Weighting Ionospheric Corrections to Improve Fast GPS Positioning Over Medium Distances

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BIOGRAPHY

Dennis Odijk is a Ph.D. student at the Department of Mathematical Geodesy and Positioning of the Delft University of Technology, where he is engaged in the development of GPS data processing strategies for medium-scaled networks, with an emphasis on ambiguity fixing and modelling of the ionospheric delays.

ABSTRACT

The success of precise GPS positioning over long baselines depends on the ability of resolving the integer phase ambiguities when short observation time spans are required. The ionosphere is the major error source for these kind of positioning problems and due to the current maximum of solar activity the ionospheric delay errors in GPS signals can be much larger than in periods of minimum activity. When measurements are carried out in the vicinity of a permanent GPS network, then from the long-time data of these permanent stations precise ionospheric delays can be estimated. Next, these ionospheric estimates at the permanent stations can be interpolated to the location of a user operating within the area to correct the user’s measurements.

In this article the performance of these ionospheric corrections is tested in terms of improvement of ambiguity resolution at the user’s site. For permanent networks with a large inter-station spacing (100-200 km), the difference between the interpolated ionospheric corrections and the real ionospheric delay in the user’s data can be too large and may hamper a fast estimation of the integer ambiguities. A way to improve this performance is to weight the ionospheric corrections in the adjustment of the observations, instead of treating them in a deterministic way. These ionospheric weights can be implemented by an extension of the stochastic model of the GPS observations. Results of this technique show that for baselines for which the distance to the nearest permanent station can be up to 75 km, the needed observation time could be reduced to only 5 minutes when a-priori ionospheric weights were set up as function of the baseline length.

1. INTRODUCTION

To obtain precise positions with GPS one has to use phase observations measured in a relative receiver-satellite setup. Resolution of the phase ambiguities at their integer values is a prerequisite to keep the observation time as short as possible. Conceptually, one can frame all GPS adjustment models in which ambiguity resolution plays a role into the following general equations:

\[ E\{y\} = A_1 x_1 + A_2 x_2, x_i \in \mathbb{Z}^n; \quad D\{y\} = Q_y \]

where \(E\{\cdot\}\) denotes the mathematical expectation operator and \(D\{\cdot\}\) the dispersion operator. The observations are denoted as \(y\) and the variance-covariance (vc-) matrix \(Q_y\) models their stochastic nature. For the unknown parameters, a distinction is made between two types. The first type of parameters is \(x_1\), for which integer constraints are imposed (these are the double-difference phase ambiguities). The second type concerns all remaining parameters \(x_2\), which are known to be real-valued. Real-valued parameters of main interest are the (baseline) coordinates. \(A_1\) and \(A_2\) contain design matrix elements.

Usually, model (1) is solved in a three-step-procedure:

1. One disregards the integer nature of the ambiguities and solves all parameters at their real values through a standard least-squares adjustment. The solution from this step is called the ambiguity-float solution and consists of the least-squares estimates (denoted with a ‘hat’-symbol) plus their vc-matrix:

\[
\hat{x}_1 \sim \begin{bmatrix} Q_{\hat{x}_1} & Q_{\hat{x}_1\hat{x}_2} \\ Q_{\hat{x}_2\hat{x}_1} & Q_{\hat{x}_2} \end{bmatrix}.
\]
2. In a second step, the float ambiguity solution of the first step (\( \hat{x}_1, Q_{x_1} \)) forms the input for the ambiguity resolution. The integer ambiguity solution (denoted with a 'check'-symbol) is obtained:

\[
\hat{x}_1 = \Xi(\hat{x}_1)
\]

with \( \Xi \) an integer mapping function (\( \Xi: \mathbb{R}^n \rightarrow \mathbb{Z}^n \)).

3. In the last step, a second standard least-squares adjustment is performed to obtain the ambiguity-fixed solution (plus vc-matrix) for the parameters of interest:

\[
\begin{align*}
\hat{x}_2 &= \hat{x}_2 - Q_{x_2}^{-1}(\hat{x}_1 - \hat{x}_1) \\
Q_{x_2} &= Q_{x_2} - Q_{x_2}^{-1}Q_{x_1} Q_{x_1}^{-1}Q_{x_2} Q_{x_2}^{-1}Q_{x_1}^{-1}
\end{align*}
\]

The success of estimating precise baseline coordinates depends on the success of estimating the correct integer ambiguities in step 2. For GPS measurements obtained with receivers at short distance from each other (typical less than 10 km), the observation time needed to estimate these correct integer ambiguities could be very short. This is because it is allowed to neglect the relative errors in the measurements due to propagation through the ionosphere. However, in order to estimate the correct ambiguities for GPS observations collected over longer distances, we need to include unknowns for the relative ionospheric delays in the parameter vector \( x_2 \) of model (1). The consequence is that the mathematical model becomes weaker and the observation time needed to estimate the correct ambiguities longer.

A permanent GPS network, which is present in the area of the measurement location, can mitigate these significant ionospheric delays. From the long-time GPS data of such a network it is possible to estimate the (relative) ionospheric delays between thepermanent stations. Next, these ionospheric delay estimates can be used to predict the ionospheric delays at the measurement location within the area, by means of an interpolation. Such an ionosphere interpolation procedure could be used as a part of the virtual GPS reference station concept, see (Van der Marel, 1998), in case of large distances between the permanent stations.

To test the performance of this ionospheric interpolation, in (Odijk, 2000) computations were carried out with data of the Southern California Integrated GPS Network (SCIGN) in the USA. Five stations of the SCIGN-network were assigned as ‘permanent’ stations in a way that ionospheric estimates were obtained and interpolated for some other stations within the area, designated as ‘user’ stations, with distances up to 100 km from the nearest ‘permanent’ station. The results, in terms of improvement in ambiguity resolution, showed that for most of these baselines the time to resolve the integer ambiguities was drastically reduced when the corrections were applied, compared to the situation without corrections. However, after ambiguity resolution, there were still residual ionospheric delays present (difference between ‘real’ and ‘interpolated’ delays) which could bias the eventual baseline estimation if no unknowns for these residual ionospheric delays were introduced.

In (Odijk, 2000) the ionospheric corrections were treated as pure deterministic quantities; for the present article it has been investigated to what extend a stochastic approach could contribute to fast integer ambiguity resolution and coordinate estimation. This so-called ‘ionosphere-weighted’ model will be discussed in the next section. In section 3 for this model planning computations have been performed to give insight into the expected success of ambiguity resolution and into the precision of the fixed baseline coordinates. Note that these planning computations could be carried out purely based on the functional and stochastic model assumptions of the ionosphere-weighted model (so no actual measurements were needed). In section 4 test results of ambiguity resolution and precise positioning will be given, this time when real ionospheric corrections are weighted together with the GPS observations. Finally, in section 5, the conclusions will be drawn.

## 2. THE IONOSPHERE-WEIGHTED GPS MODEL

In this section the mathematical model is discussed in which a-priori information on the ionospheric delays in GPS data is added as being stochastic corrections. It will be made clear what the relation is of this ionosphere-weighted model with on the one hand the model in which the ionospheric delays are treated as deterministic corrections, and on the other hand the model which treats them as completely unknown parameters.

### 2.1 The ionosphere-float model

If the ionospheric delays in GPS data are so large that they do not cancel out when differencing, it is significant to parameterize the ionospheric delays in the adjustment model.

According to the general framework (1), the (linearized) model of double-differenced (DD) dual-frequency carrier phase and code (pseudo-range) observations reads for two\(^1\) GPS receivers continuously tracking \( m \) satellites (where \( m \geq 4 \)) during \( k \) observation epochs (\( k \geq 1 \)):

\[
E \left[ \begin{array}{l} \phi \\ p \end{array} \right] = \left[ \begin{array}{l} e_{1} \\ e_{2} \end{array} \right] \otimes G \left( \begin{array}{cc} A & 0 \\ 0 & F \end{array} \right) \otimes \left[ \begin{array}{c} -\mu \\ \mu \end{array} \right] \otimes J \left[ \begin{array}{c} b \\ a \end{array} \right] ;
\]

\[
D \left[ \begin{array}{l} \phi \\ p \end{array} \right] = \left[ \begin{array}{cc} C_{\phi} & 0 \\ 0 & C_{p} \end{array} \right] \otimes R
\]

\(^1\)The models described in this article all involve a single-baseline setup for the sake of simplicity, but they could be easily extended for more than two receivers (network setup).
This model will be referred to as the ionosphere-float model: the unknown DD ionospheric delays are estimated simultaneously with the baseline coordinates and DD ambiguities. Note that this ionosphere-float model is closely related to the in practice widely used ionosphere-free combination of L1 and L2 observations. This linear combination eliminates the unknown ionospheric delays, while model (2) explicitly solves for them.

In (2), \( \otimes \) denotes the matrix Kronecker-product, which is used to write the GPS observation equations in a very compact way. For a \( q \times q \) matrix \( U \) and an arbitrary matrix \( V \) it is defined as:

\[
U \otimes V = \begin{pmatrix}
    u_{11}V & \cdots & u_{1q}V \\
    \vdots & \ddots & \vdots \\
    u_{q1}V & \cdots & u_{qq}V
\end{pmatrix}
\]

For its properties we refer to (Rao, 1973).

The other symbols used in the functional model of (2) have the following meaning:

- \( \phi = (\phi_1, \phi_2)^T \): \( 2(m-1) \times 1 \) vector of ‘observed-minus-computed’ DD phase observations on L1 and L2;
- \( p = (p_1, p_2)^T \): \( 2(m-1) \times 1 \) vector of ‘observed-minus-computed’ DD code observations on L1 and L2;
- \( b \): \( 3 \times 1 \) vector of baseline components (constant during time span);
- \( a = (a_1, a_2)^T \): \( 2(m-1) \times 1 \) vector of DD ambiguities for L1 and L2 (constant during the time span);
- \( I \): \( (m-1) \times 1 \) vector of DD ionospheric delays on L1 (for each observation epoch);
- \( (e_1^T, e_2^T)^T \): \( 4 \times 1 \) vector with ones;
- \( G \): \( (m-1) \times 3 \) design matrix for baseline coordinates (which captures the relative receiver-satellite geometry);
- \( \Lambda = \text{diag}(\lambda_1, \lambda_2) \): \( 2 \times 2 \) diagonal matrix with wavelengths on L1 and L2;
- \( F = \mathbb{I}_k \otimes I_{m-1} \): \( (m-1)k \times (m-1) \) design matrix to model time-constancy of ambiguities;
- \( (-\mu^T, \mu^T)^T \): \( 4 \times 1 \) vector with coefficients for ionospheric delays on L1, with \( \mu = (1, \lambda_1^2 / \lambda_2^2)^T \);
- \( J = \mathbb{I}_k \otimes I_{m-1} \): \( (m-1)k \times (m-1)k \) unit matrix to model DD ionospheric delays at each observation epoch;
- \( 0 \): \( 2 \times 2 \) zero matrix.

The stochastic model of (2) contains the following parts:

- \( C_p \): \( 2 \times 2 \) vc-matrix for the undifferenced code observations on L1 and L2;
- \( R \): \( (m-1)k \times (m-1)k \) cofactor matrix to model, among others, the correlation due to double differencing.

### 2.2 The ionosphere-fixed model

If we do not have a-priori knowledge of the ionospheric delays, the ionosphere-float model should be used. On the other hand, if we know in advance that the ionospheric delays in the GPS data are sufficiently small, or, if ionospheric corrections are available, we may use a model in which no unknowns at all for the ionosphere are estimated. Instead of model (2) we can use the so-called ionosphere-fixed model:

\[
E \left[ \begin{bmatrix} \phi + \mu \otimes I \\ p - \mu \otimes I \end{bmatrix} \right] = \begin{bmatrix} e_2 \otimes G \\ e_2 \otimes F \end{bmatrix} a;
\]

\[
D \left[ \begin{bmatrix} \phi + \mu \otimes I \\ p - \mu \otimes I \end{bmatrix} \right] = C_p \otimes R.
\]

So ionospheric corrections may be added/subtracted to the phase and code observations. For sufficiently ‘short’ baselines, usually shorter than 10 km, it is often allowed to completely neglect the relative ionospheric delays: \( I = 0 \).

### 2.3 The ionosphere-weighted model

Model (3) can be considered as a model in which the observations are a-priori corrected for the ionospheric delays, in a deterministic way: the corrections are supposed to correct for the complete ionospheric delay. However, often these ionospheric corrections are not precise enough, so a suggestion would be to treat them stochastically instead of deterministically, i.e. to propagate their uncertainty in the stochastic model with a proper vc-matrix. If we assume the vc-matrix of the ionospheric corrections as \( D[I] = \sigma^2 R \), the adapted ionosphere-fixed model (3) becomes:

\[
E \left[ \begin{bmatrix} \phi + \mu \otimes I \\ p - \mu \otimes I \end{bmatrix} \right] = \begin{bmatrix} e_2 \otimes G \\ e_2 \otimes F \end{bmatrix} a;
\]

\[
D \left[ \begin{bmatrix} \phi + \mu \otimes I \\ p - \mu \otimes I \end{bmatrix} \right] = \left[ C_p + \sigma^2 \mu^T \right] \otimes R.
\]

This model will be referred to as the ionosphere-weighted model (see also Teunissen, 1997). The ionospheric corrections can be regarded as pseudo-observations. This idea of ‘weighting’ ionospheric information was already used in an early article by (Bock et al., 1986).
In the computations described in this article a simple stochastic model of the ionospheric pseudo-observations is used: all DD ionospheric observations are assumed to have the same a priori standard deviation $\sigma_\phi$. Possible other effects, which might be present, such as a dependency on the elevation angle or time-correlation have not been taken into account. It is important to stress that the ionosphere-weighted model is the most general model when treating the ionospheric errors. If one sets $\sigma_\phi = 0$ (or “infinite” weight), then it is assumed that the a-priori ionospheric information is completely known and the solution of the model becomes completely equivalent with that of the ionosphere-fixed model (3). On the other hand, specifying a $\sigma_\phi = \infty$ (or “zero” weight) means that the a-priori ionospheric information does not contribute at all to the solution of the model (the ionospheric errors are assumed to be completely unknown), and the solution equals that of the ionosphere-float model (2).

3. PLANNING COMPUTATIONS WITH THE IONOSPHERE-WEIGHTED GPS MODEL

The usefulness of the ionosphere-weighted model for precise GPS positioning using short observation time spans can in a first step be analyzed without taking real data into account, by means of planning computations which are completely based on the assumptions done in the functional and stochastic model. In the first part of this section, the expected success of ambiguity resolution, which is of crucial importance for fast positioning results, is studied. In the second part, the expected precision of the baseline coordinates will be illustrated.

3.1 Ambiguity success probabilities

The success of estimating the correct integer ambiguities depends on three factors:

• the (formal) precision of the float ambiguities, estimated in the first step of solving the model (see section 1), which is contained in the vc-matrix $Q_\hat{a}$;

• the used integer ambiguity estimator;

• the size of the bias in the measurements that is propagated into the float ambiguity solution $\hat{a}$.

The third factor depends on the actual measurement data, but knowledge of the first two is sufficient to carry out planning computations.

Assuming that the observations match with the formulated mathematical model and that they are normally (Gaussian) distributed, the float ambiguity solution is unbiased and also normally distributed:

$$\hat{a} \sim N(a, Q_\hat{a})$$

On the basis of this it is possible to infer a-priori whether or not the estimated integer ambiguities (denoted by $\tilde{a}$) have enough chance to coincide with the true but unknown integer ambiguities, by means of evaluating ambiguity success probabilities. These ambiguity success probabilities will be evaluated not for the integer least-squares estimator, though we have used this in the computations with real observations (described in section 4 of this article), but for another type of integer estimator: the so-called integer bootstrapped estimator.

The integer least-squares estimator is based on a search algorithm (as part of the LAMBDA-method) and gives the largest probability of estimating the correct ambiguities (Teunissen, 1999), but unfortunately, it is very difficult to actually compute the ambiguity success probabilities. However, this integer bootstrapped estimator, for which the algorithm is described in (Teunissen et al., 1998), is less optimal in the sense of estimating the correct integers, but has a major advantage that the success probabilities can be directly computed from the vc-matrix $Q_\hat{a}$. Besides this, and very importantly, it turned out that these ‘bootstrapped’ success probabilities serve as lower bounds of the success probabilities based on the integer least-squares estimator. And these lower bounds seem to be very sharp if first the decorrelating transformation of the LAMBDA-method is applied to $Q_\hat{a}$, giving more precise ambiguities than the original (DD) ambiguities (Teunissen, 1998a). Therefore, for the planning computations, it is allowed to rely on the bootstrapped probabilities.

In figure 1 for the ionosphere-weighted model the probability of successful integer (bootstrapped) estimation is plotted as function of the a-priori ionospheric standard deviation $\sigma_\phi$. The functional model assumptions in these computations are based on the geometry of two stations of the SCIGN-network (see section 4), which continuously tracked 6 satellites during a time span of 1 hour. The stochastic model assumptions are $C_\phi = \sigma_\phi^2 I_2$ for the undifferenced dual-frequency phase observables (with $\sigma_\phi = 3$ mm), and $C_p = \sigma_p^2 I_2$ for the undifferenced dual-frequency code observables (with $\sigma_p = 30$ cm).

To investigate the influence of the time span, from the 1-hour data set time spans of 1 epoch, 5 minutes and 10 minutes were selected for which success probabilities have been computed.

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3.2 Fixed baseline precision

Ambiguity resolution is not the goal of precise GPS positioning. After the estimated integer ambiguities may be assigned as the correct ones with a sufficient amount of probability, the final baseline solution is obtained from a new (standard) least-squares adjustment in which the integer ambiguities are held fixed (see section 1). In this section we will investigate the (formal) precision of the baseline coordinates as a function of the a-priori ionospheric standard deviation.

In order to do so, it can be proved that the vc-matrix of the baseline coordinates estimated with the ambiguity-fixed ionosphere-weighted model, denoted as $Q_b(I)$, can be decomposed into a (scalar) variance-factor and a 3x3 cofactor matrix (for the proof, see Teunissen, 1998b):

$$Q_b(I) = f^2(I) \cdot Q$$

The variance-factor $f^2(I)$ depends entirely on the a-priori vc-matrices of the observations, i.e. $C_\phi$, $C_p$ and $\sigma^2_I$, while the matrix $Q$ is entirely governed by the (changing) satellite-receiver geometry. Because of this nice decomposition, the influence of weighting the ionospheric corrections is only felt through the scalar variance-factor. In figure 2 the square root of this factor is plotted as function of the ionospheric standard deviation.

From figure 2 the deteriorating effect of increasing the ionospheric standard deviation is clear. Like the ambiguity success probabilities, also the baseline precision shows S-curve behaviour: if $\sigma_I$ becomes larger than a few cm’s, the fixed baseline precision coincides with the precision of the ionosphere-float model, if it is smaller than a few mm’s, the baseline precision approximates that of the ionosphere-fixed model. The following inequality holds true for the vc-matrices of the

![Fig. 2: Square root of variance-factor $f^2(I)$ as function of $\sigma_I$, where $C_\phi = \sigma_\phi^2 I_2$ with $\sigma_\phi = 3$ mm and $C_p = \sigma_p^2 I_2$ with $\sigma_p = 30$ cm.](image-url)
fixed baseline coordinates estimated with the three types of ionosphere models:

\[ Q_{bI} \leq Q_b(I) \leq Q_{bQ} \]

Note that in this context it can be proved that the factor with which the baseline vc-matrix of the ionosphere-fixed model \( Q_{bI} \) has to be multiplied to obtain the baseline vc-matrix of the ionosphere-float model \( Q_{bQ} \), is with a good approximation independent of the a-priori phase and code variances. Assuming uncorrelated phase and code data, whereas the precision of the phase is much better than the code data (such that \( \sigma^2_\phi / \sigma^2_v \approx 0 \)), we may namely write:

\[ Q_b \approx 17.74 \ Q_{bI} \]

For the proof, see (Teunissen, 1998b).

This implies that the standard deviations of the fixed baseline components obtained with the ionosphere-weighted model are at maximum a 4.21 factor worse than those obtained with the ionosphere-fixed model.

In table 1 examples of values are given for the expected minimum and maximum fixed baseline standard deviations when the ionosphere-weighted model is used. The standard deviations are given in North, East, Up and have been computed for different time spans. Note that for an a-priori ionospheric standard deviation of a few cm’s, the baseline precision will approximate the values obtained with the ionosphere-float model. Furthermore, note the beneficial role of increasing the length of the time span.

<table>
<thead>
<tr>
<th>Standard dev.</th>
<th>( \sigma_N )</th>
<th>( \sigma_E )</th>
<th>( \sigma_U )</th>
<th>( \ldots )</th>
<th>( \sigma_N )</th>
<th>( \sigma_E )</th>
<th>( \sigma_U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous</td>
<td>0.4</td>
<td>0.3</td>
<td>1.2</td>
<td>\ldots</td>
<td>1.5</td>
<td>1.3</td>
<td>5.0</td>
</tr>
<tr>
<td>5 minutes</td>
<td>0.1</td>
<td>0.08</td>
<td>0.4</td>
<td>\ldots</td>
<td>0.4</td>
<td>0.3</td>
<td>1.7</td>
</tr>
<tr>
<td>10 minutes</td>
<td>0.07</td>
<td>0.06</td>
<td>0.3</td>
<td>\ldots</td>
<td>0.3</td>
<td>0.2</td>
<td>1.2</td>
</tr>
<tr>
<td>1 hour</td>
<td>0.03</td>
<td>0.02</td>
<td>0.1</td>
<td>\ldots</td>
<td>0.1</td>
<td>0.09</td>
<td>0.4</td>
</tr>
</tbody>
</table>

### 4. TEST COMPUTATIONS WITH THE IONOSPHERE-WEIGHTED MODEL

In this section tests of the performance of the ionosphere-weighted model for fast and precise GPS positioning are described for GPS data of the SCIGN-network in California, USA. Five permanent stations of this network have been selected to act as reference stations for which the ionospheric delays are estimated. From these reference ionospheric estimates the ionospheric delays for another six stations of this network have been interpolated and in a next step applied as stochastic ionospheric corrections in the processing for these stations as if they were user stations.

In the first part of this section, the generation of the ionospheric corrections (i.e. sample-values plus their standard deviations) will be explained. In the next part, the performance of the ambiguity resolution is analyzed for the user baselines when these corrections are applied. After that, the results of the user coordinate estimation with the ambiguities fixed will be discussed.

The software used is GPSveQ, developed at the Delft University of Technology, which includes the LAMBDA-method for optimal integer ambiguity estimation (see De Jonge, 1998). GPSveQ was recently extended with ionospheric pseudo-observables.

#### 4.1 Generation of the ionospheric corrections

![Fig. 3: Configuration of the stations of the SCIGN network in the tests: LINJ-TRAK-WIDC-SIO3-SNI1 (the ‘permanent’ stations) and LEEP-CIT1-CRFP-BILL-CAT1-SCIP (the ‘user’ stations). The lengths of the ‘user’ baselines vary between 18 and 104 km. Note that the dotted lines are dependent baselines.](image)

In figure 3 the configuration of the used stations in the tests is given. The distances between the five ‘permanent’ stations are between 100 and 200 km. The maximum distance from a ‘permanent’ to a ‘user’ station is about 100 km. All stations are equipped with Ashtech Z-XII 3 receivers.

The data set used was measured with a sampling-interval of 30 sec. on January 1\textsuperscript{st} 1999 from 20:00 to 21:00h GPS time, so 12:00-13:00h local time in California. Each of the assigned ‘permanent’ stations continuously tracked the same 6 satellites during the time span. Furthermore, in the processing final IGS orbits were used to compute the satellite positions. Also for each ‘permanent’ station a tropospheric zenith delay parameter was estimated.

The ionospheric corrections for the ‘user’ sites were obtained in two steps. In a first step, dual-frequency phase and code data of the reference stations were processed according to a network version of the ionosphere-float model (2), with the reference station coordinates held fixed. Due to the time span of 1 hour, the ambiguities could be reliably fixed. As a
consequence, very precise relative (DD) ionospheric delays between the reference stations could be estimated. In a second step, the DD ionospheric delays between the reference stations were linearly interpolated at the approximated user station positions (which were obtained from a single point positioning). Information on the algorithmic aspects of this interpolation can be found in (Van der Marel, 1998).

In the former it has been described how the sample values of the ionospheric corrections were generated. To apply the ionosphere-weighted model at the user’s processing, assumptions had to be made on the standard deviations of these corrections as well. For this purpose not only the DD ionospheric delays between the ‘permanent’ stations LINJ, SNI1, SIO3, TRAK and WIDC were used, but-as to make the assessment more reliable- also DD ionospheric delay estimates for some other baselines of the SCIGN-network. These additional stations were not assigned either as ‘permanent’ or ‘user’ baselines in the test. The used stations for the assessment of the ionospheric standard deviation are depicted in figure 4 on the left.

From the 1-hour time-series, for each baseline empirical standard deviations of the DD ionospheric delays have been computed according to the following formula:

\[
s_I = \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{k} (I - \bar{I})^2 \]

where \( I \) denotes a DD ionospheric estimate and \( \bar{I} \) the average of these DD ionospheric estimates over all epochs and satellites (note that for each baseline \( k=120 \) and \( m=6 \) have been used). The factor \( \frac{1}{2} \) is added to convert the double-differenced standard deviations to undifferenced.

![Fig. 4: Baselines of the SCIGN-network (left) for which empirical ionospheric standard deviations have been computed. These standard deviations \( s_I \) are plotted against the baseline length \( l \) (in km) in the graph on the right. Through these data points, the line \( s_I[\text{cm}] = 0.04/[\text{km}] \) is fitted.](image)

In figure 4 on the right the assessed empirical ionospheric standard deviations have been plotted against the baseline length. From this graph one can see that a more or less linear function of the baseline length holds. Therefore, for the a-priori standard deviation of the interpolated ionospheric corrections to be used at the ‘user’ baselines, the following linear function will be assumed:

\[ \sigma_I[\text{cm}] = 0.04/[\text{km}] \]

In table 2 for the user baselines these a-priori ionospheric standard deviations are given.

<table>
<thead>
<tr>
<th>User baseline</th>
<th>Length ( l ) [km]</th>
<th>A-priori ionospheric standard deviation ( \sigma_I ) [cm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEEP-CIT1</td>
<td>18</td>
<td>1</td>
</tr>
<tr>
<td>LINJ-CIT1</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>TRAK-CAT1</td>
<td>58</td>
<td>2</td>
</tr>
<tr>
<td>WIDC-CRFP</td>
<td>66</td>
<td>2</td>
</tr>
<tr>
<td>WIDC-BILL</td>
<td>74</td>
<td>3</td>
</tr>
<tr>
<td>SNI1-SCIP</td>
<td>104</td>
<td>4</td>
</tr>
</tbody>
</table>

### 4.2 Ambiguity resolution for the ‘user’ baselines

The performance of the ambiguity resolution at the user sites has been tested with four strategies:

1. ionosphere-fixed model without corrections;
2. ionosphere-fixed model with corrections;
3. ionosphere-weighted model with corrections;
4. ionosphere-float model (no difference with/without corrections as the ionospheric information is not used).

Because of the selected long ‘user’ baselines with significant ionospheric delays, ambiguity resolution with the first strategy was not expected to be very successful. Also successful ambiguity resolution with the fourth strategy was not expected either, due to the short observation time spans (see figure 1). Both of these strategies have been carried out for the sake of comparison. More success was expected from the second strategy, in which the ionospheric corrections were pure deterministic quantities, and from the third strategy, in which these ionosphere corrections were weighted according to the ionospheric standard deviations as given in table 2.

In the tables 3, 4 and 5 the results of ambiguity resolution for the strategies 1-4 are given. For each ‘user’ baseline there was a 1-hour observation time available (the same hour as for the ‘permanent’ stations) and, as to imitate real short time spans, this 1-hour time span was divided into smaller batches. For each batch it was tried to estimate the correct ambiguities without taking information from the previous windows into account. Three batch lengths have been used: single-epoch (instantaneous; table 3), 10 epochs (5 minutes time spans; table 4) and 20 epochs (10 minutes time spans; table 5). Longer batch lengths have not been used, because of the interest in fast positioning results. The success of ambiguity resolution is measured with so-called success-rates (the term ‘rate’ has been used as to distinguish from the a-priori success probability). This ambiguity success-rate is defined as:

\[ \text{success} = \frac{\text{number of batches with correct ambiguities}}{\text{total number of batches in 1 hour}} \times 100\% \]
Table 3. Ambiguity success-rates using batches of one epoch (k=1)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEEP-CIT1</td>
<td>81%</td>
<td>97%</td>
<td>100%</td>
<td>2%</td>
</tr>
<tr>
<td>LINJ-CIT1</td>
<td>7%</td>
<td>31%</td>
<td>97%</td>
<td>0%</td>
</tr>
<tr>
<td>TRAK-CAT1</td>
<td>0%</td>
<td>51%</td>
<td>92%</td>
<td>0%</td>
</tr>
<tr>
<td>WIDC-CRFP</td>
<td>0%</td>
<td>44%</td>
<td>99%</td>
<td>0%</td>
</tr>
<tr>
<td>WIDC-BILL</td>
<td>0%</td>
<td>46%</td>
<td>93%</td>
<td>0%</td>
</tr>
<tr>
<td>SNII-SCIP</td>
<td>0%</td>
<td>12%</td>
<td>82%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Table 4. Ambiguity success-rates using batches of 5 minutes (k=10)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEEP-CIT1</td>
<td>92%</td>
<td>100%</td>
<td>100%</td>
<td>25%</td>
</tr>
<tr>
<td>LINJ-CIT1</td>
<td>17%</td>
<td>67%</td>
<td>100%</td>
<td>17%</td>
</tr>
<tr>
<td>TRAK-CAT1</td>
<td>0%</td>
<td>75%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>WIDC-CRFP</td>
<td>8%</td>
<td>67%</td>
<td>100%</td>
<td>33%</td>
</tr>
<tr>
<td>WIDC-BILL</td>
<td>0%</td>
<td>67%</td>
<td>100%</td>
<td>8%</td>
</tr>
<tr>
<td>SNII-SCIP</td>
<td>0%</td>
<td>17%</td>
<td>92%</td>
<td>17%</td>
</tr>
</tbody>
</table>

Table 5. Ambiguity success-rates using batches of 10 minutes (k=20)

<table>
<thead>
<tr>
<th>Strategy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEEP-CIT1</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>67%</td>
</tr>
<tr>
<td>LINJ-CIT1</td>
<td>50%</td>
<td>100%</td>
<td>100%</td>
<td>17%</td>
</tr>
<tr>
<td>TRAK-CAT1</td>
<td>0%</td>
<td>83%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>WIDC-CRFP</td>
<td>17%</td>
<td>100%</td>
<td>100%</td>
<td>83%</td>
</tr>
<tr>
<td>WIDC-BILL</td>
<td>0%</td>
<td>67%</td>
<td>100%</td>
<td>17%</td>
</tr>
<tr>
<td>SNII-SCIP</td>
<td>0%</td>
<td>50%</td>
<td>100%</td>
<td>33%</td>
</tr>
</tbody>
</table>

From the tables 3, 4 and 5 the following may be noticed:

- As was already expected, the use of strategy 1 leads in general to very low success-rates. The DD ionospheric delays in the observations are too large and the ionosphere-fixed model is not suited for successful ambiguity resolution for these baselines. There is one exception: using a time span of at least 10 minutes for the short baseline LEEP-CIT1 the correct integer ambiguities could be resolved.
- When the observations are a-priori corrected (strategy 2), for most baselines and most time spans the success-rates increased enormously compared to the results of strategy 1, but the success-rates are in general still not high enough. Note that the poorest results are obtained for SNII-SCIP, the long baseline which lies almost ‘outside’ the ‘permanent’ network.
- Application of the stochastic corrections (strategy 3), yielded the best ambiguity success-rates: for each baseline for more than 80% of the epochs, the correct ambiguities could be resolved instantaneously (against about 10% using strategy 2). Lengthening the time span to 5 minutes lead to 100% success-rates, except for SNII-SCIP (92%).
- Finally, the success-rates obtained with the ionosphere-float model (strategy 4) are very poor, as was already expected.

Comparing these empirical success-rates with the success probabilities (section 3.1) which are based on a-priori model assumptions, then for the ionosphere-fixed model large differences may be noticed. Looking at the success probabilities one may conclude that ambiguity resolution with the ionosphere-fixed model has a larger probability to be successful than with the ionosphere-weighted model. However, one has to take into account that this only holds true when the ionospheric corrections are precisely known. This means that any discrepancy between the corrections and the real ionospheric delays in the observations should be very small, especially for short time spans. For the tested baselines however, there seem to be significant discrepancies present between the corrections and the real delays in the observations, due to the interpolation within the wide spacing of the permanent stations. As a consequence, significant residual ionospheric delays are propagated into the float ambiguity solution and bias the integer solution.

As an example, in figure 5 for the 104 km baseline SNII-SCIP the residual DD ionospheric delays (after corrections have been applied) in the data are plotted. Although it can be seen that the interpolated ionospheric delays corrected for a large amount of the real ionospheric delays, the residuals are not zero (but can be as large as 10 cm). These large residuals influenced the ambiguity resolution with strategy 2: for only 12% of the 120 single-epoch batches the DD ionospheric delays were sufficiently small such that the correct integer ambiguities could be resolved instantaneously.

When using strategy 3 instead of 2, uncertainty in the ionospheric corrections is allowed for correct ambiguity resolution. The used a-priori ionospheric standard deviations for these corrections, which are a (linear) function of the baseline length, seem to describe this ionospheric uncertainty sufficiently well for time spans of at least 5 minutes (see table 4), except for the baseline SNII-SCIP. But for this baseline the interpolation might have performed worse than for the other ‘user’ baselines because of its location outside the ‘permanent’ network.

4.3 Fixed ‘user’ baseline estimation

In the previous section it was shown that for the tested ‘user’ baselines (except one) the ambiguities could be resolved very quickly when the ionospheric corrections were properly weighted. In this section these integer ambiguities are used to estimate the coordinates of the user sites. An analysis of these (ambiguity-) fixed baseline coordinates will be given for different strategies of treating the ionospheric delay, like was done for the
ambiguity resolution described in section 4.2. In the analysis only single-epoch (instantaneous) baseline solutions will be considered. However, the conclusions to be drawn from these single-epoch solutions, are also valid for the time spans of 5 and 10 minutes, although these results are not presented here.

In figure 6 for the ‘user’ baselines the instantaneous solutions are given as North, East and Up corrections with respect to the a-priori coordinates. For each baseline these components are estimated according to the three strategies 2, 3 and 4, as discussed in section 4.2 (strategy 1 is not included). Note that to obtain the coordinates, the integer ambiguities were resolved according to strategy 3.

From the graphs at the left in figure 6 it can be seen that all the coordinates estimated using strategy 2 show a non-constant behaviour in time, despite the fact that the correct ambiguities were used. So here we have the peculiar situation that residual ionospheric delays do not harm ambiguity resolution (except for the shaded areas in fig. 6) though do bias the final coordinate solution! Note that this effect is even visible for the relatively short baseline (18 km) LEEP-CIT1. In this context, note that the ionospheric interpolation (as a part of the data generation for a virtual station) described in (Wanninger, 1999) also did not remove the complete ionospheric delay in the observations. Therefore he suggested using the ionosphere-free linear combination for the final coordinate estimation. Instead of the ionosphere-free combination, in this study we have applied the ionosphere-float model (strategy 4), which just like the ionosphere-free combination produces coordinates that are free from possible ionospheric biases. The coordinate time-series are given at the right of figure 6 and visible is a more or less time-constant behaviour indicating that residual ionospheric effects seem to be absent indeed.

Besides the coordinate estimation with strategies 2 and 4, of particular interest is the application of strategy 3: the ionosphere-weighted model. The graphs in the middle of figure 6 reflect the instantaneous baseline solutions obtained when the ionospheric corrections are weighted in the same manner as done for the ambiguity resolution described in section 4.2. Comparison of these graphs with those of the ionosphere-float model (strategy 4) turns out that there are fortunately hardly any differences: also with the ionosphere-weighted model the baseline solutions do not seem to be biased by residual ionospheric effects.

From this similarity in coordinate estimates obtained with the ionosphere-weighted and –float models, follows that the empirical precision of the estimated coordinates from both models is equivalent as well. In fact, this is in agreement with what one would expect from the planning computations described in section 3.2. And indeed, looking at figure 2, for standard deviations larger than 1 cm (which is the case here), the formal precision of the coordinates estimated with the ionosphere-weighted model approximates that of the ionosphere-float model. Of course more testing need to be done, especially with more ionosphere-disturbed data, but from these real data processing results it seems that the ionosphere-weighted strategy should be considered as a successful strategy for fast positioning for medium-length baselines.

5. SUMMARY AND CONCLUSIONS

This article dealt with high-precision positioning relative to a permanent GPS network with a rather large inter-station spacing (100-200 km). The distance to the nearest permanent station can be up to 100 km, so a user should account for ionospheric effects in his GPS processing if short observation times are required. Significant ionospheric effects may bias the integer ambiguity resolution (a prerequisite for short time spans) and the estimation of the baseline coordinates, especially in the current time of maximum solar activity.

There are several ways to account for the ionospheric delays in the GPS data:

1. One can add unknowns for the ionospheric delays in the adjustment model. The model obtained is called the ionosphere-float model. However due to the additional unknowns, the time needed for successful ambiguity resolution will be relatively long.

2. Another suggestion is to a-priori correct the GPS data with corrections interpolated from relative ionospheric delay estimates between the permanent stations. The model to use is called the ionosphere-fixed model. However, due to the large distance between the permanent stations, there may be discrepancies between the interpolated corrections and the real ionospheric delays. In this article it was shown that due to these discrepancies ambiguity resolution often fails. Also if the ambiguity resolution does not fail, the baseline estimates can still be significantly biased with residual ionospheric effects, implying that for coordinate estimation the ionosphere-float model should be used, while for ambiguity resolution the ionosphere-fixed model was used. This last seems to be very ad hoc.

In this article it has been shown that application of a more rigorous technique turned out to be much more successful: a stochastic modelling, a weighting of the ionospheric corrections (the same corrections as in 2) in the adjustment. With this method, possible residual ionospheric effects can be accounted for by introducing a variance-covariance matrix of the ionospheric corrections in the stochastic model.

To set up this ionospheric variance-covariance matrix, the ionospheric delays estimated at the permanent stations were used. For the test described in this article, the ionospheric standard deviations could be properly modelled as a linear function of the length of the baseline.
In the graphs these epochs are shaded.
The results, in terms of ambiguity resolution, with these stochastic ionospheric corrections were very good:

- with the weighted corrections applied for more than 80% of the epochs the correct integers were instantaneously resolved (against 10% when corrections were treated as deterministic quantities);
- a lengthening of the time span to 5 minutes for most of the tested baselines the ambiguity success-rate increased to 100%.

Also, after the correct ambiguities had been resolved, the same ionospheric weights could be used to estimate the baseline coordinates free from residual ionospheric biases, making the difference between the choice of models for ambiguity resolution and baseline estimation disappear.

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REFERENCES


