A New Carrier Phase Ambiguity Estimation for GNSS Attitude Determination Systems

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BIOGRAPHY

Dr. Chansik Park is associate professor of school of electrical and computer engineering. His interests include the development of GNSS receivers and data processing techniques. Dr. Peter Teunissen is professor of MGP (Mathematical Geodesy and Positioning), Delft University of Technology. He is currently engaged in the development of GNSS data processing strategies for medium scaled networks with an emphasis on carrier phase ambiguity estimation and validation. They have interest in carrier phase ambiguity resolution in GNSS attitude determination system.

ABSTRACT

In this paper, the results of a new method to fix integer ambiguity in the attitude determination are presented. The results are obtained by means of a novel LAMBDA based method of making use of all a priori information on the baseline configuration The LAMBDA based method is mathematically rigorous and compact. It dramatically improves the success rate of an integer ambiguity resolution and therefore improves the reliability of attitude determination systems. It is expected that ourmethod can be directly used to build precise and reliable attitude determination systems with off-the-shelf low cost GNSS receivers.

1. INTRODUCTION

Carrier phase measurements from more than two antennas and an integer ambiguity resolution method are used to get precise attitudes such as roll, pitch and yaw. Integer ambiguity resolution method in attitude determination needs to be highly optimized to meet the real time navigation requirement. Many researches on attitude determination systems using GNSS (Global Navigation Satellite System) have been done and there are commercial products already [Ashtech03], [Furuno03]. But, in these systems the probability of correct integer estimation is not high enough and there are still strong requirements of integer ambiguity resolution methods which have high success rates.

The LAMBDA (Least squares AMBiguity Decorrelation Adjustment) method [Teunissen93], [Teunissen94], [Teunissen98a] is one of the most famous integer ambiguity resolution method and many systems has been implemented using it. The LAMBDA method is basically an ILS (Integer Least Squares) estimator and provides an optimal solution with high reliability and efficiency. However, to apply the LAMBDA method to attitude determinations, some modifications are required so as to make use of all a priori available information to improve performances. Some other methods using the baseline length in validation or including the baseline length into the model can be used as alternatives. However, these methods are not rigorous and it is hard to determine critical thresholds.

Other methods such as LSAST (Least Squares Ambiguity Search Technique) and ARCE (Ambiguity Search using Constraint Equation) divide integer ambiguities into independent and dependent parts and perform search using an independent part only. Furthermore, using the constant baseline length constraints, the search space can be further reduced. Because of computational efficiency, these methods are popular. But, there is no theoretic guideline to divide independent and dependent parts, and to choose the threshold for a thickness of a search sphere which is critical to the performance.

In this paper the results of the LAMBDA based method for attitude determination are presented The method is implemented using MATLAB and applied to real measurements. Experimental results with various GPS receivers show the effectiveness of the proposed method.

In section 2 we give a brief review of GNSS model and attitude determination methods. In section 3, after a brief introduction of the ILS problem, the LAMBDA method is explained. Some ambiguity resolution methods in attitude determination are briefly introduced in section 4. The experimental results and analysis with various real measurements are given in section 5 . In final section, some concluding remarks are given.

2. ATTITUDE DETERMIANTION USING GNSS

2.1 The GNSS model

We will consider the following system of linear(ized) observation equations

(1) $$y = Aa + Bb + e$$

where y is the given GNSS data vector of order m, a and b are the unknown parameter vectors respectively of order n and p and where e is the noise vector. In principle all the GNSS models can be cast in this frame of observation equations. The data vector y will usually consist of the 'observed minus computed' single or dual frequency double difference (DD) carrier phase and/or pseudorange (code) observations. The entries of vector a are then the DD carrier phase ambiguities, expressed in units of cycles rather than range. They are known to be *integers*, $a \in Z^n$. The entries of vector b will consist of baseline components (coordinates) and possibly atmospheric delay parameters. They are known to be real valued, $b \in R^n$. In attitude determination system, usually the baseline length is short enough to neglect the effect of atmospheric delay. Furthermore, to meet the real time requirement single epoch's measurements are considered which results in p=3.

The procedure which is usually followed for solving the GNSS model (1) can be divided into three steps. In the first step one simply disregards the integer constraints $a \in Z^n$ on the ambiguities and performs a standard least-squares adjustment. As a result one obtains the (real valued) estimates of a and b, together with their variance-covariance (vc-) matrix

(2) $$\begin{bmatrix} \hat{b} \\ \hat{a} \end{bmatrix}, \quad \begin{bmatrix} Q_{\hat{a}} & Q_{\hat{a}\hat{b}} \\ Q_{\hat{b}\hat{a}} & Q_{\hat{b}} \end{bmatrix}$$

This solution is referred to as the 'float' solution. In the second step the 'float' ambiguity estimate $\hat{a}$ is used to compute the corresponding integer ambiguity estimate $\breve{a}$. This implies that a mapping $S{:}R^n \rightarrow Z^n$, from the n-dimensional space of reals to the n-dimensional space of integers, is introduced such that

(3) $$\breve{a} = S(\hat{a})$$

Once the integer ambiguities are computed, they used in the third step to finally correct the 'float' estimate of b. As a result one obtains the 'fixed' solution

(4) $$\breve{b} = \hat{b} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} (\hat{a} - \breve{a})$$

with vc-matrix $Q_{\breve{b}} = Q_{\hat{b}} - Q_{\hat{b}\hat{a}} Q_{\hat{a}}^{-1} Q_{\hat{a}\hat{b}}$.

In the present contribution we will show how the above procedure needs to be modified in case of attitude determination systems after a brief description of the attitude determination procedure.

2.2 Attitude determination

Attitudes of a vehicle can be determined using GNSS measurements from more than two antennas attached to a vehicle [Ashtech03], [Cohen92], [Lu95], [Park00], [Ziebart03]. The fixed baseline vector in (4) is expressed in the ECEF (Earth Centred Earth Fixed) frame. The baseline vector in the local level navigation frame, $\hat{b}^n$, is obtained by transforming the fixed baseline in the ECEF frame. By comparing $\hat{b}^n$ with a known baseline vector in the body frame, and by using the assumption that a vehicle is rigid body, attitudes can be estimated. The baseline vector in the body fame, $\hat{b}^b$, can be precisely measured at the antenna installation time. The baseline vectors in the two coordinate systems are related as follows:

(5) $$\hat{b}^b = C_\phi C_\theta C_\psi \hat{b}^n$$

where C_ψ represents a rotation about the vertical axis (heading), C_θ represents a rotation about the horizontal plane (elevation or pitch) and C_ϕ represents a rotation about the axis parallel to a forward direction (roll). The transformation matrix $C_n^b = C_\phi C_\theta C_\psi$ is a 3×3 matrix containing the nine direction cosine elements or three Euler angles (ϕ, θ and ψ).

If there are n_a antennas (or n_a -1 baselines) are attached to a vehicle the transformation matrix or attitudes can be determined using

(6) $$\hat{B}^b = C_n^b \hat{B}^n$$

where the matrix $\hat{B}^b = \begin{bmatrix} \hat{b}_1^b & \dots & \hat{b}_{n_a-1}^b \end{bmatrix}^T$ and $\hat{B}^n = \begin{bmatrix} \hat{b}_1^n & \dots & \hat{b}_{n_a-1}^n \end{bmatrix}^T$. A simple least squares or optimization techniques [Bar97] can be applied to (6) to determine the transformation matrix, C_n^b, or attitudes if there are more than 3 antennas not in coplanar.

In the GPS compass type attitude determination systems [Furuno03] [Tu96], only one baseline vector is used to determine heading and elevation. Thus (6) can not be directly applied. Another method called 'direct method' is usually used in this case. Let one antenna is located at the centre of vehicle $\begin{bmatrix}0 & 0 & 0\end{bmatrix}^T$, and the other is at $\begin{bmatrix}L & 0 & 0\end{bmatrix}^T$ in the body frame. Let the computed baseline using the GNSS be expressed as $\hat{b}^n = \begin{bmatrix}\hat{x}^n & \hat{y}^n & \hat{z}^n\end{bmatrix}^T$. The heading and elevation can be determined using

$$(7) \qquad \psi = -\tan^{-1}\frac{\hat{y}^n}{\hat{x}^n}$$

$$(8) \qquad \theta = -\tan^{-1}\frac{\hat{z}^n}{\sqrt{(\hat{x}^n)^2+(\hat{y}^n)^2}}$$

In this contribution, attitude determination using GPS compass is considered since it is a basic building block of attitude determination systems.

3. AMBIGUITY RESOLUTION

3.1 THE LAMBDA method

There are many ways of computing an integer ambiguity vector $\breve{a}$ from its real-valued counterpart $\hat{a}$. Integer rounding, integer bootstrapping and ILS are examples of the admissible integer estimation. Having the problem of GNSS ambiguity resolution in mind, one is particularly interested in the estimator which maximizes the probability of correct integer estimation.

When using the least-squares principle, the GNSS model (1) can be solved by means of the minimization problem

$$(9) \qquad \min_{a,b}\|y-Aa-Bb\|^2_{Q_y}, a\in Z^n, b\in R^p$$

with Q_y the vc-matrix of the GNSS observables. This type of least squares problem was first introduced in [Teunissen93] and has been coined with the term '*integer least-squares*'. It is a nonstandard least-squares problem due to the integer constraints $a\in Z^n$. The solution of (9) is consistent with the three solution steps of section 2.1. This can be seen as follows. It follows from the orthogonal decomposition

$$(10) \quad \|y-Aa-Bb\|^2_{Q_y} = \|\hat{e}\|^2_{Q_y} + \|\hat{a}-a\|^2_{Q_{\hat{a}}} + \|\hat{b}(a)-b\|^2_{Q_{\hat{b}|a}}$$

with $\hat{e}=y-A\hat{a}-B\hat{b}$ and $\hat{b}(a)=\hat{b}-Q_{\hat{b}\hat{a}}Q_{\hat{a}}^{-1}(\hat{a}-a)$, that the sought for minimum is obtained when the second term on the right-hand side is minimized for $a\in Z^n$ and the last term is set to zero. The integer least-squares (ILS) estimator of the ambiguities is therefore defined as follows.

Definition 1 (*Integer least-squares*)
Let $\hat{a}=\begin{pmatrix}\hat{a}_1 & \cdots & \hat{a}_n\end{pmatrix}^T\in R^T$ be the ambiguity 'float' solution and let $\breve{a}_{LS}\in Z^n$ denote the corresponding integer least-squares solution. Then

$$(11) \qquad \breve{a}_{LS} = \arg\min_{z\in Z^n}\|\hat{a}-z\|^2_{Q_{\hat{a}}}$$

In contrast to integer rounding and integer bootstrapping, an integer search is needed to compute $\breve{a}_{LS}$. Although we will refrain from discussing the computational intricacies of ILS estimation, the conceptual steps of the computational procedure will be described briefly. The ILS procedure is mechanized in the GNSS LAMBDA (Least-squares AMBiguity Decorrelation Adjustment) method, which is currently one of the most applied methods for GNSS carrier phase ambiguity resolution. More information and practical results on the LAMBDA methods can be found, for example, in [Hatch94], [Jonge96], [Joosten01], [Teunissen93], [Teunissen94], [Teunissen98a], [Teunissen98b], [Teunissen01], [Teunissen02], [Tiberius98] and [Verhagen02].

The main steps as implemented in the LAMBDA method are as follows. One starts by defining the ambiguity search space

$$(12) \qquad \Omega_a = \left\{a\in Z^n \mid (\hat{a}-a)^T Q_{\hat{a}}^{-1}(\hat{a}-a)\le \chi^2\right\}$$

with χ^2 a to be chosen positive constant. The boundary of this search space is ellipsoidal. It is centred at $\hat{a}$, its shape is governed by the vc-matrix $Q_{\hat{a}}$ and its size is determined by χ^2. In case of GNSS, the search space is usually extremely elongated, due to the high correlations between the ambiguities. Since this extreme elongation usually hinders the computational efficiency of the search, the search space is first transformed to a more spherical shape,

$$(13) \qquad \Omega_z = \left\{z\in Z^n \mid (\hat{z}-z)^T Q_{\hat{z}}^{-1}(\hat{z}-z)\le \chi^2\right\}$$

using the admissible ambiguity transformations $\hat{z}=Z^T\hat{a}$, $Q_{\hat{z}}=Z^TQ_{\hat{a}}Z$. Ambiguity transformations Z are said to be admissible when both Z and its inverse Z^{-1} have integer entries. Such matrices preserve the integer nature of the ambiguities. In order for the transformed search space to become more spherical, the volume preserving

Z-transformation is constructed as a transformation that decorrelates the ambiguities as much as possible. Using the triangular decomposition of $Q_{\hat{z}}$, the left-hand side of the quadratic inequality in (13) is then written as a sum-of-squares:

$$\sum_{i=1}^{n} \frac{(\hat{z}_{i|I} - z_i)^2}{\sigma_{i|I}^2} \leq \chi^2 \tag{14}$$

On the left-hand side one recognizes the conditional least-squares estimator $\hat{z}_{i|I}$, which follows when the conditioning takes place on the integers $z_1, z_2, \ldots, z_{n-1}$. Using the sum-of-squares structure, one can finally set up the n intervals which are used for the search. These sequential intervals are given as

$$\begin{aligned} &(\hat{z}_1 - z_1)^2 \leq \sigma_1^2 \chi^2 \\ &(\hat{z}_{2|1} - z_2)^2 \leq \sigma_{2|1}^2 (\chi^2 - \frac{(\hat{z}_1 - z_1)^2}{\sigma_1^2}) \\ &\quad \vdots \end{aligned} \tag{15}$$

In order for the search to be efficient, one not only would like the vc- matrix $Q_{\hat{z}}$ to be as close as possible to a diagonal matrix, but also that the search space does not contain too many integer grid points. This requires the choice of a small value for χ^2, but one that still guarantees that the search space contains at least one integer grid point. Since the bootstrapped estimator is so easy to compute and at the same time gives a good approximation to the ILS estimator, the bootstrapped solution is an excellent candidate for setting the size of the ambiguity search space. Following the decorrelation step $\hat{z} = Z^T \hat{a}$, the LAMBDA-method therefore uses, as one of its options, the bootstrapped solution $\breve{z}_B$ for setting the size of the ambiguity search space as

$$\chi^2 = (\hat{z} - \breve{z}_B)^T Q_{\hat{z}}^{-1} (\hat{z} - \breve{z}_B) \tag{16}$$

In this way one can work with a very small search space and still guarantee that the sought for integer least-squares solution is contained in it.

3.2 The ILS success rate

The estimation procedure will always result in integer values for the phase ambiguities. These values, however, will not always be correct. A simple measure to predict the probability of estimating the unknown phase ambiguities at their correct values is called the success-rate, introduced in [Teunissen02]. This is a single number between 0 and 1 (or 0% and 100%). It is a design measure, which means that actual observations are not needed to compute this number. As the success-rate is the probability of correct integer ambiguity estimation, it equals the probability that the estimated float ambiguities will be mapped onto the correct integers. The actual ambiguity success-rate depends on three contributing factors: the observation equations (functional model), the precision of the observables (the stochastic model), and the chosen method of integer estimation. Changes in any one of these will affect the success-rate. The first two contributing factors reflect the strength of the data model and they are known once the measurement set-up is known. As to the method of integer estimation, a variety of options is available. Since different methods of integer estimation will generally result in different success-rates, one might wish to use the method that maximizes the success-rate. It has been proven that the integer least-squares estimator has the largest success-rate of all admissible integer estimators. The success-rate of the LAMBDA method is therefore larger than, or at least as large as any other integer ambiguity estimator. No analytical expressions are available for the integer least squares ambiguity success-rate. However, in a lower bound for the success-rate is given, based on integer bootstrapping. The expression for the lowerbound of the ambiguity success-rate reads

$$P(\breve{z} = z) \geq \prod_{i=1}^{n} (2\Phi(\frac{1}{2\sigma_{\hat{z}_{i|I}}}) - 1) \tag{17}$$

with the integration of the Gaussian probability density function

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} z^2) dz \tag{18}$$

The standard deviation $\sigma_{\hat{z}_{i|I}}$ is the square root of the conditional variance of the i^{th} ambiguity (conditioned on the previous $I = 1, \ldots, i-1$ ambiguities). It can be obtained from the diagonal matrix D after an LDL^T decomposition of the matrix $Q_{\hat{z}}$. The success rate should be sufficiently high, i.e. very close to one, in order to guarantee that the ambiguity will be fixed to the correct values.

It should be noted that this lower bound depends on the parametrization of the ambiguities. If computed for the LAMBDA-decorrelated ambiguities, it is a tight lower bound, i.e., the success-rate for integer bootstrapping is very close to the integer least squares success-rate. The ambiguity success-rate can be evaluated once the GNSS functional and stochastic models are known. Similar to the usage of Dilution Of Precision (DOP) measures, it can be computed without having the actual measurements available, thus prior to actual field operation. By means of the success-rate the user is given a rigorous way of assessing how often he or she can expect ambiguity resolution to be successful.

4. INTEGER AMBIGUITY RESOLUTION IN ATTITUDE DETERMINATION SYSTEMS

The baseline length constraint, $|b| = \ell$, can be expressed also as

$$(19) \qquad \|b\|_I^2 = \ell^2$$

where ℓ is the known baseline length and I is (3×3) identity matrix.

There may be many approaches to handle this constraint. The easiest one is applying constraint in validation phased. Because a vehicle is assumed as rigid body, the length of fixed baseline vector $\hat{b}(a)$ should satisfies

$$(20) \qquad \ell - \delta\ell \le \left|\hat{b}(a)\right| \le \ell + \delta\ell$$

with very small $\delta\ell$. For all candidates in the search space, baseline lengths are computed and compared with predefined threshold $\delta\ell$. If a baseline length is not in tolerable region, the ambiguity candidates a is rejected. It is not easy to define reliable $\delta\ell$. Because of noise, sometimes true a may be rejected with small $\delta\ell$ while big $\delta\ell$ has no effects. It is simple method but it requires more computational power because $\left|\hat{b}(a)\right|$ should be computed for every candidates. Sometimes validation procedure is applied to the first N small candidates in Ω_a to reduce computations.

In other approaches like LSAST [Hatch94] and ARCE [Park96, Park97], integer ambiguity vector is divided into independent and dependent parts as

$$(21) \qquad a = \begin{bmatrix} a_I & a_D \end{bmatrix}^T$$

where $a_I \in Z^3$ and $a_D \in Z^{n-3}$. Let carrier phase measurements are precise enough then we can assume

$$(22) \qquad \left|\hat{b}_I(a_I)\right| = \left|H_I^{-1}(l_I - \lambda a_I)\right| \approx \ell \, .$$

It implies that not n-dimensional but 3-dimensional search can be applied to ambiguity resolution. Furthermore, if $|b| = \ell$ is added as pseudo-measurement, it becomes 2-dimensional search. In other words, candidates on the surface of sphere are searched. It provides computational efficiency, however, because of noise, $\left|\hat{b}_I(a_I)\right| = \ell$ is not valid and also pseudo-measurement $|b| = \ell$ is not valid. Therefore, boundary $\ell - \delta\ell \le \left|\hat{b}_I(a_I)\right| \le \ell + \delta\ell$ is adopted. The size of search space increases for longer baseline since the surface of sphere is larger. It is not easy to determine $\delta\ell$ and there is no theoretic guide to divide a_I and a_D.

Not only known baseline lengths but also configuration of baselines can be utilized in the modified LAMBDA. This can be done by expanding the existing ILS problem with the information. The LAMBDA method is modified in order to make use of all a priori available information.

5. EXPERIMENTAL RESULTS

5.1 Short rotating baseline

Total of 2100 epochs measurements from two NovAtel 12 channels single frequency C/A code GPS receivers are used to evaluate the performance of heuristic methods. It is assumed that true attitudes and integer ambiguity vector are known. The attitude obtained using the integer ambiguity vector gives consistent result or a long times so that it is considered as a true integer ambiguity vector. Experimental success rates are computed by comparing the true integer ambiguity vector and an obtained integer ambiguity vector at every epoch. Baseline length is 40cm and it starts rotations in the horizontal plane using a stepping motor after 1000 epochs.

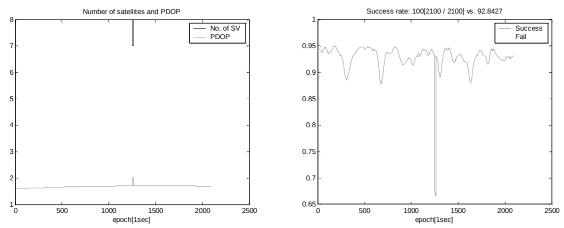
Table 1 summarized experimental results. In table LAMBDA means the original Lambda method which does not consider baseline length is used. In this case, it is not surprising that the success rate is very poor. LAMBDA+VALIDATION means that validation procedure using $\ell - \delta\ell \le \left|\hat{b}(a)\right| \le \ell + \delta\ell$ is added to the original Lambda method. By adding this validation procedure, some improvement in success rate is achieved. ARCE gives much more performance improvements. Because of measurement noise and uncertainties in true baseline length, there are 90 failed epochs where the baseline lengths with true integer ambiguity are out of search space. The modified LAMBDA method gives dramatic improvement. It gives 100% success rate which is almost double of the original Lambda method.

Fig. 1 shows the number of satellites, PDOP during the experiment, and experimental and theoretic success rates computed at every epoch. The average of theoretic success rate is 92.8%. It is around 90% when there are 8 satellites but it suddenly decreases to less than 70% when number of satellites drops to 7. In the figure, the red dots will indicate failure epochs but because we have 100% success rate with the proposed method there are no red dots.

Fig. 2 show obtained heading, pitch (elevation) and baseline length. Blue line indicates attitude using floating ambiguity while red line is obtained using fixed ambiguity. Figure also represents horizontal trajectory with floating and fixed ambiguity. The reference antenna is represented by a circle at (0,0) point. These figures clearly show why ambiguity resolutions are required. Figures explain movements of the baseline: it stayed at first 1000 epochs and then it starts rotation. Obtained pitches reveal the baseline is not well leveled and it oscillates as rotations of the baseline. For the first 1000 static epochs, the mean of heading, pitch and length are 49.6511deg, -0.7037deg and 0.4025m. The standard deviation of heading, pitch and length are 0.2899deg, 0.5533deg and 0.0019m, respectively. These results are exactly consistent with the analysis in [Park00].

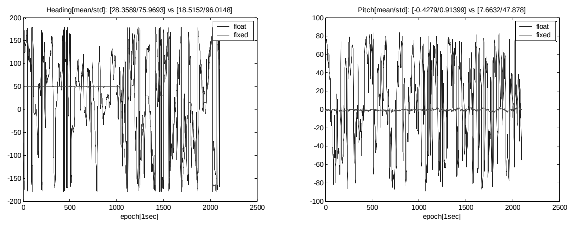
Table 1. Experimental success rate [40cm baseline]

Method	Success Rate
LAMBDA	51.1% (1073/2100)
LAMBDA+VALIDATION	78.7% (1653/2100)
ARCE	95.7% (2010/2100)
LAMBDA MODIFIED	100% (2100/2100)

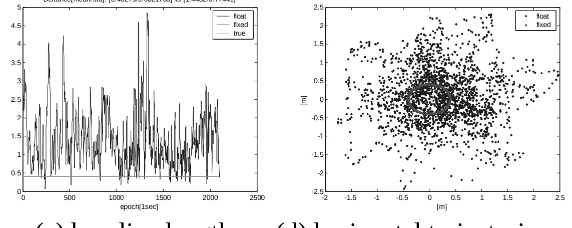


(a) No Sat and PDOP (b) Success Rate

Fig. 1 Number of satellites, PDOP and Success Rates



(a) headings (b) elevations



(c) baseline lengths (d) horizontal trajectories

Fig. 2 Floating and fixed attitudes

5.2 Van test with 2.5m baseline

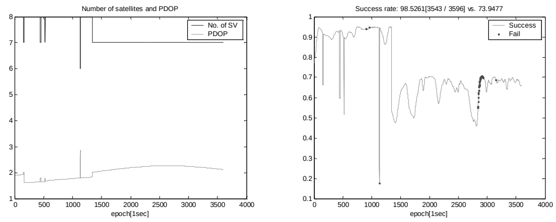
The stack of data collected at December 22, 1996 with Trimble 4000 SSI Geodetic Surveyor receivers in the Flevo-polder [Tiberius98] are re-invited for attitude determination processing. Measurements from two receivers (Stations 38 and 39 with distance of 2.563m) installed on a van is used to obtain attitudes. Test starts at 08:10 and lasts for an hour. Among 3600 epoch's data, 4 epoch measurements are removed because of cycle slips. To compare the performance, gathered measurements are processed in two ways, i) single frequency and ii) dual frequency measurements.

Numbers of satellites, PDOP and success rate using single frequency processing are shown in Fig. 3. The average of theoretic success rate is 73.95% while the experimental success rate is obtained as 98.53% (3543 success / 3596 trials). In Fig. 4, numbers of satellites, PDOP and success rate using dual frequency processing are shown. Average number of satellites for L2 is 7.3184 and for sum of L1 and L2 are 14.6813. The average of theoretic success rate becomes 99.9997% as numbers of satellites increase, and the experimental success rate is obtained as 100%. That is, using the modified LAMBDA method, integer ambiguities are always found.

The horizontal trajectories in Fig. 5 clearly show the relative trajectories of two antennas. The reference antenna is located at the center of figure while the rover antenna's locations are given on the circle. It reflects the test driving that a 10 km stretch on a dike has been traveled forth and back with a few minutes of measurements taken stationary at the start, the turning-point and at the end.

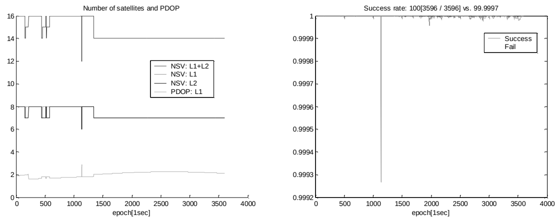
Fig. 6 show floating and fixed attitudes of a van. Determined headings clearly show when the van takes turns. And determined pitch shows how turns are done. The van is slanted when it turns. It also can be clearly distinguished that an integer ambiguity is correctly resolved or not by seeing pitches or horizontal trajectories obtained using fixed integer ambiguity. Because an integer ambiguity which gives a position on the surface of a sphere with radius of baseline length is found in the ambiguity resolution, it is easy to check the correctness of obtained attitudes. However, in this paper, it is not applied yet since the purpose of this research is on the integer ambiguity resolution itself.

Furthermore, it is interesting to note that using the determined pitches it can be possible to distinguish stationary and moving operations. According to the log [Tiberius98], the van was in static modes for intervals of (0 - 180), (708 - 888) and (3290 - 3599). The deviation of pitch during the stationary operation is much less than that of moving.



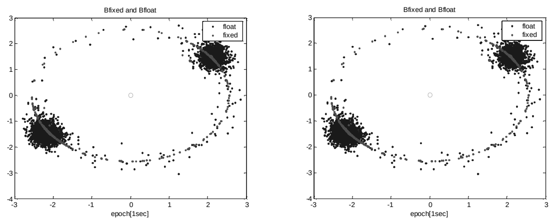
(a) No Sat and PDOP (b) Success Rate

Fig. 3 Number of satellites, PDOP and Success Rates (Single Frequency)



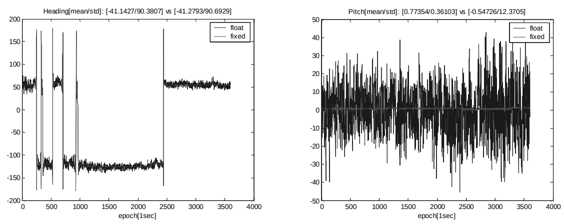
(a) No Sat and PDOP (b) Success Rate

Fig. 4 Number of satellites, PDOP and Success Rates (Dual Frequency)



(a) Single Freq. (b) Dual Freq.

Fig. 5 Horizontal trajectories with fixed ambiguities



(a) headings (b) elevations

Fig. 6 Floating and fixed attitudes (Dual Frequency)

6. CONCLUSIONS

In this paper the results of a novel LAMBDA based method for attitude determination are presented. The modified LAMBDA method makes use of all available information for attitude determination. Compared to other methods such as heuristic validation method or ARCE, the LAMBDA based method is mathematically rigorous and compact. Experimental results with real measurements show that it dramatically improves the success rate of integer ambiguity resolution and therefore improves the reliability of attitude determination systems.

It is expected that our method can be directly used to build precise and reliable attitude determination systems with off-the-shelf low cost GNSS receivers.

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