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ORIGINAL ARTICLE

Characterization of between-receiver GPS-Galileo inter-system biases and their effect on mixed ambiguity resolution

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Abstract The Global Positioning System (GPS) and Galileo will transmit signals on similar frequencies, that is, the L1–E1 and L5–E5a frequencies. This will be beneficial for mixed GPS and Galileo applications in which the integer carrier phase ambiguities need to be resolved, in order to estimate the positioning unknowns with centimeter accuracy or better. In this contribution, we derive the mixed GPS + Galileo model that is based on "inter-system" double differencing, that is, differencing the Galileo phase and code observations relative to those corresponding to the reference or pivot satellite of GPS. As a consequence of this, additional between-receiver inter-system bias (ISB) parameters need to be solved as well for both phase and code data. We investigate the size and variability of these between-receiver ISBs, estimated from L1 and L5 observations of GPS, as well as E1 and E5a observations of the two experimental Galileo In-Orbit Validation Element (GIOVE) satellites. The data were collected using high-grade multi-GNSS receivers of different manufacturers for several zero- and short-baseline setups in Australia and the USA. From this analysis, it follows that differential ISBs are only significant for receivers of different types and manufacturers; for baselines formed by identical receiver types, no differential ISBs have shown up; thus, implying that the GPS and GIOVE data are then fully interoperable. Fortunately, in case of different receiver types, our analysis also indicates that the phase and code ISBs may be calibrated, since their estimates, based on several datasets separated in time, are shown to be very stable. When the single-frequency (E1) GIOVE phase and code data of different receiver types are a priori corrected for the differential ISBs, the short-baseline instantaneous ambiguity success rate increases significantly and becomes comparable to the success rate of mixed GPS + GIOVE ambiguity resolution based on identical receiver types.

Keywords GPS-Galileo interoperability · Between-receiver inter-system bias · Integer ambiguity resolution · GIOVE

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Introduction

The two experimental Galileo satellites, GIOVE (Galileo In-Orbit Validation Element) A and B, have been in orbit since 2005 and 2008, respectively. One of the objectives of the GIOVE mission is to characterize the novel features of the Galileo signal design (see http://www.esa.int). In addition, two new satellites were launched on October 21, 2011, and an additional two on October 10, 2012, as part of the In-Orbit Validation (IOV) stage of Europe's Galileo project. Both the GIOVE and IOV satellites broadcast signals at the proposed Galileo frequencies, which are the E1, E5a, E5b, E5 (E5a + E5b), and E6 frequencies. It is remarked that the E6 frequency will only be received as part of Galileo's Commercial Service, while all other frequencies will be used in the (free) Open Service. From this set of Galileo frequencies, the E1 (1,575.42 MHz) and E5a (1,176.45 MHz) are overlapping the L1 and L5 frequencies of the US Global Positioning System (GPS). At present (2012), L5 signals are only transmitted by the GPS Block IIF satellites, that is, G25 (SVN62), which was launched in

2010, by G01 (SVN63), launched in 2011, and by SVN65 as launched on October 4, 2012. This overlap in frequencies between GPS and Galileo is foreseen to be beneficial for mixed dual-constellation high-precision real-time kinematic (RTK) positioning applications (Tiberius et al. 2002). The benefit as compared to GPS-only is basically due to (1) the roughly doubling of the number satellites and (2) the expected improved precision of the code or pseudorange data. Initial analysis has already demonstrated that the L5 code data are of better quality than those of current GPS (de Bakker et al. 2012). Also the precision of the Galileo code data is expected to be promising, see Colomina et al. (2011). This improved code precision will enhance carrier phase integer ambiguity resolution (Teunissen 1997; Milbert 2005), an essential process for centimeter-level positioning or better. Initial results of integrated GPS + GIOVE ambiguity resolution are reported in Simsky et al. (2008) and de Bakker et al. (2009, 2012); however, these results have all been obtained using the *geometry-free* model, the model that solves for the receiver-satellite ranges instead of the receiver position. We base the ambiguity resolution on the *geometry-based* model, the usual model for positioning.

In principle, with GPS and Galileo tracking data on the same frequencies, double differencing between both constellations (i.e., Galileo double differences having a pivot satellite of GPS and vice versa) must be possible (Sleewaegen 2012). However, when combining signals of different constellations and different receiver types one have to take care of *inter-system biases*, despite that the signals are tracked on the same frequency (Hegarty et al. 2004). The GPS-to-Galileo inter-system bias is the difference between the delay in the receiver that the GPS and Galileo signals experience. It depends on the correlation inside the receiver and depending on the receiver type it may reach up to several hundreds of nanoseconds (Montenbruck et al. 2011). We will characterize the inter-system biases between, on the one hand, GPS L1 and L5 and, on the other hand, Galileo E1 and E5a for different types of multi-GNSS receivers. Furthermore, the possibility of calibrating these inter-system biases and the effect on mixed GPS + Galileo integer ambiguity resolution is analyzed. This is done for both zero- and short-baseline models, for which relative atmospheric effects cancel out and thus do not bias the estimation of the differential inter-system biases.

Section "GPS+Galileo mixed observation models" presents the model of mixed GPS-Galileo observation equations, with a special focus on the parameterization of GPS-Galileo differential inter-system biases. Section "Results of differential ISB estimation and their effect on GPS+GIOVE ambiguity resolution" presents results of the analysis of differential inter-system biases for a number of high-end geodetic receiver types. The analysis is restricted to data of GIOVE-A/B, since at the time of the writing the navigation messages broadcast by the new Galileo IOV satellites were not yet available. Results of the effect of a priori correcting the mixed receiver observations for the inter-system biases on the performance of integer ambiguity resolution are presented in "Results of differential ISB estimation and their effect on GPS+GIOVE ambiguity resolution" as well. Section "Summary and conclusions" provides conclusions.

GPS + Galileo mixed observation models

This section discusses the observation equations and models for combined GPS + Galileo real-time kinematic relative positioning. As the focus is on the parameterization of inter-system biases, our derivations apply to the overlapping GPS and Galileo frequencies only.

GPS + Galileo phase and code observation equations

Let us assume that a mixed receiver tracks both GPS and Galileo phase and code observations at the same time instants in the time system of one of the constellations, for example, GPS Time (GPST). A GPS satellite broadcasts its position and clock information also in GPST; however, a Galileo satellite broadcasts its position and clock information in the time system of Galileo, that is, Galileo System Time (GST). Hence, in order to setup the Galileo observation equation in GPST as well, a GPS-to-Galileo Time Offset (GGTO; Hahn and Powers 2005) needs to be applied to convert the time of transmission at the Galileo satellite to GPST.

The observation equation for a phase observable of receiver r and GPS satellite s tracked at frequency j then reads (Hofmann-Wellenhof et al. 2008):

$$\phi^s_{r,j} = \rho^s_r + dt_r - dt^s + \alpha^s_{r,j} + \lambda_j\left(\delta^G_{r,j} + \varphi_{r,j} - \delta^s_{,j} - \varphi^s_{,j} + N^s_{r,j}\right) + \epsilon^s_{r,j} \quad (1)$$

with $\phi^s_{r,j}$ the phase observable, ρ^s_r the range between the receiver and the GPS satellite, dt_r the receiver clock error, dt^s the GPS satellite clock error, $\alpha^s_{r,j}$ the (frequency-dependent) phase atmospheric delay, λ_j the wavelength, $\delta^G_{r,j}$ the hardware GPS phase delay in the receiver, $\varphi_{r,j}$ the initial phase in the receiver, $\delta^s_{,j}$ the hardware phase delay in the satellite, $\varphi^s_{,j}$ the initial phase in the satellite, $N^s_{r,j}$ the (integer) phase ambiguity, and $\epsilon^s_{r,j}$ all other errors including measurement noise. Note that all variables are expressed in meters; except the hardware delays and the ambiguity, which are expressed in cycles. In the notation of the receiver hardware phase delay, we use the superscript "G" (of GPS), as to distinguish it from the phase delay a Galileo signal is experiencing inside a mixed receiver (which is denoted using the superscript "E" of Europe). As the Galileo frequencies are assumed to overlap those of GPS, the Galileo phase observation equation is very similar to that of GPS, with the only difference being the GGTO:

$$\phi^q_{r,j} = \rho^q_r + dt_r - dt^q + \alpha^q_{r,j} + \lambda_j\left(\delta^E_{r,j} + \varphi_{r,j} - \delta^q_j - \varphi^s_j + N^q_{r,j}\right) - GGTO + \in^q_{r,j} \quad (2)$$

The superscript q is used for a Galileo satellite as to distinguish it from superscript s for a GPS satellite. The GGTO is expressed here in meters. The code observation equation at frequency j for GPS, denoted as $p^s_{r,j}$, reads

$$p^s_{r,j} = \rho^s_r + dt_r - dt^s + a^s_{r,j} + d^G_{r,j} - d^s_j + e^s_{r,j} \quad (3)$$

where the following variables are different from the GPS phase observation equation: $d^G_{r,j}$, the hardware code delay in the receiver, d^s_j the satellite hardware code delay, $a^s_{r,j}$ the (frequency-dependent) code atmospheric delay, and $e^s_{r,j}$ the remaining code errors including measurement noise. All variables in the code observation equation are expressed in meter. For Galileo, it reads

$$p^q_{r,j} = \rho^q_r + dt_r - dt^q + a^q_{r,j} + d^E_{r,j} - d^q_j - GGTO + e^q_{r,j} \quad (4)$$

where $d^E_{r,j}$ denotes the receiver Galileo code hardware delay, d^q_j the satellite hardware delay, $a^q_{r,j}$ the code atmospheric delay, and $e^q_{r,j}$ the remaining code errors including measurement noise. If we compare the above Galileo equations with those of GPS, we note that the combined receiver clock error plus GGTO, that is, $dt_r - GGTO$ can be considered a receiver clock parameter for Galileo, in addition to the GPS receiver clock error. Because of this, in case of mixed standard point positioning one would then estimate a separate receiver clock error for GPS as well as Galileo. Results of the combined use of GPS and GIOVE data for the purpose of single and precise point positioning are not given in this article; they can be found in Bonhoure et al. (2009) and Cao et al. (2010).

Mixed model with unknown differential ISBs

Single-differencing relative to the observations of a pivot receiver (denoted using subscript 1; the rover receiver is denoted using subscript 2) results in the following observation equations in which the satellite-specific biases are eliminated. For GPS phase and code this yields:

$$\begin{aligned} \phi^s_{12,j} &= \phi^s_{2,j} - \phi^s_{1,j} = \rho^s_{12} + dt_{12} + \alpha^s_{12,j} + \lambda_j(\delta^G_{12,j} + \varphi_{12,j} + N^s_{12,j}) + \in^s_{12,j} \\ p^s_{12,j} &= p^s_{2,j} - p^s_{1,j} = \rho^s_{12} + dt_{12} + a^s_{12,j} + d^G_{12,j} + e^s_{12,j} \end{aligned} \quad (5)$$

with $j = 1,\ldots,f$ and $s = 1_G,\ldots,m_G$, where f denotes the number of frequencies and m_G the number of GPS satellites tracked. Similar, for Galileo phase and code, the following single differences can be formed

$$\begin{aligned} \phi^q_{12,j} &= \phi^q_{2,j} - \phi^q_{1,j} = \rho^q_{12} + dt_{12} + \alpha^q_{12,j} + \lambda_j(\delta^E_{12,j} + \varphi_{12,j} + N^q_{12,j}) + \in^q_{12,j} \\ p^q_{12,j} &= p^q_{2,j} - p^q_{1,j} = \rho^q_{12} + dt_{12} + a^q_{12,j} + d^E_{12,j} + e^q_{12,j} \end{aligned} \quad (6)$$

with $j = 1,\ldots,f$ frequencies overlapping with GPS and $q = 1_E,\ldots,m_E$, where m_E denotes the number of Galileo satellites. It can be seen that in the above observation equations the GGTO is removed. For this paper, we only consider short-baseline applications for which it can be assumed that the differential atmospheric delays can be ignored, that is, $\alpha^s_{12,j} = a^s_{12,j} = 0\,\forall j, s$ and $\alpha^q_{12,j} = a^q_{12,j} = 0\,\forall j, q$. In a next step, with "classical" double differencing, we form per constellation double differences to eliminate the receiver-dependent biases. If we fix $s = 1_G$ as pivot satellite for GPS, we obtain

$$\begin{aligned} \phi^{1_G s}_{12,j} &= \phi^s_{12,j} - \phi^{1_G}_{12,j} = \rho^{1_G s}_{12} + \lambda_j N^{1_G s}_{12,j} + \in^{1_G s}_{12,j} \\ p^{1_G s}_{12,j} &= p^s_{12,j} - p^{1_G}_{12,j} = \rho^{1_G s}_{12} + e^{1_G s}_{12,j} \end{aligned} \quad (7)$$

for $s = 2_G,\ldots,m_G$. The Galileo double-difference observation equations are, fixing $q = 1_E$ as pivot satellite:

$$\begin{aligned} \phi^{1_E q}_{12,j} &= \phi^q_{12,j} - \phi^{1_E}_{12,j} = \rho^{1_E q}_{12} + \lambda_j N^{1_E q}_{12,j} + \in^{1_E q}_{12,j} \\ p^{1_E q}_{12,j} &= p^q_{12,j} - p^{1_E}_{12,j} = \rho^{1_E q}_{12} + e^{1_E q}_{12,j} \end{aligned} \quad (8)$$

for $q = 2_E,\ldots,m_E$. As consequence, the double-differenced ambiguities for GPS as well as Galileo are integers. For position estimation, the above double-differenced observation equations are linearized, since the receiver-satellite ranges are nonlinear in both receiver and satellite positions. In addition, for short-baseline applications, the position of the pivot receiver 1 is held fixed, as well as the satellite positions, the latter being computed from the satellite's navigation message.

Because the GPS and Galileo frequencies overlap, instead of forming Galileo double differences as above, we can also form Galileo double differences with respect to the *GPS* pivot satellite:

$$\begin{aligned} \phi^{1_G q}_{12,j} &= \phi^q_{12,j} - \phi^{1_G}_{12,j} = \rho^{1_G q}_{12} + \lambda_j(\delta^{GE}_{12,j} + N^{1_G q}_{12,j}) + \in^{1_G q}_{12,j} \\ p^{1_G q}_{12,j} &= p^q_{12,j} - p^{1_G}_{12,j} = \rho^{1_G q}_{12} + d^{GE}_{12,j} + e^{1_G q}_{12,j} \end{aligned} \quad (9)$$

for $q = 1_E,\ldots,m_E$. Thus, for phase as well as code, one double difference more can be formed as compared to classical double differencing. On the other hand, the following terms are not eliminated in this "inter-system" double differencing: $\delta_{12,j}^{GE} = \delta_{2,j}^{GE} - \delta_{1,j}^{GE}$ and $d_{12,j}^{GE} = d_{2,j}^{GE} - d_{1,j}^{GE}$, denoting the between-receiver differential GPS-Galileo *inter-system biases* (*ISB*) for, respectively, phase and code. The ISB equals the difference in hardware delay between the GPS and Galileo signals on identical frequencies inside the same receiver, that is, $\delta_{r,j}^{GE} = \delta_{r,j}^{E} - \delta_{r,j}^{G}$ for phase and $d_{r,j}^{GE} = d_{r,j}^{E} - d_{r,j}^{G}$ for code, with $r = 1, 2$. Both phase and code ISBs are only estimable as *differential* parameters, that is, relative to those of the reference receiver.

Unfortunately, the system of mixed observation equations formed by (7) and (9) contains a *rank deficiency*: it is not possible to simultaneously estimate the differential phase ISB parameters and the double-difference Galileo ambiguities with respect to GPS. This rank deficiency is of size f (equal to the number of frequencies) and can be eliminated by making use of the following identity:

$$N_{12,j}^{1_G q} = \left(N_{12,j}^{q} - N_{12,j}^{1_E}\right) + \left(N_{12,j}^{1_E} - N_{12,j}^{1_G}\right) = N_{12,j}^{1_E q} + N_{12,j}^{1_G 1_E}, \quad q = 1_E, \ldots, m_E \tag{10}$$

that is, we can reparameterize the Galileo ambiguities relative to the GPS pivot satellite into ambiguities relative to the Galileo pivot satellite, plus the ambiguity of the Galileo pivot satellite with respect to the GPS pivot satellite (i.e., $N_{12,j}^{1_G 1_E}$). This last ambiguity is then estimable lumped to the differential phase ISB term. The phase and code observation equations for Galileo corresponding to a *full-rank* GPS + Galileo model then read:

$$\begin{aligned} \phi_{12,j}^{1_G q} &= \phi_{12,j}^{q} - \phi_{12,j}^{1_G} = \rho_{12}^{1_G q} + \lambda_j(\bar{\delta}_{12,j}^{GE} + N_{12,j}^{1_E q}) + \epsilon_{12,j}^{1_G q} \\ p_{12,j}^{1_G q} &= p_{12,j}^{q} - p_{12,j}^{1_G} = \rho_{12}^{1_G q} + d_{12,j}^{GE} + e_{12,j}^{1_G q} \end{aligned} \tag{11}$$

for $q = 1_E, \ldots, m_E$, with the estimable differential phase ISB parameter $\bar{\delta}_{12,j}^{GE} = \delta_{12,j}^{GE} + N_{12,j}^{1_G 1_E}$. Thus, although the estimable differential code ISB is a truly unbiased parameter, the estimable ISB parameter for phase is *biased* by the inter-system ambiguity between the pivot satellites.

Although differencing the Galileo observations with respect to GPS, see (11), yields a system having more observations than when they are differenced relative to a Galileo pivot satellite, see (8), the system based on (11) has more unknowns as well, that is, the phase and code ISB parameters. It can be shown that the redundancy (number of observations minus number of estimable parameters) of the model based on (7–11) is, however, identical to that of the model based on (7–8). In single-epoch mode, this redundancy equals $f(m_G + m_E - 2) - v$, where v denotes the number of coordinate parameters ($v = 3$ for a single-baseline positioning model). Thus, redundancy appears with the combined number of satellites $m_G + m_E \geq v + 2$. To compare, the redundancy of the GPS-only model reads $f(m_G - 1) - v$, so $m_G \geq v + 1$. Thus, adding Galileo increases the redundancy with $f(m_E - 1)$, which means that adding only one Galileo satellite does not contribute, however, at least two do.

Mixed model with known differential ISBs

From the previous subsection, it follows that the alternative double differencing between constellations does not strengthen the mixed GPS + Galileo model in terms of ambiguity resolution and positioning compared to classical double differencing per constellation, since the benefit of having additional double differences is annihilated by the presence of the additional differential ISB parameters. This, however, changes if one has knowledge on the behavior of the differential ISBs. For example, if they can be assumed to be constant in time instead of treating them as time-varying parameters, this would already strengthen the model. In the case they would be completely known (deterministic), they can be a priori subtracted from the phase and code observations.

In this subsection, we will evaluate the inter-system Galileo double differences assuming the differential phase and code ISBs, that is, $\bar{\delta}_{12,j}^{GE}$ and $d_{12,j}^{GE}$ are *known*. At first sight, in case of the phase ISB, there seems to be a problem, because of its lumping with the ambiguity between the pivot satellites of both constellations, which is integer but unknown. Fortunately, this is not an issue and this can be shown as follows. Let us denote the phase and code ISB corrections using a tilde on top of it, such that:

$$\begin{aligned} \tilde{\delta}_{12,j}^{GE} &= \delta_{12,j}^{GE} + z_{12,,j}, \quad \text{with } z_{12,j} \in \mathbb{Z} \\ \tilde{d}_{12,j}^{GE} &= d_{12,j}^{GE} \end{aligned} \tag{12}$$

Now integer $z_{12,j}$ denotes the ambiguity as part of the correction, which is in principle *different* from the ambiguity $N_{12,j}^{1_G 1_E}$, that is present in the GPS + Galileo dataset we would like to correct. Denoting the phase ISB parameter of the dataset to be corrected as $\bar{\delta}_{12,j}^{GE} = \delta_{12,j}^{GE} + N_{12,j}^{1_G 1_E}$, then the phase ISB correction can be rewritten as:

$$\tilde{\delta}_{12,j}^{GE} = \bar{\delta}_{12,j}^{GE} - \left(N_{12,j}^{1_G 1_E} - z_{12,j}\right) \tag{13}$$

Thus, the difference in ISB parameter and correction is due to a difference of ambiguities between the observations that are corrected and the observations from which the correction is determined. Correcting the observations for the phase ISB then implies that this ambiguity difference needs to be added to the estimable Galileo integer ambiguity $N_{12,j}^{1_E q}$, such that the estimable ambiguity becomes $N_{12,j}^{1_E q} + \left(N_{12,j}^{1_G 1_E} - z_{12,j}\right) = N_{12,j}^{1_G q} - z_{12,j}$. This ambiguity is a combination of the Galileo double-difference ambiguity, but now relative to the *pivot satellite of GPS*, minus the integer ambiguity as lumped in the biased phase ISB correction. The ISB-corrected phase and code Galileo observation equations (denoted using a tilde on top of the observables) can then be given as:

$$\begin{array}{lllll} \tilde{\phi}_{12,j}^{1_G q} & = & \phi_{12,j}^{1_G q} - \tilde{\delta}_{12,j}^{GE} & = & \rho_{12}^{1_G q} + \lambda_j \tilde{N}_{12,j}^{1_G q} + \epsilon_{12,j}^{1_G q} \\ \tilde{p}_{12,j}^{1_G q} & = & p_{12,j}^{1_G} - \tilde{d}_{12,j}^{GE} & = & \rho_{12}^{1_G q} + e_{12,j}^{1_G q} \end{array} \quad (14)$$

for $q = 1_E, \ldots, m_E$ and $j = 1, \ldots, f$ and with estimable integer ambiguity $\tilde{N}_{12,j}^{1_G q} = N_{12,j}^{1_G q} - z_{12,j}$. It is thus not problematic that there is an additional integer $z_{12,j}$, since only the *combined* integer $\tilde{N}_{12,j}^{1_G q}$ is estimated in the processing. This integer ambiguity is relative to the pivot satellite of GPS, similar to the Galileo double-differenced observations. From this follows the important conclusion that when the inter-system biases are corrected for, the Galileo data can be processed as if they were GPS data.

This ISB-corrected GPS + Galileo model is stronger than the mixed model in which the double-difference ambiguities are defined with respect to a pivot satellite in its own system. In terms of redundancy, it is increased with f compared to the model having the ISBs as unknown parameters. Compared to GPS-only, the redundancy is increased with fm_E. This means that already one Galileo satellite contributes to a strengthening of the model, while this was at least two for the model with the unknown ISB parameters.

Results of differential ISB estimation and their effect on GPS + GIOVE ambiguity resolution

In this section, we will analyze differential ISBs from multiple GPS + GIOVE datasets consisting of identical and different types of geodetic multi-GNSS receivers. We will determine phase and code ISBs for L1–E1 and L5–E5a based on the observation model formulation in (7) and (11), for several zero- and short-baseline cases. For the purpose of differential ISB estimation, we do not estimate the rover position, but keep it fixed to known ground-truth values. Besides strengthening the model, this fixing of the rover coordinates is advantageous since it requires less satellites then would be required in case of solving the position unknowns. The coordinate-fixed mixed model ($v = 0$, see "Mixed model with unknown differential ISBs") is already solvable with just one GPS and one Galileo satellite, which is relevant for the estimation of the ISBs for L5–E5a, since at present there are only few GPS satellites transmitting the L5 signal. The stability of the ISBs is analyzed by processing data time spans of full passes (between rising and setting) of the GIOVE satellites, if possible. For all cases, we use broadcast navigation message data to compute both GPS and GIOVE orbits. Although there may be small (i.e., cm level) differences in the reference frames of the GPS and GIOVE satellite positions, due to the high altitudes of the orbits they can be safely ignored for the cases as discussed here.

It is emphasized that in all cases the differential ISB estimation is based on *epoch-by-epoch* processing, without any link between the epochs. For visualization purposes, we subtracted an arbitrary but same *integer* from the phase ISB estimates for all epochs, such that the residual or fractional ISB estimate is varying close to zero. This way of presenting the estimated phase ISB is done for all numerical examples throughout this article.

The estimated differential ISBs for L1–E1 are in a next step applied to other (in time) datasets of zero/short baselines, so as to investigate the possibility of calibration. With the GIOVE data a priori corrected for the ISBs, the *positioning model* is now solved (i.e., $v = 3$), and the performance of GPS-only versus GPS + GIOVE geometry-based single-frequency ($f = 1$) ambiguity resolution is analyzed (except for L5–E5a because of too few satellites). In all cases, ambiguity resolution is performed by means of the LAMBDA method (Teunissen 1995). Integer estimation is conducted for each epoch individually, meaning that no information from previous or forthcoming epochs is taken into account, as to test the performance of the fastest mode of ambiguity resolution. The integer outcomes of each epoch are compared to a "ground-truth" solution, obtained from a multi-epoch (Kalman filter) processing for which the ambiguities are kept constant in time. An (empirical) success rate of the instantaneous ambiguity resolution process is obtained by dividing the number of correctly resolved epochs with the total number of epochs of the time span. In the final step, for epochs with correctly resolved integer ambiguities, the unknown parameters of interest are solved with high precision.

Curtin University Bentley/Muresk experiment: short baselines

The first GPS + GIOVE experiments have been carried out at two campuses of Curtin University: (1) the campus in Muresk, about 90 km east of Perth, Western Australia, on June 8, 2011 (18:20–24:00 UTC); and (2) Curtin's main campus in Bentley (Perth), on June 9, 2011 (04:07–10:21 UTC). In both experiments, very short (0.5–1 m) baselines were occupied using a combination of identical Javad Quattro-G3D Delta receivers, or a combination of one of

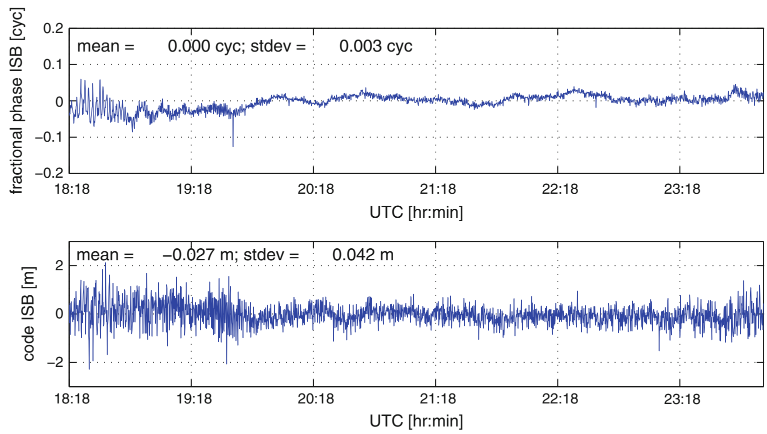

Fig. 1 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for the Muresk experiment (08/06/11), for two Javad receivers separated by 0.5 m, based on GPS and GIOVE-A data

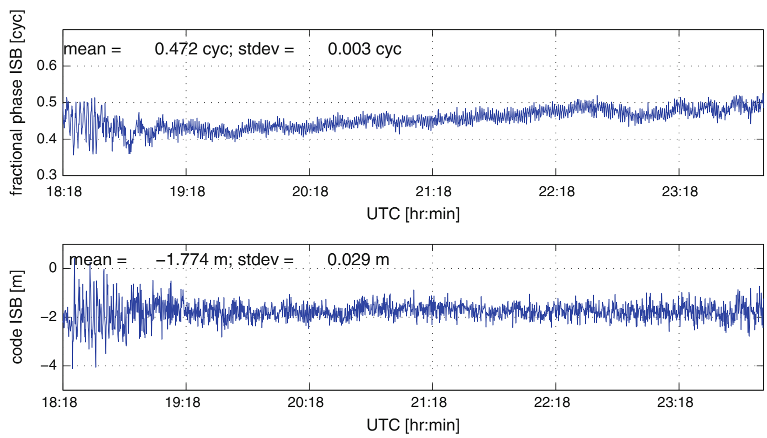

Fig. 2 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for the Muresk experiment (08/06/11), for Javad and Trimble receivers, separated by 0.5 m, based on GPS and GIOVE-A data

these Javad receivers with a Trimble Net R9 receiver. The measurement sampling interval was 10 s and the cutoff elevation 15 deg. While for both days the number of GPS satellites varied between 6 and 10, for each of the 2 days only one of the GIOVE satellites was in view: GIOVE-A (E51) on the first day and GIOVE-B (E52) on the second day. Since the Javad receiver only collects GIOVE data on E1, in this experiment we will only focus on a presence of any L1–E1 inter-system biases.

Figure 1 depicts for the Muresk experiment the estimated differential ISBs for the baseline consisting of the identical Javad receivers and Fig. 2 those for the Javad–Trimble baselines. The graphs show that the differential ISBs for the Javad–Javad baseline are estimated with a mean of practically zero for the (fractional) phase ISB and a mean of about -3 cm for the code ISB, over the almost 6-h time span. On the other hand, the estimated ISBs for the Javad–Trimble baseline have a mean of 0.47 cyc for the phase and a mean of -1.77 m for the code. Considering the measurement noise, which is at the level of few millimeters for phase and a few decimeters for code, the estimated differential ISB estimates fall safely within the noise of the observations for the baseline consisting of identical Javad receivers. The higher noise at the beginning of the time series in both figures is due to the relatively low elevation of the GIOVE satellite, causing the observations to be less precise. For the mixed Javad–Trimble baseline, the estimated differential ISBs seem to be very significant. The standard deviation at the top of each graph in the figures is the standard deviation corresponding to the differential ISB *assuming it as constant* during the time span in the figure. As can be seen from the right graphs, the standard deviation for the phase ISB of the Javad–Trimble baseline is 0.003 cyc, while that for the code ISB is about 3 cm. Both these standard deviations are much lower than the measurement noise of phase and code, and this stability suggests that the estimated differential ISBs can be used for calibration of other datasets consisting of the same receiver types.

To verify this, we have used these estimated ISBs to a priori correct the phase and code data for GIOVE-B, collected in the Bentley experiment of the next day (note that the ISBs have been determined based on GIOVE-A). In the Bentley experiment again two baselines were observed, one consisting of identical Javad receivers and the other of mixed Javad–Trimble receivers. Only the GIOVE data of the Javad–Trimble baseline were corrected for the differential ISBs. For both baselines in the Bentley experiment it

Table 1 Ambiguity resolution success rates for baselines in the Curtin Bentley experiment

	GPS-only	GPS + GIOVE-B ISBs uncorrected	GPS + GIOVE-B ISBs corrected
Javad–Javad (1 m)	1,795/2,245 = 80.0 %	2,054/2,245 = 91.5 %	2,054/2,245 = 91.5 %
Javad–Trimble (0.5 m)	2,023/2,245 = 90.1 %	0/2,245 = 0 %	2,148/2,245 = 95.7 %

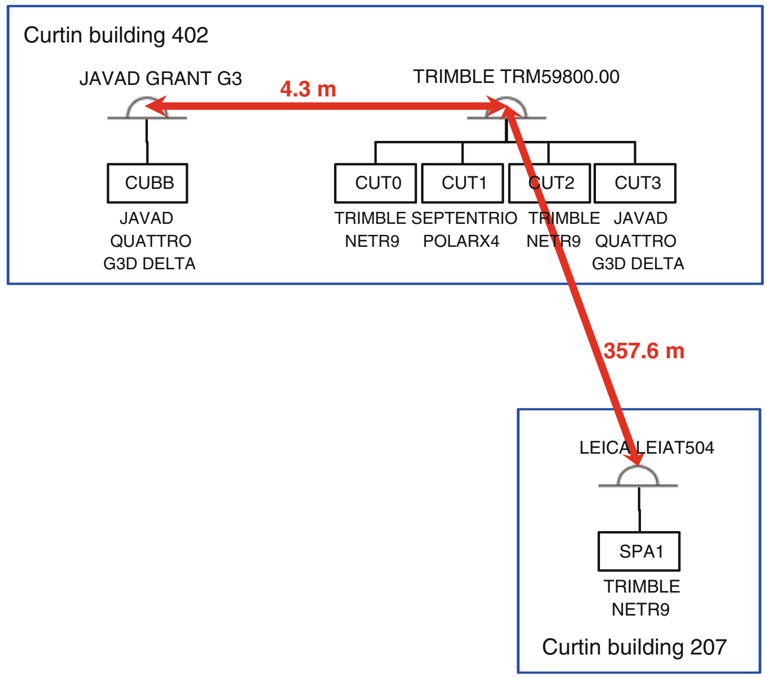

Fig. 3 Overview of multi-GNSS receivers and antennas as used in the Curtin Bentley tests

was then tried to resolve the integer ambiguities on an epoch-by-epoch basis, based on the single-frequency positioning model, first for GPS-only and then for GPS + GIOVE, see Table 1. As to demonstrate the benefit of correcting for the ISBs, in this table also the success rates are shown for GPS + GIOVE with the ambiguities for GIOVE relative to the pivot satellite of GPS, but with the GIOVE data *uncorrected* for the differential ISBs.

The table shows that the success rates of instantaneous ambiguity resolution for GPS-only are for both baselines at least 80 %. The higher success rate for the Javad–Trimble baseline than the Javad–Javad baseline is a consequence of the better code precision of the Trimble data as compared to Javad. Combining the GPS data with those of GIOVE, processed according to (14) without ISB correction for both baselines, resulted in a 91.5 % success rate for the Javad–Javad baseline; however, a 0 % success rate was found for the Javad–Trimble baseline, so for none of the epochs the correct integer ambiguities could be resolved. Correcting the GIOVE data of this baseline using the ISB estimates of the previous day, increased the success rate over 90 % for this baseline as well. This demonstrates that we have been able to successfully calibrate the Javad–Trimble baseline, based on the differential ISBs determined one day before.

Curtin University Bentley campus: zero/short baselines

A next series of experiments has been carried out on the roof of building 402 at the Bentley campus of Curtin University, where several multi-GNSS receivers are connected to a Trimble TRM59800.00 antenna, thereby forming zero baselines consisting of different receiver types:

- CUT0: Trimble Net R9;
- CUT1: Septentrio PolaRx4;
- CUT2: Trimble Net R9;
- CUT3: Javad Quattro-G3D Delta.

On the same roof, at 4.3 m of the Trimble antenna, a Javad Grant G3T antenna is setup, connected to a Javad Quattro-G3D Delta receiver (CUBB), which is an identical receiver as the one at CUT3. Furthermore, on the roof of the campus building 207 another Trimble Net R9 receiver is setup (SPA1), connected to a Leica LEIAT504 antenna. This location is about 357.6 m from the building with the

CUT receivers. See Fig. 3 for a schematic overview of all receivers and antennas involved.

From approximately 22:00 UTC on May 21, 2011, until 04:00 UTC, the next day GIOVE-B (E52) was tracked simultaneously with GPS satellite SVN62 (G25) and all receivers (except the Javad receivers CUBB and CUT3, which were not yet operational at that time) were tracking E1 + E5a of this GIOVE satellite and L1 + L5 of the GPS satellite. Figures 4, 5, 6, 7 show the estimated differential phase and code ISBs for L1–E1 and L5–E5a for the baseline CUT0–CUT2, formed by identical Trimble receivers, as well as the baseline CUT0–CUT1, formed by the Trimble and Septentrio receivers. These results are based on a sampling interval of 30 s and an elevation cutoff of 10 deg. From the figures follows that for CUT0–CUT2, the differential ISBs for both L1–E1 and L5–E5a phase are estimated having a mean close to zero, while the counterparts for code have a mean of −6 cm for L1–E1 and a mean of −1 cm for L5–E5a. These values are much smaller than the measurement noise, and hence, this confirms our observation with the Javad receivers in the experiment of "Curtin University Bentley/Muresk experiment: short baselines", namely that differential ISBs for pairs of receivers of the *same* type are absent.

For the baseline consisting of different receiver types, CUT0–CUT1, the estimated differential ISBs are, however, significant: the fractional phase ISB is estimated with a mean of −0.22 cyc for L1–E1 and with a mean of −0.48 cyc for L5–E5a, while the code ISB for L1–E1 has a mean of 1.8 m and for L5–E5a a large mean of −434.1 m. In this context, it is remarked that large values for the L5–E5a differential ISBs were also observed by Montenbruck et al. (2011). In that article, the authors state that this may be related to an anomalous bias in the GIOVE E5a signal tracking that has been first reported for observations collected with the Septentrio GeNeRx1 receiver. This has led to different ad-hoc corrections to the E5 code measurements in receivers by various manufacturers. When these corrected observations are combined with uncorrected observations provided by the GeNeRx1 receiver, this may result in large inter-system biases between the different receivers. We suspect that this is also the case for the E5a code data tracked by the Septentrio PolaRx4 relative to the other manufacturer's receivers.

Although the estimated differential ISBs are significant, they seem to be very stable during the 6-h time span considering their standard deviations, which is only 0.001 cyc for phase and about 2 cm for code, for both L1–E1 and

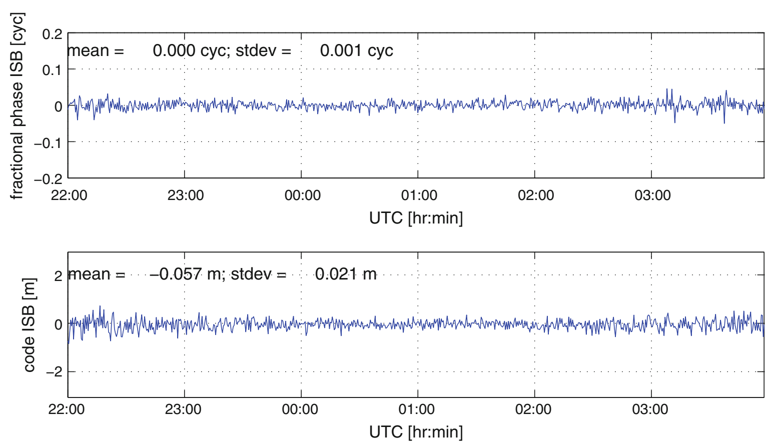

Fig. 4 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for CUT0–CUT2 (21–22/05/11), two Trimble receivers in a zero-baseline setup, based on GPS and GIOVE-B data

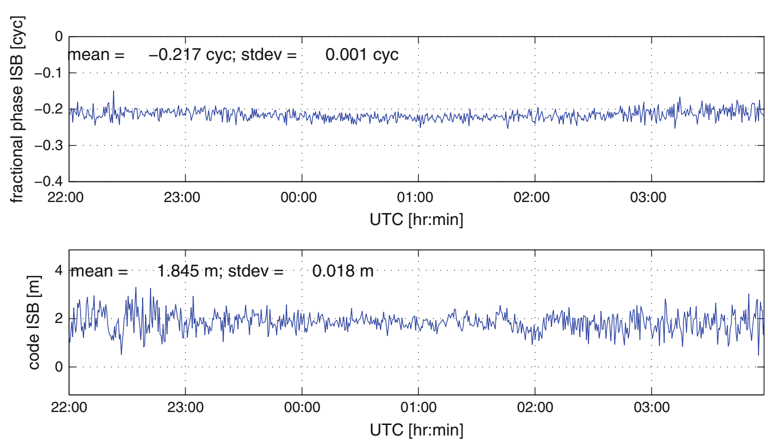

Fig. 5 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for CUT0–CUT1 (21–22/05/11), a Trimble and Septentrio receiver in a zero-baseline setup, based on GPS and GIOVE-B data

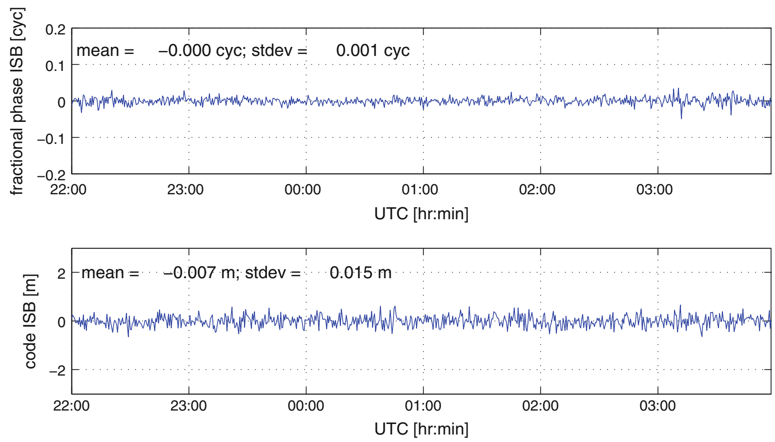

Fig. 6 Estimated differential L5–E5a ISBs for phase (*top*) and code (*bottom*) for CUT0–CUT2 (21–22/05/11), two Trimble receivers in a zero-baseline setup, based on GPS G25 and GIOVE-B data

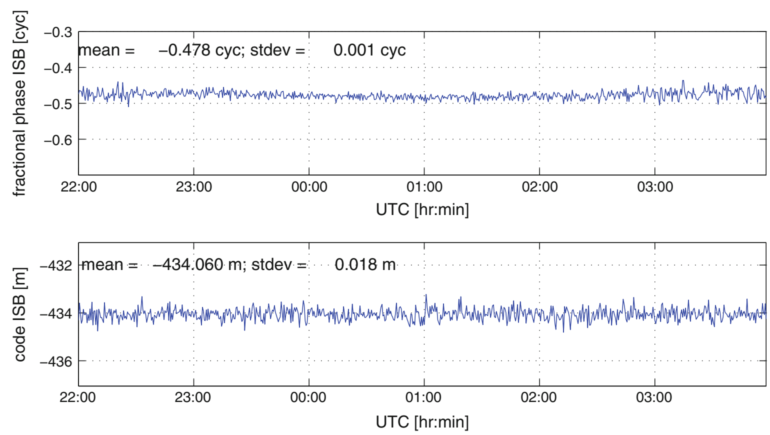

Fig. 7 Estimated differential L5–E5a ISBs for phase (*top*) and code (*bottom*) for CUT0–CUT1 (21–22/05/11), a Trimble and Septentrio receiver in a zero-baseline setup, based on GPS G25 and GIOVE-B data

Table 2 Ambiguity resolution success rates for February 17, 2011, applying the ISBs from May 21/22, 2011

357.6 m baselines	GPS-only	GPS + GIOVE-A&B ISBs uncorrected	GPS + GIOVE-A&B ISBs zero/corrected
SPA1–CUT1 (Trimble–Septentrio)	868/960 = 90.4 %	30/960 = 3.1 %	946/960 = 98.5 %
SPA1–CUT2 (Trimble–Trimble)	860/960 = 89.6 %	948/960 = 98.8 %	948/960 = 98.8 %

L5–E5a combinations. Assuming that the ISBs are stable in time, the differential ISBs as estimated from the 21/22 May datasets are in a next step applied *backwards* in time, so as to investigate whether it is possible to calibrate the L1–E1 GPS + GIOVE observations of February 17, 2011 (08:00–16:00 UTC), thus three months earlier. At that day, both GIOVE-A/B were simultaneously tracked by the Curtin receivers. Table 2 presents the empirical L1–E1 single-frequency geometry-based ambiguity success rates for the two baselines, for GPS-only, as well as for GPS combined with the two GIOVE satellites. The success rates for GPS-only are for both baselines close to 90 %, in agreement with the GPS-only results in Table 1. When adding the GIOVE data, first assuming no differential ISBs for both baselines, the ambiguity success rate is a low 3 % for the Trimble–Septentrio baseline, but close to 99 % for the baseline formed by the two Trimble receivers. Applying the differential ISBs estimated from the May dataset to the Trimble–Septentrio baseline, increased the ambiguity success rates to almost 99 %, comparable to the success rate of the Trimble–Trimble baseline. Note that these success rates are higher than the GPS + GIOVE results in Table 1, because of the *two* GIOVE satellites we included here, instead of just one in Table 1. These results confirm that the applied differential L1–E1 ISBs have successfully calibrated the GIOVE observations from months earlier and therefore seem to be very stable in time. This stability also applies to the differential ISBs of the L5–E5a combination: for the dataset of February 17, 2011, the means of the (fractional) phase and code ISBs are 0.000 cyc and 0.005 m, respectively, for the Trimble–Trimble baseline, while the means for the Trimble–Septentrio baseline are

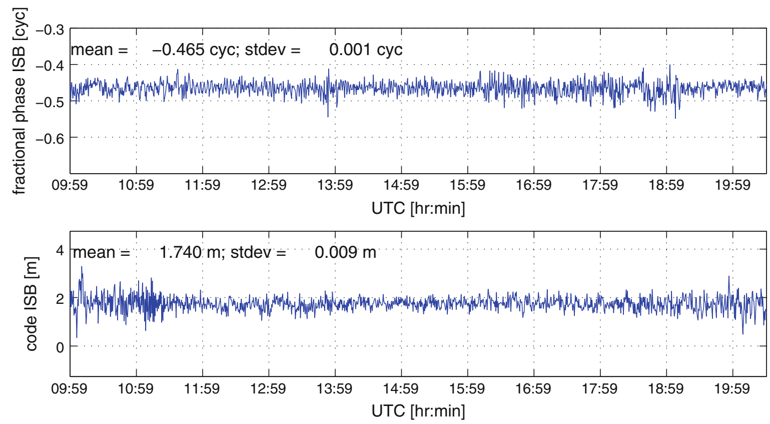

Fig. 8 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for CUT0–CUT3 (07/11/11), a Trimble and Javad receiver in a zero-baseline setup, based on GPS and GIOVE-A&B data

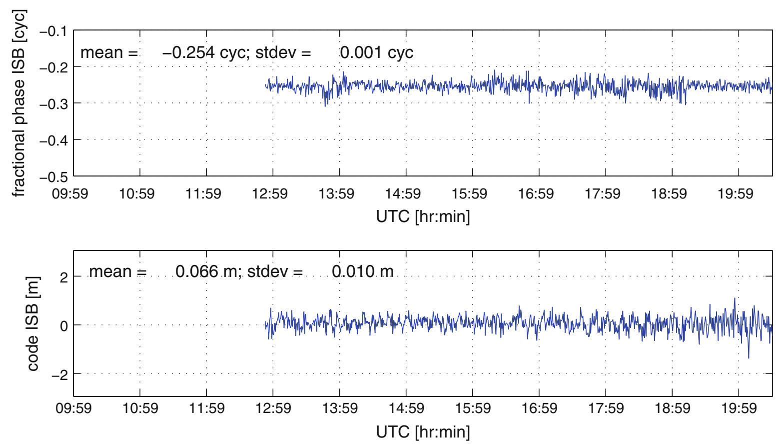

Fig. 9 Estimated differential L1–E1 ISB results for 07/11/11: CUT1–CUT3 (Septentrio–Javad). The time series start later than in Fig. 8, since CUT1 was only able to track GIOVE-A much later than CUT3 (only at the time GIOVE-B was observed as well)

−0.460 cyc and −434.005 m, respectively, for the phase and code ISBs. These values are well in agreement with the means of the May 21–22, 2011, dataset (see Figs. 6, 7).

A similar experiment as described in the preceding has been carried out, but now including both Javad receivers CUBB and CUT3 (see Fig. 3). These Javad receivers only track GIOVE data on the E1 frequency. The receiver at CUT3 is connected to the Trimble antenna, while the receiver at CUBB is connected to a Javad Grant G3 antenna, both located at the same building at about 4.3 m from each other, see Fig. 3. From 09:59:00 UTC until 20:29:30 UTC on November 7, 2011, the differential ISBs have been estimated from GPS and GIOVE-A/B L1–E1 data at the zero baselines formed by CUT0–CUT3 (Trimble–Javad) and CUT1–CUT3 (Septentrio–Javad). Figure 8 shows the estimated differential ISBs for both baselines. For CUT0–CUT3, the differential phase ISB is estimated at −0.47 cyc, and the differential code ISB at 1.74 m. Note that both these values are well in agreement with the values found in the experiment in "Curtin University Bentley/Muresk experiment: short baselines", in which the same types of receivers were involved. This confirms the stability of the differential ISBs. The differential ISBs for CUT1–CUT3 (Septentrio–Javad) are estimated at −0.25 cyc for the phase ISB and close to zero for the code ISB, see Fig. 9. Thus, for this combination of receivers, the differential ISBs seem to be absent for the code, but not for phase. In a next step, these determined ISB values have been applied as a priori correction for another GPS + GIOVE dataset, collected on December 26, 2011, almost 2 months later.

For this 26-December dataset, geometry-based ambiguity resolution is executed for the 4.3-m baselines between CUBB (Javad) and CUT0 (Trimble), CUT1 (Septentrio) and CUT3 (Javad). The GIOVE data of Javad–Javad baseline CUBB–CUT3 are *not* corrected for differential ISBs. The empirical ambiguity success rates are depicted in Table 3, for GPS-only as well as GPS + GIOVE-A (GIOVE-B was not tracked). The GPS-only success rates for the three baselines are close together, in the range of 85–90 %, which is in agreement with other GPS-only results in this article. The success rate for the baseline consisting of two Javad receivers is slightly lower than the mixed receiver baselines, which is due to a slightly poorer quality of the code data of the Javad compared to the other two receivers. Not correcting for the differential ISBs for the mixed receiver baselines, resulted in a low success rate of even 0 % for the Javad–Trimble baseline and 4 % for the Javad–Septentrio

Table 3 Ambiguity resolution success rates for December 26, 2011, applying the ISBs from November 7, 2011

4.3 m baselines	GPS-only	GPS + GIOVE-A ISBs uncorrected	GPS + GIOVE-A ISBs corrected
CUBB–CUT0 (Javad–Trimble)	744/831 = 89.5 %	0/831 = 0 %	787/831 = 94.7 %
CUBB–CUT1 (Javad–Septentrio)	718/831 = 86.4 %	36/831 = 4.3 %	764/831 = 91.9 %
CUBB–CUT3 (Javad–Javad)	707/831 = 85.1 %	756/831 = 91.0 %	756/831 = 91.0 %

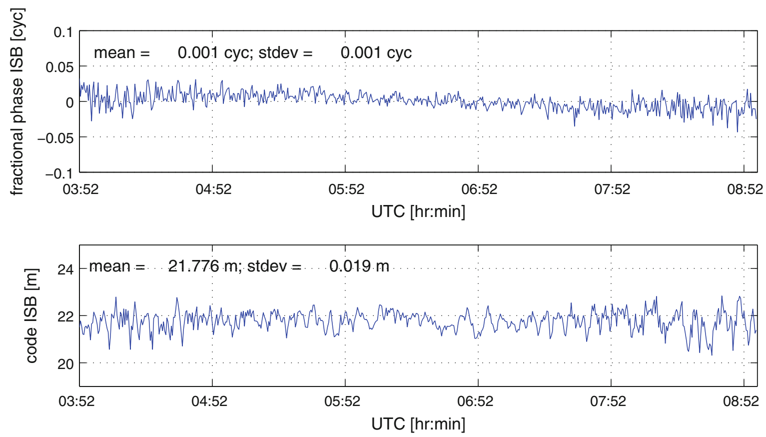

Fig. 10 Estimated differential L1–E1 ISBs for phase (*top*) and code (*bottom*) for USN4–USN5 (04/03/12), a Novatel and Septentrio receiver in a zero-baseline setup, based on GPS and GIOVE-A data

baseline. Addition of the (calibrated) GIOVE data increased the ambiguity success rate to values between 91 and 95 %.

United States Naval Observatory: zero baseline

The last experiment has been carried out using zero-baseline data collected at the United States Naval Observatory in Washington DC. The zero baseline is formed by two different receiver types, a Septentrio PolaRx4 (USN4) and a Novatel OEM6 (UNS5), and both receivers are connected to an AOAD/M_T (Dorne & Margolin) geodetic antenna. Both receivers track the same GPS and GIOVE observables: the GPS C/A signal on L1 and the GIOVE signal on E1 and moreover the GPS L5 and GIOVE E5a signals.

For a time span of approximately 5 h on March 4, 2012, GIOVE-A (E51) was simultaneously tracked with GPS SVN63 (G01) and differential ISBs were estimated for both L1–E1 and L5–E5a, see Figs. 10 and 11, based on a measurement sampling interval of 30 s and an elevation cutoff of 10 deg. From the graphs follows that for both L1–E1 and L5–E5a, the differential ISBs for the phase data are estimated with a mean close to zero, indicating that they are absent. The estimated differential ISBs for the code data cannot be ignored, having a mean of 21.8 m for L1–E1 and a mean of 188.3 m for L5–E5a. Again, they seem to be very stable, as can be concluded from the values of the standard deviations.

These differential ISB estimates have been applied in a next step to calibrate the GIOVE code data on E1 for two other datasets of the same zero baseline, a first dataset collected during May 8–9, 2012 (23:24–03:34 UTC), in which GIOVE-B was tracked, and a second dataset collected on May 9, 2012 (09:33–13:42 UTC), as well, but now includes GIOVE-A instead of GIOVE-B. Again, sampling interval was 30 s and elevation cutoff 10 deg. Similar to the experiments described in the previous subsections, for the two zero-baseline datasets single-frequency geometry-based ambiguity resolution was executed on an epoch-by-epoch basis for GPS-only and for GPS + GIOVE with the GIOVE data a priori corrected for the differential code ISB as estimated from the 4 March dataset, see Table 4 for the results. For GPS-only the success rates are 82 % for the first dataset and 72 % for the second. This lower success rate for the second dataset is due to the fewer GPS satellites tracked in that dataset. Because of the relatively large code ISB, adding uncalibrated GIOVE data resulted in 0 % ambiguity success rates for both datasets. Calibration of the GIOVE data for the differential code ISBs increased the success rate with 12–13 %. These results are in agreement with the results in the previous subsections.

Summary and conclusions

We investigated one aspect of GPS + Galileo interoperability. Both GNSSs will transmit signals at the overlapping L1–E1 and L5–E5a frequencies, which potentially benefits

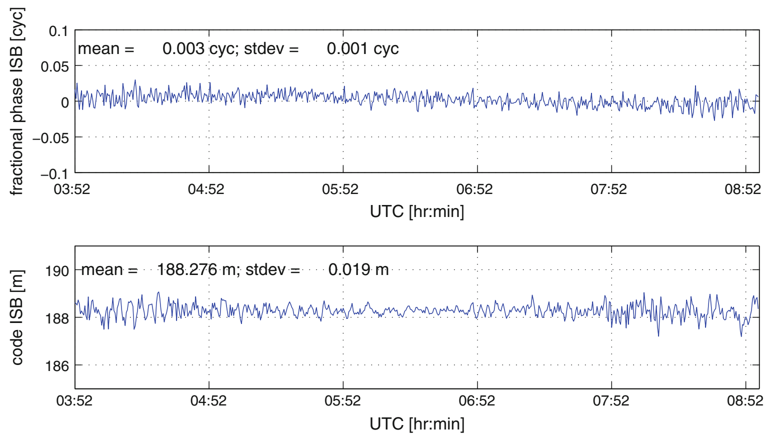

Fig. 11 Estimated differential L5–E5a ISBs for phase (*top*) and code (*bottom*) for USN4–USN5 (04/03/12), a Novatel and Septentrio receiver in a zero-baseline setup, based on GPS and GIOVE-A data

Table 4 Ambiguity resolution success rates for May 8/9, 2012, applying the ISBs from March 4, 2012

Zero baseline	GPS-only	GPS + GIOVE ISBs uncorrected	GPS + GIOVE ISBs corrected
USN4-USN5 (Novatel–Trimble); GIOVE-B	412/500 = 82.4 %	0/500 = 0 %	470/500 = 94.0 %
USN4-USN5 (Novatel–Trimble); GIOVE-A	360/500 = 72.0 %	0/500 = 0 %	429/500 = 85.8 %

Table 5 Summary of differential phase and code ISB corrections (after subtraction of an integer) for L1–E1 and L5–E5a for different multi-GNSS receiver types, estimated from GPS + GIOVE baselines

	Trimble Net R9 ($r = 2$)		Novatel OEM6 ($r = 2$)		Javad Quattro-G3D Delta ($r = 2$)
	L1–E1	L5–E5a	L1–E1	L5–E5a	L1–E1
Septentrio PolaRx4 ($r = 1$)	$\tilde{\delta}_{12,j}^{GE} = 0.22\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = -1.8\,\text{m}$	$\tilde{\delta}_{12,j}^{GE} = 0.48\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = 434.1\,\text{m}$	$\tilde{\delta}_{12,j}^{GE} = 0\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = 21.8\,\text{m}$	$\tilde{\delta}_{12,j}^{GE} = 0\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = 188.3\,\text{m}$	$\tilde{\delta}_{12,j}^{GE} = 0.25\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = 0\,\text{m}$
Javad Quattro-G3D Delta ($r = 1$)	$\tilde{\delta}_{12,j}^{GE} = 0.47\,\text{cyc}$ $\tilde{d}_{12,j}^{GE} = -1.8\,\text{m}$	–	–	–	Zero

Since the Javad receiver tracks GIOVE only on E1, no L5–E5a ISB corrections are given. For the Javad–Novatel combination, no data were available

high-precision positioning applications relying on integer carrier phase ambiguity resolution. Traditionally, in the mixed GPS + Galileo relative positioning model, each system defines its own reference or pivot satellite, such that the only common parameters between both systems are the baseline components. Instead of this, for observations at overlapping frequencies, we may perform "inter-system" double differencing, which means that the observations of one GNSS are double differenced with respect to the pivot satellite of the other GNSS. At first sight, this introduces additional parameters to be estimated due to a difference in the delay the GPS and Galileo signals experience: the so-called inter-system biases (ISBs). These ISBs are in principle different for phase as well as code and can only be estimated in differential mode, that is, between receivers.

Fortunately, by means of extensive processing based on GPS data as well as data of the two experimental Galileo (GIOVE) satellites, we have demonstrated that for baselines consisting of *identical* receiver types there is no reason to expect differential ISBs to be present and hence they do not have to be taken into account in a mixed model. On the other hand, for baselines formed by *different* receiver types, the differential ISBs are usually significant, but seem to be very stable in time. Table 5 summarizes for some receiver combinations the differential phase and code ISBs we estimated from GPS + GIOVE observations in the various zero- and short-baseline setups.

Because of this stability of the differential ISBs it should be possible to calibrate them and a priori correct the Galileo/GIOVE observations based on different receiver combinations. This potential calibration has been tested by applying the differential ISBs we estimated to other datasets that are significantly separated in time. Results were measured in terms of improvement of the success rate of instantaneous single-frequency (L1–E1) GPS + GIOVE ambiguity resolution, based on the short-baseline positioning model. The results demonstrated that while the instantaneous success rate based on GPS-only is in the range of 70–90 % (which mainly depends on the quality of the code data and the number of satellites), the success rate increases to 86–99 % based on GPS + GIOVE data, either assuming the differential ISBs to be absent (for identical receiver types) or present but calibrated (for different receiver types). Future research will focus on the determination of the differential ISBs based on data of the recently launched Galileo IOV satellites.

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