First results of instantaneous GPS/Galileo/COMPASS attitude determination

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***Abstract*—The advent of modernized and new global navigation satellite systems (GNSS) has enhanced the availability of satellite based positioning, navigation, and timing (PNT) solutions. Specifically, it increases redundancy and yields operational back-up or independence in case of failure or unavailability of one system. Among existing GNSS, the Chinese COMPASS navigation satellite system (CNSS) and the European Galileo system are being developed. In this contribution, a COMPASS/Galileo/GPS robustness analysis is carried out for instantaneous, unaided attitude determination.**

Precise attitude determination using multiple GNSS antennas mounted on a platform relies on the successful resolution of the integer carrier phase ambiguities. The constrained Least-squares AMBiguity Decorrelation Adjustment (C-LAMBDA) method has been developed for the quadratically constrained GNSS compass model that incorporates the known baseline length. In this contribution the method is used to analyse the attitude determination performance when using the GPS, the Galileo, and COMPASS systems. The attitude determination performance is evaluated using GPS/Galileo/COMPASS data sets from a real data campaign in Australia. The study includes the performance analyses of both stand-alone and mixed constellation (GPS/Galileo/COMPASS) attitude estimation under satellite outage. We demonstrate and quantify the improved availability, reliability, and accuracy of attitude determination using the combined constellation.

***Index Terms*—GNSS, GPS, COMPASS, attitude determination, constrained integer least-squares, C-LAMBDA, carrier phase ambiguity resolution**

I. Introduction

The advent of modernized and new global navigation satellite systems (GNSS) has enhanced the availability of satellite based positioning, navigation, and timing (PNT) solutions. Specifically, it increases redundancy and yields operational back-up or independence in case of failure or unavailability of one system. Among existing GNSS, the Chinese COMPASS navigation satellite system (CNSS) and the European Galileo system are being developed. In this contribution, a robustness analysis of attitude determination using the standalone and the combined GPS, Galileo, and COMPASS systems is carried out.

Multiple GNSS receivers/antennas rigidly mounted on a platform can be used to determine platform attitude (orientation) [1]–[4]. Precise attitude determination, however, relies on successful resolution of the integer carrier phase ambiguities. The Least squares AMBiguity Decorrelation Adjustment (LAMBDA) method [5] is currently the standard method for solving unconstrained and linearly constrained GNSS ambiguity resolution problems [6]–[8]. For such models, the method is known to be numerically efficient and optimal in the sense that it provides integer ambiguity solutions with the highest possible success-rate [9], [10]. To exploit the known baseline length, we make use of the constrained (C-)LAMBDA method [11]–[15]. Due to the rigorous inclusion of the known baseline length, significantly higher success rates will be demonstrated.

Our analyses are carried out using data sets from real data campaign in Australia. Since satellite navigation data is not yet publicly available for the COMPASS system and not continuously available for Galileo test satellites, we use off-line navigation from post-processed orbit and clock information derived from an experimental regional network of monitoring stations in Australia, Asia, and Russia, and the Cooperative Network for GIOVE Observation (CONGO) [16], [17]. We evaluate the epoch-by-epoch, single- and multi-frequency integer ambiguity resolution performance of the C-LAMBDA method under satellite outage. For mixed constellation attitude determination, we consider inter-system double differencing by taking advantage of common frequencies namely, GPS L1 - Galileo E1, GPS L5 - Galileo E5a, and Galileo E5b - COMPASS B3 that results in the highest possible redundancy. Since we deployed two identical receivers in the real data campaign, the differential inter-system biases (DISBs) are known to be absent [18]. Hence, the GNSS observations at a common frequency is considered as if they are from a single system. Our analyses are the first reported results of GNSS attitude determination using real data from the triple system and demonstrate the increased availability of GNSS-based attitude solution by the inclusion of Galileo and COMPASS systems.

This contribution is organized as follows. Section II presents our attitude determination method using multi-constellation GNSS data. First, it describes the phase and code observation equations for short-baseline GPS/Galileo/COMPASS positioning. Then, it formulates the quadratically constrained GPS/Galileo/COMPASS model, followed by a description of the C-LAMBDA method for attitude determination. Section III presents the results of the performance evaluation for combined constellation ambiguity resolution and attitude determination under satellite outage environment, while Section IV draws conclusions of this contribution.

II. The GNSS-Based Attitude Determination

In this section we present our attitude determination method using the combined GPS/Galileo/COMPASS system. First we describe the functional and stochastic model for the combined observations and then we present the steps for solving the baseline constrained, mixed-integer attitude model.

A. GPS/Galileo/COMPASS Observations

Since these systems have a number of common frequencies, namely, GPS L1 - Galileo E1, GPS L5 - Galileo E5a, and Galileo E5b - COMPASS B3, we consider inter-system double differencing [18] whenever possible. For common frequency observations, after inter-system bias calibration [18], observations are considered as if they are from the first system. Let us consider two GPS/Galileo/COMPASS receivers r and 1 forming a short baseline and collecting observations from all three systems. The double difference (DD) pseudo-range and carrier-phase observations at frequency j for satellite pairs 1-s of GNSS system Φ (G for GPS, E for Galileo, and C for COMPASS), denoted as $p_{1r,j}^{1s,\Phi}$ and $\phi_{1r,j}^{1s,\Phi}$ respectively, are given as [19]

$$\begin{aligned} \mathrm{E}\left(p_{1r,j}^{1s,\Phi}\right) &= \rho_{1r}^{1s,\Phi}, \quad s=2,\ldots,(m_j^{\Phi}+1) \quad (1)\\ \mathrm{E}\left(\phi_{1r,j}^{1s,\Phi}\right) &= \rho_{1r}^{1s,\Phi}+\lambda_j^{\Phi}N_{1r,j}^{1s,\Phi}, \quad (2)\end{aligned}$$

where $\mathrm{E}(\cdot)$ denotes the expectation operator, $\rho_{1r}^{1s,\Phi}$ is the DD topocentric distance, λ_j^{Φ} is the wave length, $N_{1r,j}^{1s,\Phi}$ is the time-invariant *integer* DD carrier-phase ambiguity, and $(m_j^{\Phi}+1)$ is the number of satellites at frequency j.

The linearized DD observation equations corresponding to (1) and (2), read

$$\begin{aligned} \mathrm{E}\left(\Delta p_{1r,j}^{1s,\Phi}\right) &= g_j^{1s,\Phi^T}b, \quad s=2,\ldots,(m_j^{\Phi}+1) \quad (3)\\ \mathrm{E}\left(\Delta\phi_{1r,j}^{1s,\Phi}\right) &= g_j^{1s,\Phi^T}b+\lambda_j^{\Phi}N_{1r,j}^{1s,\Phi}, \quad (4)\end{aligned}$$

where $\Delta p_{1r,j}^{1s,\Phi}$ and $\Delta\phi_{1r,j}^{1s,\Phi}$ are the observed-minus-computed code and phase observations, b is the baseline vector containing relative position components, and $g_j^{1s,\Phi}$ is the geometry vector given as $g_j^{1s,\Phi}=e_{1,j}^{1,\Phi}-e_{1,j}^{s,\Phi}$ with $e_{r,j}^{s,\Phi}$ the unit line-of-sight vector between receiver-satellite pair $r-s$. The vector form of the DD observation equation for the jth frequency read

$$\begin{aligned} \mathrm{E}(y_{p;j}^{\Phi}) &= G_j^{\Phi}b \quad (5)\\ \mathrm{E}(y_{\phi;j}^{\Phi}) &= G_j^{\Phi}b+\lambda_j^{\Phi}z_{r,j}^{\Phi} \quad (6)\end{aligned}$$

with $y_{p;j}^{\Phi}=[\Delta p_{1r,j}^{12,\Phi}\ \ldots\ \Delta p_{1r,j}^{1(m_j^{\Phi}+1),\Phi}]^T$, $y_{\phi;j}^{\Phi}=[\Delta\phi_{1r,j}^{12,\Phi}\ \ldots\ \Delta\phi_{1r,j}^{1(m_j^{\Phi}+1),\Phi}]^T$, $G_j^{\Phi}=[g_j^{12,\Phi}\ \ldots\ g_j^{1(m_j^{\Phi}+1),\Phi}]^T$, $z_{r,j}^{\Phi}=[N_{1r,j}^{12,\Phi}\ \ldots\ N_{1r,j}^{1(m_j^{\Phi}+1),\Phi}]^T$.

For stochastic modeling, we assume elevation dependent noise characteristics [20]. That is, the standard deviation of the undifferenced observable ς can be written as

$$\sigma_{\varsigma}(\epsilon) = \sigma_{\varsigma_0}\left(1+a_{\varsigma_0}\exp\left(\frac{-\epsilon}{\epsilon_{\varsigma_0}}\right)\right) \quad (7)$$

where ϵ is the elevation angle of the corresponding satellite, and σ_{ς_0}, a_{ς_0}, and ϵ_{ς_0} are the elevation dependent model parameters. We further assume that the receivers have similar characteristics and that the observation noise standard deviations can be decomposed as follows:

$$\begin{aligned} \sigma_{\phi_{r,j}^{s,\Phi}} &= \sigma_r\sigma_{\phi_0}\sigma_{,j}\sigma^{,\Phi}\nu^{s,\Phi}\\ \sigma_{p_{r,j}^{s,\Phi}} &= \sigma_r\sigma_{p_0}\sigma_{,j}\sigma^{,\Phi}\nu^{s,\Phi} \quad (8)\\ \nu^{s,\Phi} &= \left(1+a_0\exp\left(\frac{-\epsilon^{s,\Phi}}{\epsilon_0}\right)\right)\end{aligned}$$

where σ_r, $\sigma^{,\Phi}$, and $\sigma_{,j}$ are the receiver, the system, and the frequency dependent weightings, respectively, and σ_{ϕ_0} and σ_{p_0} are observation dependent weightings.

B. The GPS/Galileo/COMPASS Attitude Model

When combining the single-epoch, multi-frequency linearized DD GNSS code and phase observation equations of (5) and (6), we obtain the mixed integer model of observation equations:

$$\mathrm{E}(y)=Az+Gb, \quad \begin{array}{l} z\in\mathbb{Z}^m \\ b\in\mathbb{R}^3 \end{array} \quad \text{with } m=\sum_{\Phi\in\{G,E,C\}}\sum_{j=1}^{f^{\Phi}}m_j^{\Phi} \quad (9)$$

where $y=[y_{\phi}^T\ y_p^T]^T$ is the $2m\times 1$ vector of linearized (observed-minus-computed) DD observations with $y_{\phi}=[y_{\phi}^{G^T}\ y_{\phi}^{E^T}\ y_{\phi}^{C^T}]^T$, $y_{\phi}^{\Phi}=[y_{\phi;1}^{\Phi^T}\ \ldots\ y_{\phi;f^{\Phi}}^{\Phi^T}]^T$, $y_p=[y_p^{G^T}\ y_p^{E^T}\ y_p^{C^T}]^T$, and $y_p^{\Phi}=[y_{p;1}^{\Phi^T}\ \ldots\ y_{p;f^{\Phi}}^{\Phi^T}]^T$, $z=[z^{G^T}\ z^{E^T}\ z^{C^T}]^T$ is the $m\times 1$ vector of unknown DD integer ambiguities with $z^{\Phi}=[z_1^{\Phi^T}\ \ldots\ z_{f^{\Phi}}^{\Phi^T}]^T$, b is 3×1 vector unknown baseline parameters, $G=e_2\otimes[G^{G^T}\ G^{E^T}\ G^{C^T}]^T$ is the $2m\times 3$ geometry matrix with $G^{\Phi}=[G_1^{\Phi^T}\ \ldots\ G_{f^{\Phi}}^{\Phi^T}]^T$ and e_n the $n\times 1$ vector of 1's, $A=[L^T\ 0^T]^T$ is the $2m\times m$ design matrix with $m\times m$ matrix $L=\mathrm{blockdiag}(\Lambda^G,\ \Lambda^E,\ \Lambda^C)$ and $\Lambda^{\Phi}=\mathrm{blockdiag}(\lambda_1^{\Phi}I_{m_1^{\Phi}},\ldots,\lambda_{f^{\Phi}}^{\Phi}I_{m_{f^{\Phi}}^{\Phi}})$ the diagonal wavelength matrix, with $\otimes$ denoting the Kronecker product [21], [22].

To construct the stochastic model for the observations in (9), consider the undifferenced observations reading

$$\zeta=[\zeta_1^T\ \zeta_2^T]^T \quad (10)$$

where $\zeta_r=[\phi_r^T\ p_r^T]^T$, $\phi_r=[\phi_r^{G^T}\ \phi_r^{E^T}\ \phi_r^{C^T}]^T$, $\phi_r^{\Phi}=[\phi_{r,1}^{\Phi^T}\ \ldots\ \phi_{r,f^{\Phi}}^{\Phi^T}]^T$, $\phi_{r,j}^{\Phi}=[\phi_{r,j}^{1,\Phi}\ \ldots\phi_{r,j}^{m_{f^{\Phi}}^{\Phi}+1,\Phi}]^T$, $p_r=[p_r^{G^T}\ p_r^{E^T}\ p_r^{C^T}]^T$, $p_r^{\Phi}=[p_{r,1}^{\Phi^T}\ \ldots\ p_{r,f^{\Phi}}^{\Phi^T}]^T$, $p_{r,j}^{\Phi}=[p_{r,j}^{1,\Phi}\ \ldots p_{r,j}^{m_{f^{\Phi}}^{\Phi}+1,\Phi}]^T$, and $p_{r,j}^{s,\Phi}$ and $\phi_{r,j}^{s,\Phi}$ are the undifferenced code and phase observations for $r-s$ receiver-satellite pair at jth frequency. Using the noise characteristics of (8) and assuming that the observables are normally distributed and mutually uncorrelated, the dispersion matrix of the observation vector ζ can be written as

$$\mathrm{D}(\zeta) = Q_r\otimes Q_t\otimes\mathrm{blockdiag}(Q_G,Q_E,Q_C) \quad (11)$$

where $\mathrm{D}(\cdot)$ denotes the dispersion operator, $Q_r=\mathrm{diag}[\sigma_1^2\ \ \sigma_2^2]$, $Q_t=\mathrm{diag}[\sigma_{\phi_0}^2\ \ \sigma_{p_0}^2]$, $Q_{\Phi}=\sigma_{,\Phi}{}^2\mathrm{blockdiag}(Q_{\Phi,1}\ \ \ldots\ \ Q_{\Phi,m_j^{\Phi}+1})$, and

$Q_{\Phi,j} = \sigma_{,j}{}^2 \text{diag}[\nu^{1,\Phi^2} \; \dots \; \nu^{m_j^\Phi+1,\Phi^2}]$ are the co-factor matrices. The dispersion matrix of the DD observations is then given as

$$\mathrm{D}(y) = \mathrm{D}(\mathcal{D}^T\zeta) = Q_{yy} \quad (12)$$

with the DD operator $\mathcal{D}^T = D_1^T \otimes I_2 \otimes \text{blockdiag}\left(D_{m^G}^T,\ D_{m^E}^T,\ D_{m^C}^T\right)$, in which $D_n^T = [-e_n,\ I_n]$ is the differencing matrix and $m^\Phi = \sum_{j=1}^{f^\Phi} m_j^\Phi$.

The DD observation equations of (9) form, together with the dispersion matrix of (12), a *mixed-integer* Gauss-Markov model with unknown integer vector $z \in \mathbb{Z}^m$ and unknown baseline vector $b \in \mathbb{R}^3$. This model can be strengthened with the known baseline length. With the inclusion of the baseline length constraint, we obtain the GNSS compass model [12], [13]

$$\begin{aligned} \mathrm{E}(y) &= Az + Gb \qquad \|b\| = l, z \in \mathbb{Z}^m, b \in \mathbb{R}^3 \quad (13)\\ \mathrm{D}(y) &= Q_{yy} \quad (14) \end{aligned}$$

where l is the known baseline length and $\|\cdot\|$ denotes the unweighted norm. Hence, the baseline is thus now constrained to lie on a sphere with radius l ($\mathbb{S}_l = \left\{b \in \mathbb{R}^3 |\ \|b\| = l\right\}$). Our objective is to solve for b in a least-squares sense, thereby taking the integer constraints on z and the quadratic constraint on vector b into account. Hence, the least-squares minimization problem that will be solved reads

$$\min_{z\in\mathbb{Z}^m, b\in\mathbb{S}_l} \|y - Az - Gb\|^2_{Q_{yy}} \quad (15)$$

with $\|\cdot\|_Q^2 = (\cdot)^T Q^{-1}(\cdot)$. It is a quadratically constrained (mixed) integer least-squares (QC-ILS) problem [11], for which no closed-form solution is available. In the following sections, we describe the method for solving (15).

C. The Ambiguity Resolved Attitude

We now describe the steps for computing the integer ambiguity resolved attitude angles.

1) The real-valued float solution: The float solution is defined as the solution of (15) without the constraints. When we ignore the integer constraints on z and the quadratic constraint on b, the float solutions $\hat{z}$ and $\hat{b}$, and their variance-covariance matrices are obtained as follows:

$$\begin{bmatrix} Q_{\hat{z}\hat{z}} & Q_{\hat{z}\hat{b}} \\ Q_{\hat{b}\hat{z}} & Q_{\hat{b}\hat{b}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \hat{z} \\ \hat{b} \end{bmatrix} = \begin{bmatrix} A^T \\ G^T \end{bmatrix} Q_{yy}^{-1} y \quad (16)$$

with

$$\begin{bmatrix} Q_{\hat{z}\hat{z}} & Q_{\hat{z}\hat{b}} \\ Q_{\hat{b}\hat{z}} & Q_{\hat{b}\hat{b}} \end{bmatrix} = \left(\begin{bmatrix} A^T \\ G^T \end{bmatrix} Q_{yy}^{-1} \begin{bmatrix} A & G \end{bmatrix} \right)^{-1} \quad (17)$$

The z-constrained solution of b and its variance-covariance matrix can be obtained from the float solution as follows

$$\begin{aligned} \hat{b}(z) &= \hat{b} - Q_{\hat{b}\hat{z}} Q_{\hat{z}\hat{z}}^{-1} (\hat{z} - z) \quad (18)\\ Q_{\hat{b}(z)\hat{b}(z)} &= Q_{\hat{b}\hat{b}} - Q_{\hat{b}\hat{z}} Q_{\hat{z}\hat{z}}^{-1} Q_{\hat{z}\hat{b}} \\ &= \left(G^T Q_{yy}^{-1} G\right)^{-1} \quad (19) \end{aligned}$$

Using the above estimators, the original problem in (15) can be decomposed as

$$\begin{aligned} &\min_{z\in\mathbb{Z}^m, b\in\mathbb{S}_l} \|y - Az - Gb\|^2_{Q_{yy}} \\ &= \|\hat{e}\|^2_{Q_{yy}} + \min_{z\in\mathbb{Z}^m} \left(\|\hat{z} - z\|^2_{Q_{\hat{z}\hat{z}}} + \min_{b\in\mathbb{S}_l} \left\|\hat{b}(z) - b\right\|^2_{Q_{\hat{b}(z)\hat{b}(z)}} \right) \quad (20) \end{aligned}$$

with $\hat{e} = y - A\hat{z} - G\hat{b}$ being the vector of least-squares residuals. Note that the first term on the right hand side of (20) does not depend on the unknown parameters z and b and is therefore constant.

2) The integer ambiguity resolution: Based on the orthogonal decomposition (20), the quadratic constrained integer minimization can be formulated as:

$$\check{z} = \arg\min_{z\in\mathbb{Z}^m} C(z) \quad (21)$$

with ambiguity objective function

$$C(z) = \|\hat{z} - z\|^2_{Q_{\hat{z}\hat{z}}} + \left\|\hat{b}(z) - \check{b}(z)\right\|^2_{Q_{\hat{b}(z)\hat{b}(z)}} \quad (22)$$

where

$$\check{b}(z) = \arg\min_{b\in\mathbb{S}_l} \left\|\hat{b}(z) - b\right\|^2_{Q_{\hat{b}(z)\hat{b}(z)}} \quad (23)$$

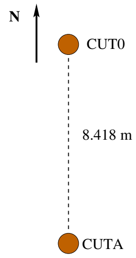
The cost function $C(z)$ is the sum of two coupled terms: the first weighs the distance from the float ambiguity vector $\hat{z}$ to the nearest integer vector z in the metric of $Q_{\hat{z}\hat{z}}$, while the second weighs the distance from the conditional float solution $\hat{b}(z)$ to the nearest point on the sphere $\mathbb{S}_l$ in the metric of $Q_{\hat{b}(z)\hat{b}(z)}$.

Unlike with the standard LAMBDA method [5], the search space of the above minimization problem is non-ellipsoidal due to the presence of the second term in the ambiguity objective function. Moreover, its solution requires the computation of a nonlinear constrained least-squares problem (23) for every integer vector in the search space. In the C-LAMBDA method, this problem is mitigated through the use of easy-to-evaluate bounding functions [13]. Using these bounding functions, two strategies, namely the *Expansion* and the *Search and Shrink* strategies, were developed, see e.g. [11], [23]. These techniques avoid the computation of (23) for every integer vector in the search space, and compute the integer minimizer $\check{z}$ in an efficient manner.

3) The ambiguity resolved attitude: Finally, we obtain the ambiguity resolved attitude solution by substituting $\check{z}$ into (18), thus giving $\hat{b}(\check{z})$. For a single baseline, b is related to the Euler-angles $\xi = [\phi\ \theta]^T$, with ϕ the heading and θ the elevation, as $b(\xi) = lu(\xi)$, where $u(\xi) = [c_\theta c_\phi,\ c_\theta s_\phi,\ -s_\theta]^T$ with $s_\alpha = \sin(\alpha)$ and $c_\alpha = \cos(\alpha)$. Hence, the sought-for attitude angles $\xi\left(\check{z}\right)$ are then obtained by solving the following nonlinear least squares problem:

$$\begin{aligned} \mathrm{E}\left(\hat{b}(\check{z})\right) &= lu(\xi) \\ \mathrm{D}\left(\hat{b}(\check{z})\right) &= Q_{\hat{b}(z)\hat{b}(z)} \end{aligned} \quad (24)$$

(a) Antenna setup



(b) Antenna geometry

Fig. 1. Curtin GNSS antennas used for the real data campaign

Using a first order approximation, the formal variance-covariance matrix of the ambiguity resolved, least-squares estimated heading and elevation angles is given by

$$Q_{\hat{\xi}\hat{\xi}} \approx \frac{1}{l^2}\left(J_{u,\xi}(\hat{\xi})^T Q^{-1}_{\hat{b}(z)\hat{b}(z)} J_{u,\xi}(\hat{\xi})\right)^{-1} \tag{25}$$

with Jacobian matrix

$$J_{u,\xi}(\xi) \quad = \quad \begin{bmatrix} -s_\phi c_\theta & -c_\phi s_\theta \\ c_\phi c_\theta & -s_\phi s_\theta \\ 0 & -c_\theta \end{bmatrix} \tag{26}$$

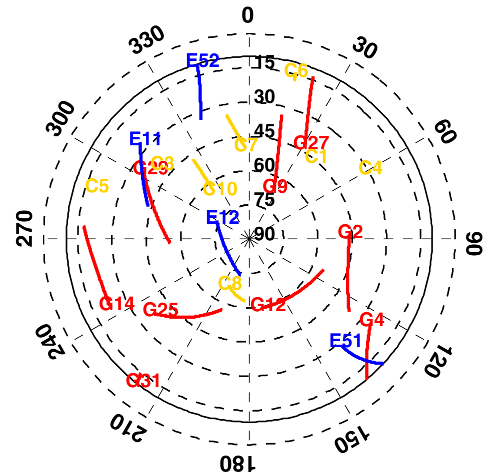
As the results in the next section show, this first order approximation works well. This is due to the fact that the ambiguity resolved solution is driven by the high precision of the carrier phase observables.

III. Performance of GPS/Galileo/COMPASS Attitude Determination

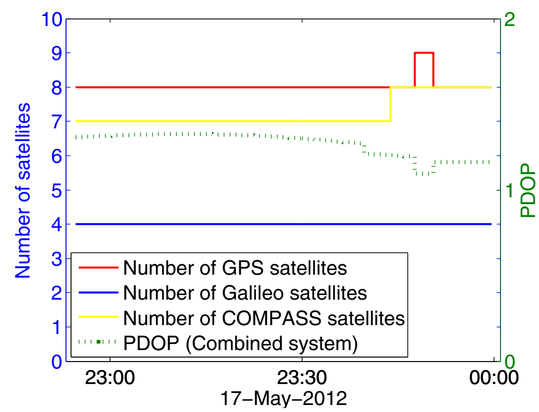
In this section the performance analyses of GPS/Galileo/COMPASS attitude determination are presented. The data was collected from two TRM59800.00-SCIS antennas mounted on the roof of the Bentley campus building 402 of Curtin University in Perth, Australia. As shown in Figure 1(a), they form a short baseline ($B_0 = 8.418$ m, Figure 1(b)). These antennas are connected to two TRIMBLE NETR9 GNSS receivers continuously tracking all available GNSS satellites.

Due to orbital parameters, co-visibility of Galileo experimental satellites does not exist for every day and hence, we chose a unique period (May 17, 2012 between 22:54:36 and 23:59:35) having co-visibility of all four Galileo experimental satellites and collected the data at a rate of 1 Hz. Figure 2 shows the GPS/Galileo/COMPASS satellite visibility (the skyplot, the number of satellites, and the PDOP value) during the period. The stochastic model parameters of the elevation dependent model (7) for the receivers are reported in Table I.

Our robustness analysis is based on single- (L1, E1, and/or B1), dual- (L1 and L2; E1 and E5a; and/or B1 and B2), and triple- (E1, E5a, and E5b; and/or B1, B2, and B3) frequency attitude determination under satellite outage by arbitrarily removing a number of visible GPS, Galileo, and/or COMPASS satellites. Note that, for triple-frequency processing, we only considered Galileo/COMPASS combination as the third



(a) Skyplot



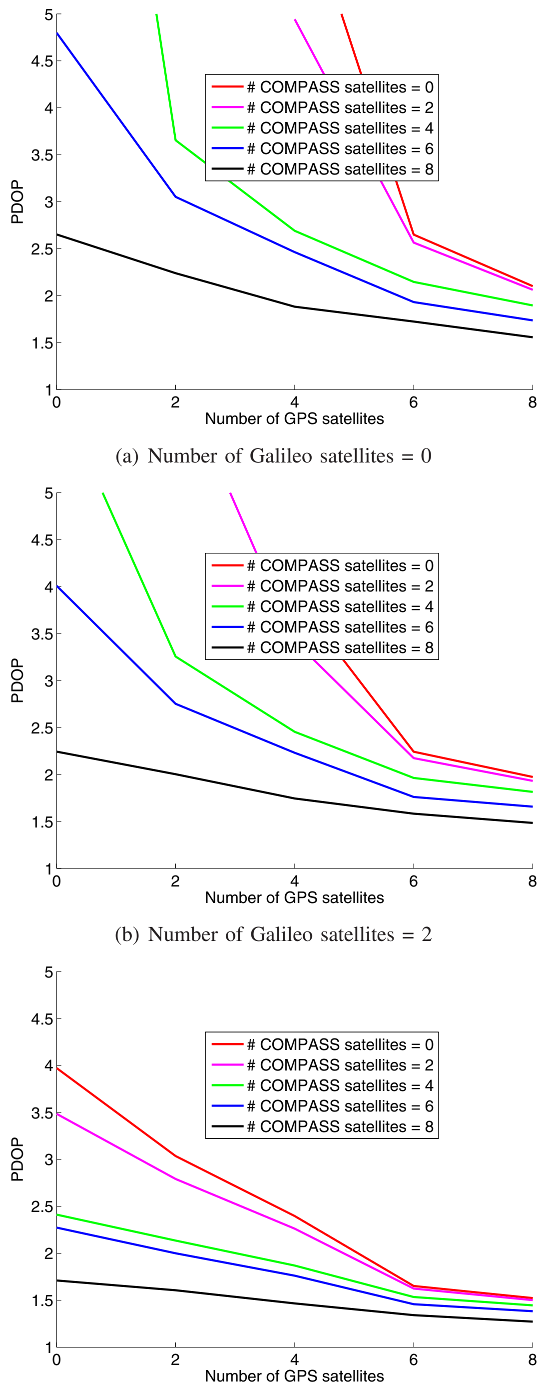
(b) Number of satellites and PDOP

Fig. 2. Satellite visibility of GPS (red), Galileo (blue), and COMPASS (yellow) constellations for $10°$ elevation cut-off

System	Frequency	Code			Phase		
		σ_{p_0} [cm]	a_{p_0}	ϵ_{p_0} [deg]	σ_{ϕ_0} [mm]	a_{ϕ_0}	ϵ_{ϕ_0} [deg]
GPS	L1	15	5	20	1	5	20
	L2	20	2	15	2	6	15
	L5	10	2	15	1	6	15
Galileo	E1	15	5	20	1	5	20
	E5a	10	2	15	1	6	15
	E5b	10	2	15	1	6	15
COMPASS	B1	20	5	15	1	5	15
	B2	20	5	15	2	5	15
	B3	20	5	15	3	5	15

TABLE I

Elevation dependent stochastic model parameters (cf., equation 7) for Curtin GNSS stations



(a) Number of Galileo satellites = 0

(b) Number of Galileo satellites = 2

(c) Number of Galileo satellites = 4

Fig. 3. PDOP values for simulated satellite outage

frequency (L5) of GPS system is only available from PRN 25 among the visible GPS satellites during the period considered. Figure 3 depicts the variation of PDOP values with number of satellites under simulated satellite outage.

First we evaluate the empirical instantaneous ambiguity success fraction (relative frequency), which is defined as

$$\text{success fraction} = \frac{\text{number of correctly fixed epochs}}{\text{total number of epochs}} \qquad (27)$$

where the true ambiguities are computed using known antenna coordinates in WGS84 as the antennas used in the real data campaign are part of Curtin's permanent stations. However, only length information is used for C-LAMBDA processing. Tables II, III, and IV report the LAMBDA and C-LAMBDA ambiguity success fractions for single-, dual-, and triple-frequency processing, respectively. The benefits of using C-LAMBDA are highlighted using bold text. Using multi-frequency processing, the C-LAMBDA method yields instantaneous attitude determination with as few as six satellites from GPS, Galileo and/or COMPASS constellations.

Next we evaluate ambiguity resolved angular accuracies (standard deviation). Due to baseline length and relatively poor precision of the second and third frequency observables (Table I), improvement of angular accuracies using multi-frequency precessing is not significant. Hence, only the angular standard deviations for single-frequency processing are given in Figures 4 and 5. The formal standard deviations (in dotted lines) are well in line with the empirical standard deviations (in continuous lines) confirming the assumed stochastic model parameters in Table I. A slight degradation of the angular accuracy with the number of satellites can be observed.

IV. Conclusions

In this contribution we studied the use of the combined GPS-Galileo-COMPASS constellation for C-LAMBDA attitude determination. In comparing the performances of LAMBDA and C-LAMBDA, we also studied the impact of using the known baseline length on ambiguity resolution. We demonstrated improved availability and angular accuracy due to use of triple system (GPS+Galileo+COMPASS). Using simulated satellite outages, we showed that instantaneous multi-frequency ambiguity resolution using the C-LAMBDA method is possible with as few as six satellites from GPS, Galileo, and/or COMPASS constellations.

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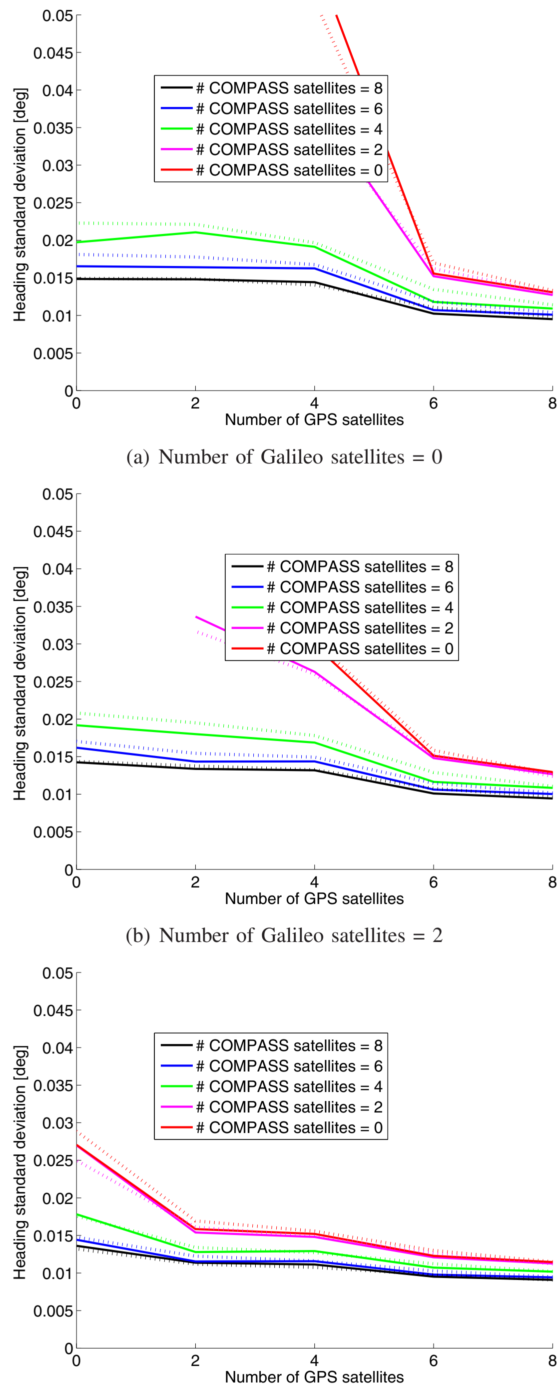
Number of satellites (PDOP)			Success fraction	
GPS	Galileo	COMPASS	LAMBDA	MC-LAMBDA
0	0	4 (12.02)	0.00	0.10
		6 (4.80)	0.21	0.80
		8 (2.65)	0.63	**1.00**
	2	4 (6.10)	0.01	0.33
		6 (4.01)	0.50	0.92
		8 (2.24)	0.90	**1.00**
	4	0 (3.97)	0.00	0.08
		2 (3.48)	0.01	0.42
		4 (2.41)	0.40	0.97
		6 (2.27)	0.94	**1.00**
		8 (1.71)	1.00	1.00
2	0	4 (3.66)	0.01	0.32
		6 (3.05)	0.47	0.95
		8 (2.24)	0.89	**1.00**
	2	2 (6.34)	0.00	0.28
		4 (3.26)	0.28	0.88
		6 (2.75)	0.91	0.99
		8 (2.00)	1.00	1.00
	4	0 (3.04)	0.11	0.88
		2 (2.79)	0.42	0.96
		4 (2.14)	0.92	**1.00**
		$\geq$ 6 (1.80)	1.00	1.00
4	0	0 (6.52)	0.00	0.08
		2 (4.94)	0.01	0.34
		4 (2.69)	0.39	0.90
		6 (2.46)	0.92	**1.00**
		8 (1.88)	1.00	1.00
	2	0 (3.90)	0.07	0.72
		2 (3.42)	0.30	0.93
		4 (2.45)	0.84	1.00
		6 (2.23)	0.99	**1.00**
		8 (1.74)	1.00	1.00
	4	0 (2.40)	0.81	**1.00**
		2 (2.26)	0.92	**1.00**
		$\geq$ 4 (1.70)	1.00	1.00
6	0	0 (2.65)	0.09	0.78
		2 (2.56)	0.35	0.91
		4 (2.15)	0.90	**1.00**
		$\geq$ 6 (1.82)	1.00	1.00
	2	0 (2.24)	0.65	0.99
		2 (2.17)	0.86	**1.00**
		4 (1.96)	0.99	**1.00**
		$\geq$ 6 (1.67)	1.00	1.00
	4	0 (1.65)	0.98	**1.00**
		2 (1.62)	0.99	**1.00**
		$\geq$ 4 (1.44)	1.00	1.00
8	0	0 (2.10)	0.69	0.99
		2 (2.06)	0.89	**1.00**
		$\geq$ 4 (1.73)	1.00	1.00
	2	0 (1.97)	0.97	**1.00**
		2 (1.93)	0.99	**1.00**
		$\geq$ 4 (1.65)	1.00	1.00
	4	$\geq$ 0 (1.42)	1.00	1.00

TABLE II
INSTANTANEOUS SINGLE-FREQUENCY AMBIGUITY SUCCESS FRACTIONS (RELATIVE FREQUENCIES) FOR SIMULATED SATELLITE OUTAGE (HERE, '$\geq s$' REFERS TO s OR MORE SATELLITES)

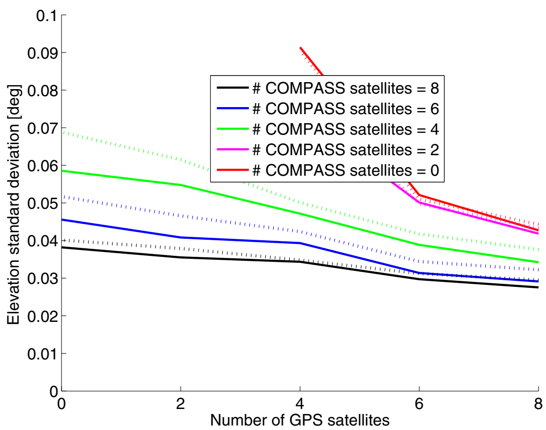


(a) Number of Galileo satellites = 0

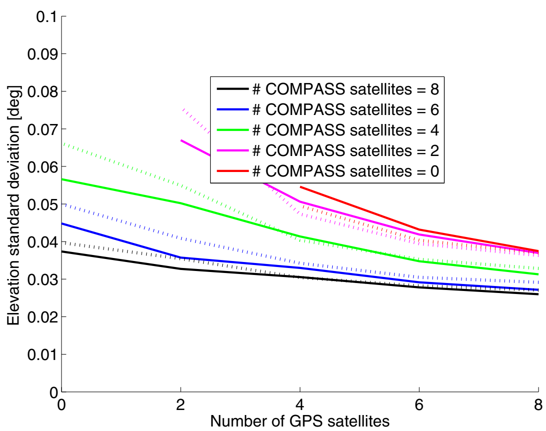
(b) Number of Galileo satellites = 2

(c) Number of Galileo satellites = 4

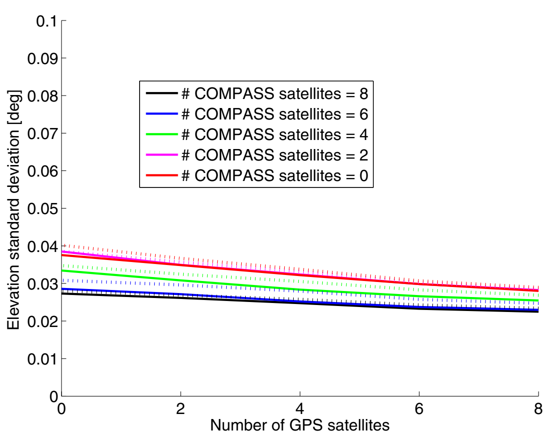
Fig. 4. Heading standard deviations (based on correctly fixed epochs) for simulated satellite outage: —— empirical and $\cdots\cdot$ formal



(a) Number of Galileo satellites = 0



(b) Number of Galileo satellites = 2



(c) Number of Galileo satellites = 4

Fig. 5. Elevation standard deviations (based on correctly fixed epochs) for simulated satellite outage: —— empirical and $\cdots\cdot$ formal

Number of satellites (PDOP)			Success fraction	
GPS	Galileo	COMPASS	LAMBDA	MC-LAMBDA
0	0	4 (12.02)	0.57	0.99
		$\geq$ 6(4.80)	1.00	1.00
	2	4 (6.10)	0.93	0.99
		$\geq$ 6 (4.01)	1.00	1.00
	4	0 (3.97)	0.20	0.96
		2 (3.48)	0.88	**1.00**
		$\geq$ 4 (2.41)	1.00	1.00
2	0	4 (3.66)	0.95	**1.00**
		$\geq$ 6 (3.05)	1.00	1.00
	2	2 (6.34)	0.63	0.93
		$\geq$ 4 (3.26)	1.00	1.00
	4	0 (3.04)	0.98	**1.00**
		$\geq$ 2 (2.79)	1.00	1.00
4	0	0 (6.52)	0.56	0.99
		2 (4.94)	0.94	**1.00**
		$\geq$ 4 (2.69)	1.00	1.00
	2	0 (3.90)	0.99	**1.00**
		$\geq$ 2 (3.42)	1.00	1.00
	4	$\geq$ 0 (2.40)	1.00	1.00
$\geq$ 6	$\geq$ 0	$\geq$ 0 (2.65)	1.00	1.00

TABLE III

INSTANTANEOUS DUAL-FREQUENCY AMBIGUITY SUCCESS FRACTIONS (RELATIVE FREQUENCIES) FOR SIMULATED SATELLITE OUTAGE (HERE, '$\geq s$' REFERS TO s OR MORE SATELLITES)

Number of satellites (PDOP)			Success fraction	
GPS	Galileo	COMPASS	LAMBDA	MC-LAMBDA
0	0	4 (12.02)	0.82	0.99
		$\geq$ 6 (4.80)	1.00	1.00
	2	4 (6.10)	0.98	**1.00**
		$\geq$ 6 (4.01)	1.00	1.00
	4	0 (3.97)	0.33	0.95
		2 (3.48)	0.97	**1.00**
		$\geq$ 4 (2.41)	1.00	1.00

TABLE IV

INSTANTANEOUS TRIPLE-FREQUENCY AMBIGUITY SUCCESS FRACTIONS (RELATIVE FREQUENCIES) FOR SIMULATED SATELLITE OUTAGE (HERE, '$\geq s$' REFERS TO s OR MORE SATELLITES)

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