{v}=(\mathbf{v}_1^T\mathbf{v}_2^T\cdots\mathbf{v}_m^T)^T$, $\mathbf{A}=(\mathbf{A}_1^T\mathbf{A}_2^T\cdots\mathbf{A}_m^T)^T$ $\mathbf{B}=\lambda(\mathbf{I}\mathbf{I}\cdots\mathbf{I})^T$, and $\mathbf{I}=(n\times n)$ identity matrix. Because the integer ambiguities z are not continuous variables, we cannot follow the conventional technique to derive the normal equations. Instead, we follow the two-step approach of Xu et al. (1995) to derive the normal equations for the ambiguity parameters. The two-step approach is based on the following least squares criterion:

$$\min:(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{l})^T\mathbf{P}(\mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{z} - \mathbf{l}) \tag{3}$$

As the first step, we compute the partial derivative of the cost function Eq. (3) with respect to the continuous variables $\mathbf{x}$ and equal them to zeros. As a result, we can represent the continuous variables of coordinate corrections in terms of integer ambiguity parameters

$$\mathbf{x} = (\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T\mathbf{P}(\mathbf{l} - \mathbf{B}\mathbf{z}) \tag{4}$$

As the second step, we substitute Eq. (4) into Eq. (3) and obtain

$$\min:(\mathbf{B}\mathbf{z} - \mathbf{l})^T\mathbf{P}\mathbf{Q}\mathbf{P}(\mathbf{B}\mathbf{z} - \mathbf{l}) \tag{5}$$

where $\mathbf{P}$=weight matrix of double difference phase observables, and

$$\mathbf{Q} = \mathbf{P}^{-1} - \mathbf{A}(\mathbf{A}^T\mathbf{P}\mathbf{A})^{-1}\mathbf{A}^T \tag{6}$$

The float solution of the ambiguity parameters can be computed by using the following normal equation:

$$\mathbf{B}^T\mathbf{P}\mathbf{Q}\mathbf{P}\mathbf{B}\mathbf{z} = \mathbf{B}^T\mathbf{P}\mathbf{Q}\mathbf{P}\mathbf{l} \tag{7}$$

In fast or kinematic GPS positioning, the normal matrix $\mathbf{B}^T\mathbf{PQPB}$ is almost always ill-conditioned, which is the motivation for the development of GPS decorrelation techniques (see e.g., Teunissen 1995; Xu 2001; Liu et al. 1999; Grafarend 2000). As is well known, inverse problem theory is a powerful tool to solve unstable/ill-conditioned problems. In order to obtain a stabilized solution from the point of view of inverse problem theory, we will apply the regularization technique to solve the above GPS ill-conditioned problem. The regularization method is to minimize the following cost function

$$\min:\mathbf{\Phi}^\alpha(\mathbf{z}) = \|\mathbf{B}\mathbf{z} - \mathbf{l}\|^2_{\mathbf{PQP}} + \alpha\|\mathbf{z}\|^2 \tag{8}$$

where α=regularization parameter; and $\|\cdot\|^2$ denotes the 2-norm, and the subscript $\mathbf{PQP}$=kernel of the norm. The solution $\mathbf{z}_\alpha$ to Eq. (8) and its bias $\mathbf{b}_\alpha$ can be represented below

$$\mathbf{z}_\alpha = (\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\mathbf{B}^T\mathbf{PQPl} \tag{9a}$$

$$\mathbf{b}_\alpha = -\alpha(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\bar{\mathbf{z}} \tag{9b}$$

where $\bar{\mathbf{z}}$=vector of true ambiguities. Once the regularization parameter α is properly estimated, the regularized solution can be uniquely determined by Eq. (9a), whereas the determination of $\mathbf{b}_\alpha$ needs the true value $\bar{\mathbf{z}}$, which is usually substituted with its least squares estimator. The covariance of $\mathbf{z}_\alpha$ can be derived from Eq. (9a) via error propagation

$$\mathbf{D}_\mathbf{z} = \sigma_0^2(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\mathbf{B}^T\mathbf{PQPB}(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1} \tag{10}$$

where σ_0^2=variance of unit weight. The accuracy of a regularized solution is normally evaluated with the mean squared error, which includes the covariance and bias, and the mean squared error $\mathbf{M}_z$ of float ambiguity solution can be represented as follows

$$\begin{aligned}\mathbf{M}_\mathbf{z} &= \mathbf{D}_\mathbf{z} + \mathbf{b}_\alpha\mathbf{b}_\alpha^T \\ &= \sigma_0^2(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\mathbf{B}^T\mathbf{PQPB}(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1} \\ &\quad + \alpha^2(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\bar{\mathbf{z}}\bar{\mathbf{z}}^T(\mathbf{B}^T\mathbf{PQPB} + \alpha\mathbf{I})^{-1}\end{aligned} \tag{11}$$

If the regularization parameter α is properly estimated, the better float ambiguities can be computed by Eq. (9a), their bias and accuracy are computed by Eqs. (9b) and (11), respectively.

Estimation of Regularization Parameter

The regularization parameter can be determined by using the L-curve approach (see, e.g., Hansen 1992), however, this method has been shown to often produce oversmoothed solutions (Xu 1998b). We compute the regularization parameter α based on the criterion of minimizing the trace of mean squared error as follows:

$$\min:\mathrm{Tr}(\mathbf{M}_z) \tag{12}$$

where $\mathrm{Tr}(\mathbf{M}_z)$ denotes the trace of $\mathbf{M}_z$. The true value $\bar{\mathbf{z}}$ for computing $\mathbf{M}_z$ can be estimated by the least squares adjustment. Since the least squares estimator will approach to zero in the iterated adjustment process, we can use the accuracy of the last iteration as the estimator of true value for computing the regularization parameter. Because $\mathbf{B}^T\mathbf{PQPB}$ is a real-valued symmetric matrix, it can be decomposed as follows:

$$\mathbf{B}^T\mathbf{PQPB} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T \tag{13}$$

where $\mathbf{\Lambda}=\mathrm{diag}(\lambda_1\lambda_2\cdots\lambda_n)$=diagonal matrix; and $\mathbf{U}$=orthogonal matrix; i.e., $\mathbf{U}\mathbf{U}^T=\mathbf{U}^T\mathbf{U}=\mathbf{I}$. Substituting Eq. (13) into Eq. (11), we obtain the trace of mean squared error as follows:

$$\mathrm{Tr}(\mathbf{M}_\mathbf{z}) = \sigma_0^2\sum_{i=1}^{n}\frac{\lambda_i}{(\lambda_i+\alpha)^2} + \alpha^2\sum_{i=1}^{n}\frac{(\mathbf{u}_i^T\bar{\mathbf{z}})^2}{(\lambda_i+\alpha)^2} \tag{14}$$

where $\mathbf{u}_i$=column vector of $\mathbf{U}$. The second order derivative of Eq. (14) with respect to α is derived and represented as

$$\frac{d^2\,\mathrm{Tr}(\mathbf{M}_z)}{d\alpha^2} = \sum_{i=1}^{n}\frac{\lambda_i(\lambda_i(\mathbf{u}_i^T\bar{\mathbf{z}})^2 + 3\sigma_0^2)}{(\lambda_i+\alpha)^4} \tag{15}$$

Since $\alpha>0$ and $\lambda_i>0$, Eq. (15) is always larger than 0, which means that the criterion Eq. (12) only exists in the unique solution and it can be solved by the following algebraic expression of equating the first-order derivative to 0:

$$\frac{d\,\mathrm{Tr}(\mathbf{M}_z)}{d\alpha} = \sum_{i=1}^{n} \frac{\lambda_i((\mathbf{u}_i^T\bar{\mathbf{z}})^2\alpha - \sigma_0^2)}{(\lambda_i + \alpha)^3} = 0 \qquad (16)$$

Once the regularization parameter is properly determined by Eq. (16), Eq. (9*a*) can estimate the float ambiguities that are normally closer to the true integer ambiguities than that of the least squares solution, and Eq. (11) can compute the mean squared error that will be less correlated than the covariance matrix of the least squares solution. Therefore, searching the integer ambiguity resolution will be more efficient by minimizing the following cost function:

$$\min:(\mathbf{z} - \mathbf{z}_\alpha)^T\mathbf{M}_z(\mathbf{z} - \mathbf{z}_\alpha) \qquad (17)$$

where $\mathbf{z}$=vector of integer ambiguities to be determined. The same search algorithms as Xu (2006) and Teunissen (1995) can be applied to determine integer ambiguities and will not be discussed in this paper. Since fixing the ambiguities to incorrect integers means no precise positioning, and because the regularized solution is biased, the hypothetical tests proposed by Xu (2006) should be further used to confirm the solved ambiguities.

As long as the integer ambiguities are properly determined, we can compute the coordinate parameters with Eq. (4) and the correspondent covariance matrix with the following expression:

$$\mathbf{D}_x = \sigma_0^2(\mathbf{A}^T\mathbf{PA})^{-1} \qquad (18)$$

If σ_0^2 is unknown, it can be estimated from the residuals of fixed ambiguity solution.

Numerical Experiments and Analysis

The data used in the experiments was collected with two single-frequency Ashtech GPS receivers on July 12, 2004, and 113 epochs of observations were collected with the sample rate of 10 s. With all of the data, the integer ambiguities are successfully determined and the baseline is solved as 2.084.802 m, and these results will be used as the "true" values in the following experiments.

Since the variance of unit weight estimated from the residuals of double difference carrier phase observations is usually too small, the estimated variance of unit weight is needed to multiply a scale factor in order that the estimated accuracy is reasonable, and the scale factor is the ratio of the variance of "true" ambiguity error over the variance of estimated ambiguity error. The mean value of the "true" ambiguity error is computed as follows:

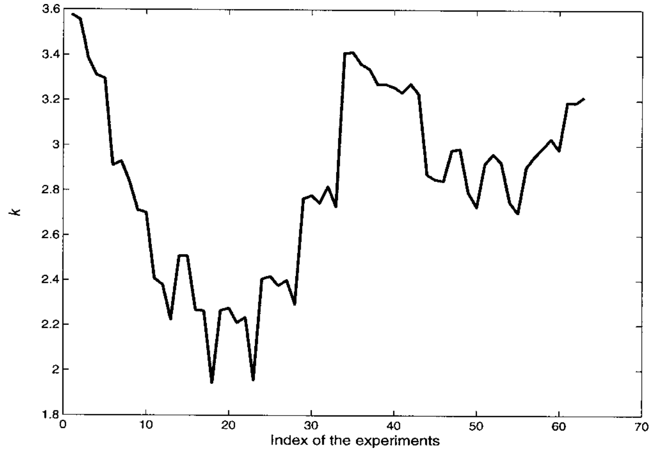

Fig. 1. Estimated scales

$$\Delta_z = \frac{\sum_{i=1}^{n} |\hat{z}_i - \bar{z}_i|}{n} \qquad (19)$$

where $\hat{z}_i$=float ambiguity solution; $\bar{z}_i$="true" ambiguity; and n=number of ambiguities. The mean value of estimated ambiguity error is as follows:

$$m_z = \hat{\sigma}_0 \frac{\sqrt{\sum_{i=1}^{n} d_{ii}}}{n} \qquad (20)$$

where d_{ii}=diagonal element of covariance matrix of least squares solution; and $\hat{\sigma}_0$=square root of estimated variance of unit weight. If $\hat{\sigma}_0$ is properly estimated, m_z should approach Δ_z. Therefore the scale k can be estimated by

$$k = \Delta_z / m_z \qquad (21)$$

We select the sequence of 50 epochs of observations to compute the scale, and shift the selected sequence for 1 epoch in each time. With a total of 113 epochs of observations, we can compute 64 scales and present them in Fig. 1, and the mean of the 64 scales is 2.828. Therefore, we multiply to the estimated variance of unit weight with the scale factor of $2.828^2 \approx 8$ in our numerical experiments. The coefficient matrix and the constant vector of the normal equation of the last 50 epochs of observations are shown in Table 1 and the five eigenvalues are in Table 2

Table 1. Normal Matrix and Constant Vector

$\mathbf{B}^T\mathbf{PQPB}$					$\mathbf{B}^T\mathbf{PQPl}$
2.0355117183	1.0825454698	0.1024318899	−0.6463949674	−0.6910577133	1.6527115349
—	0.7724941136	0.2278084620	0.3157312232	−0.5281319022	1.4431092797
—	—	0.1989688017	0.6340150845	−0.1929641632	0.6467196294
—	—	—	2.6106051857	−0.3576355873	1.5095940681
—	—	—	—	0.3740458020	−1.0497857788

Table 2. Eigenvalues of Normal Matrix

3.1685221669	2.8010173402	0.0196554049	0.0019499091	0.0004808001

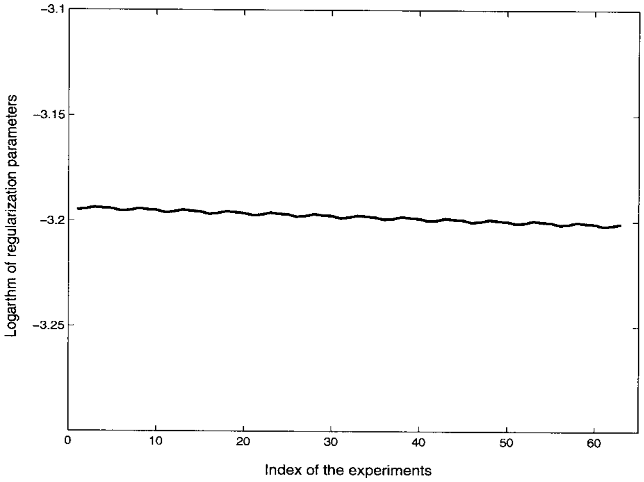

Fig. 2. Logarithm of regularization parameters

where the condition number is 6,590.10, which is really very large for a five-dimensional equation. Regularization is needed for a better result.

We do 64 numerical experiments with 50 epochs of observations selected from 113 epochs by shifting 1 epoch each time. If the ambiguities could be known to 2−3 cycles, we could compute the regularization parameters presented in Fig. 2. The condition number and the trace of the covariance matrix and the matrix of mean squared error for the least squares solution and the regularization solution are plotted in Figs. 3 and 4, where the blue solid line and the red dash line represent the least squares solution and the regularization solution, respectively. The condition number of the regularization solution is about half of that of the least squares solution. Therefore, regularization can significantly reduce the correlation of the solved float ambiguities, which can be used as an alternative method to decorrelation. Since the regularization solution is biased, the biases of five double difference ambiguities in 64 experiments are shown in Fig. 5, in which the biggest bias is near 1 cycle. We should take the biases into account in fixing the integer ambiguities.

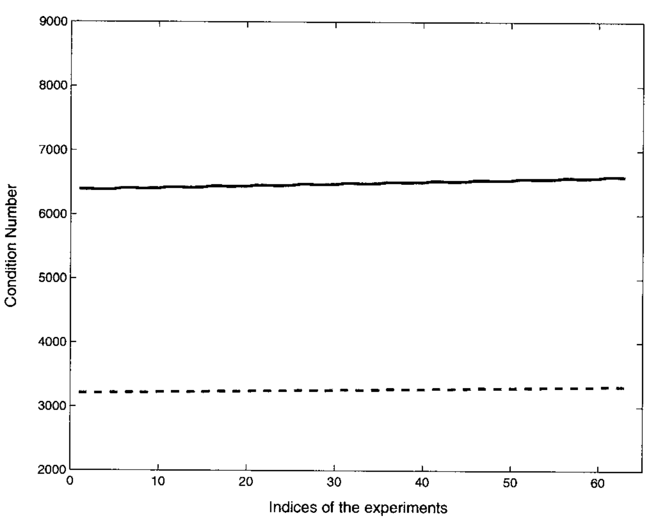

Fig. 3. Condition numbers of covariance matrix of least squares solution (blue solid line) and mean squared error matrix of regularization solution (red dash line)

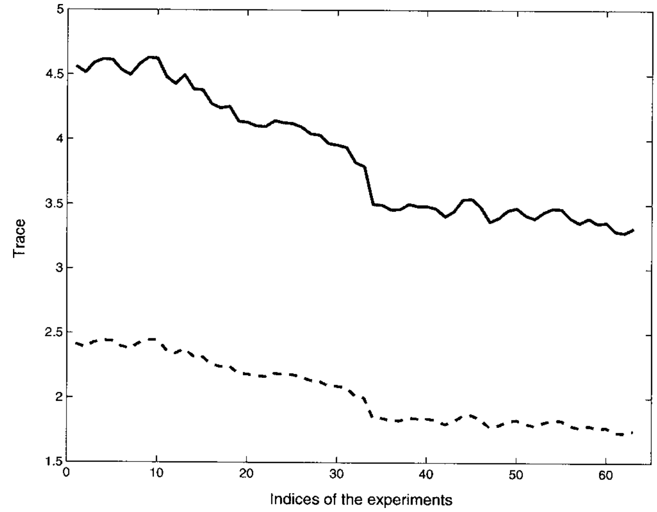

Fig. 4. Trace of covariance matrix of least squares solution (blue solid line) and mean squared error matrix of regularization solution (red dash line)

Conclusions

It is well known that regularization has been successfully applied to solve inverse ill-posed problems by significantly improving the condition number of normal matrices. Since the floating solution of GPS ambiguities in fast positioning is highly correlated, we have investigated regularization as an alternative approach to GPS decorrelation and derived all the computational formulae. The numerical experiments with single frequency phase measurements have demonstrated that the regularization algorithm can efficiently reduce correlation of the float solution and, as a result, can be applied to fast GPS ambiguity resolution. Our experiments have also confirmed the well-known fact that the accuracy of the float ambiguities has been too optimistically estimated. By comparing the floating solution with the resolved integer ambiguities, we have found that a scale factor of about 8 is needed to output the reasonable accuracy of float ambiguities. Although regularization is useful to reduce the condition number, the estimated solution is biased as a byproduct, which can be as large as 1 cycle in our experiments. Thus it should be noted that the regularized

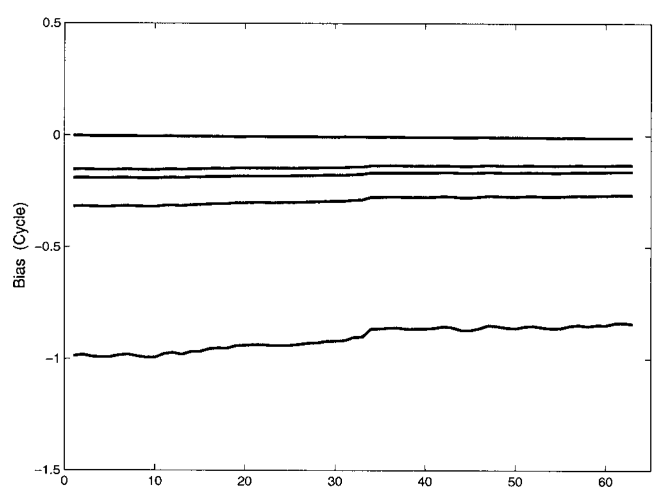

Fig. 5. Biases of regulation solution

integer ambiguity solution must be carefully checked by using conservative hypothetical tests proposed by Xu (2006).

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