International Global Navigation Satellite Systems Society
IGNSS Symposium 2009

Holiday Inn, Surfers Paradise, Qld, Australia
1 - 3 December, 2009

CORS local-site finger-printing using undifferenced least squares GNSS phase residuals

Lennard Huisman (1)
Department of Spatial Sciences, Curtin University of Technology, Australia
Tel: +61 8 9266 2218, Fax: +61 9266 2703, Email: l.huisman@curtin.edu.au
Hans van der Marel (2)
Delft Institute of Earth Observation & Space Systems, Delft University of Technology, The Netherlands
Tel: +31 15 278 4907, Fax: +31 15 278 3711, Email: h.vandermarel@tudelft.nl
Peter Teunissen (3)
Department of Spatial Sciences, Curtin University of Technology, Australia
Tel: +61 8 9266 7676, Fax: +61 9266 2703, Email: p.teunissen@curtin.edu.au
Delft Institute of Earth Observation & Space Systems, Delft University of Technology, The Netherlands
Tel: +31 15 278 3546, Fax: +31 15 278 3711, Email: p.j.g.teunissen@tudelft.nl

ABSTRACT

CORS local-site dependent effects, such as multipath, can be separated from atmospheric delays by stacking undifferenced least squares residuals of a network processing over a period of two to three weeks. In the case of double difference processing an extra step is required in order to compute the undifferenced residuals, which are mathematically equivalent to the residuals that would have been obtained from undifferenced processing. Analysing the undifferenced phase residuals at a single site can reveal an azimuth and elevation dependency of the residuals. Expressed in the form of an azimuth and elevation depended map, a so-called multipath map, the undifferenced residuals form a “finger-print” for a local-site. In this contribution local-site finger-prints are estimated from the undifferenced phase residuals for the newly installed GNSS sites in Western-Australia of the AuScope/Landgate CORS network. Different time periods are used to give insight into the repeatability and stability of the estimated finger-prints. Introducing the local-site finger-prints into CORS network processing does mitigate to some extend the local-site multipath and antenna effects, which results in more consistent empirical standard deviations for the undifferenced residuals.

KEYWORDS: CORS station finger-printing, multipath mapping, GNSS undifferenced least-squares phase residuals

1. INTRODUCTION

Despite modelling and cancelling out effects that result in systematic errors of the GNSS code and phase observations, the undifferenced phase residuals of a network processing contain remaining systematic errors. Under the assumption that satellite orbits, satellite clock-errors, mean antenna phase centre variations, mean atmospheric delays and carrier phase ambiguities were correctly modelled in the data processing, the undifferenced residuals are dominated by receiver carrier phase multipath, un-modelled antenna phase centre variations and un-modelled atmospheric delays (Alber et al., 2000, Van der Marel and Gündlich, 2006). The GPS system has the property that the satellite to receiver geometry repeats itself in one sidereal day (23h56m). Therefore when the local environment of a receiver does not change, receiver multipath shows the same pattern in azimuth and elevation for consecutive days (Leick, 2004). Assuming the un-modelled atmospheric delays vary from day to day and are zero on average they will cancel out when undifferenced residuals of multiple days are stacked. Stacking the undifferenced phase residuals of multiple days will therefore give the systematic errors in the residuals that mainly consist of receiver multipath and un-modelled antenna phase centre variations in the phase observations. Rocken et al. (2004) used this approach to analyse and explain systematic single epoch positioning errors at CORS network sites. Another application of knowledge of the receiver multipath is used in environmental GNSS research to improve the estimation of slant wet delays, for example by Braun et al. (2001), De Haan et al. (2003), Van der Marel and Gündlich (2006) and Marcias et al. (2007). The receiver multipath and un-modelled antenna phase centre variations can be visualized by a so-called multipath map (Braun et al., 1999) where the values of systematic phase residuals are plotted in a spherical map against azimuth an elevation. Undifferenced residuals, and in particular multipath maps, are used in this paper to show the feasibility of using this data to analyse systematic errors at CORS network sites in Western Australia. Although the multipath maps have been developed and used in the last ten years there is no motivation of the number of days needed to generate a reliable multipath map that only contains systematic errors in the phase residuals. This paper describes how multipath maps are generated and how multipath maps can be used to obtain local site characteristics.

2. RESEARCH METHODOLOGY

2.1 Network Description

Currently a CORS network with approximately 100 sites equipped with GPS/GLONASS receivers is set up in Australia as part of the AuScope programme (AuScope, 2009). This paper focuses on newly established sites in Western Australia. To analyse new CORS stations a network has been selected that consists of new stations and stations that were established several years ago. The stations, visualized in Figure 1, are part of the AuScope network, the Australian Regional GPS Network and/or the IGS network. Stations indicated in the figure with a triangle are IGS reference frame sites and have coordinates known in the IGS05 reference frame determined by IGS. The IGS05 reference frame has been used for this research because the precise satellite orbits are also given in the IGS05 frame. In this way both the reference stations used for fixing the network coordinates and the satellite orbits are given in the same reference frame. From Figure 1 and Table1 it can be seen that the stations in the network are not spread evenly geographically and that the network contains long baselines, but also does contain several IGS reference frame sites. A smaller network, consisting of stations located in South-Western Australia, has also been processed. The stations in the smaller network are the stations inside the black rectangle in Figure 1. This smaller network has one station that has known IGS05 coordinates and to which the network

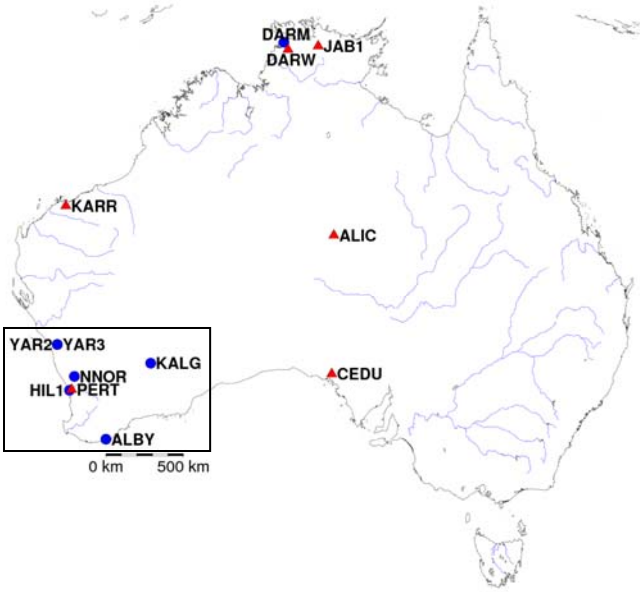


Figure 1: Selected CORS network to obtain undifferenced residuals, stations inside the black rectangle form a smaller sub network. Red triangles indicate IGS reference frame sites.

Table 1: Distances between the stations in the CORS network in kilometres.

[km]	ALBY	ALIC	CEDU	DARM	DARW	HIL1	JAB1	KALG	KARR	NNOR	PERT	YAR2
ALIC	1988											
CEDU	1522	908										
DARM	2800	1282	2163									
DARW	2770	1231	2115	53								
HIL1	396	1992	1703	2631	2608							
JAB1	2875	1221	2119	219	192	2733						
KALG	574	1457	1179	2241	2210	556	2313					
KARR	1546	1749	2044	1742	1738	1207	1910	1169				
NNOR	458	1921	1670	2539	2516	96	2643	504	1118			
PERT	392	1979	1690	2620	2597	14	2722	542	1203	88		
YAR2	694	1934	1792	2430	2412	310	2550	621	910	236	310	
YAR3	694	1934	1792	2430	2411	310	2550	621	910	236	310	0.02

can be constrained. Figure 2 shows the coordinate differences of the baseline NNOR-PERT as a result of the processing of the large network and the small network. The small network is more sensitive to missing data at the stations, which results in jumps in the coordinate time series, due to missing data. The coordinate differences of the large network are more stable over time. Therefore the choice was made to use the results of the large network, which includes all stations of the small network as well. Both networks were processed with the Bernese GPS Software version 5.0 (Dach *et al.*, 2007).

Since the network processing in the Bernese GPS software is based on double differenced batch processing, the software delivers double differenced residuals. The double differenced residuals are residuals of the ionosphere free linear combination of the L1 and L2 GPS observables (Teunissen and Kleusberg, 1998). The method to obtain undifferenced residuals from the double differenced residuals introduced by Alber *et al.* (2000) was implemented in the Bernese GPS software. This method to obtain the undifferenced residuals is based on two assumptions. The first assumption is that in the transformation from undifferenced observations to single difference observations between receivers the satellite clock errors

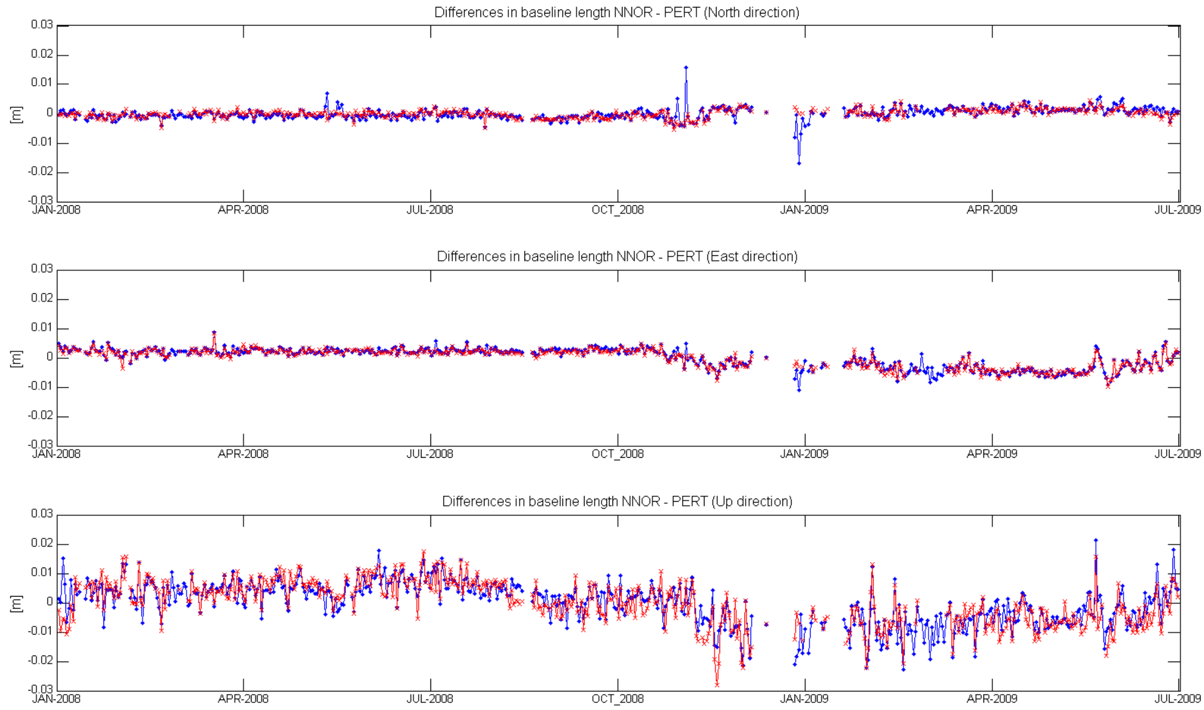


Figure 2: North, East and Up baseline length differences NNOR-PERT compared to the mean baseline length for the complete network (red) and the small network (blue).

cancel out. Since the single epoch satellite clock error is the same for all observations to that satellite the weighted mean of the undifferenced residuals should be zero. The second assumption is similar to the first assumption, when doubled differences are formed receiver clock errors cancel out. Since the single epoch receiver clock error is the same for all observations at a receiver the weighted mean of the single differenced residuals should be zero. By using these zero mean conditions, double difference CORS network-processed phase residuals can be transformed to single difference phase residuals and these single difference phase residuals can be transformed to undifferenced phase residuals of the ionosphere free linear combination. Iwabuchi *et al.* (2004) have shown that applying the zero-mean condition on the double differenced residuals from the Bernese GPS Software (and also the GAMIT Software) gives similar results as the undifferenced residuals from Precise Point Positioning with the GIPSY Software. For more information see also the discussion in Van der Marel and Gündlich (2006) and Teunissen (2007) where it is shown that the undifferenced residuals are computed as a BLUP (Best Linear Unbiased Predictor) from the double differenced residuals when the zero-mean condition is applied with the correct weights.

2.2 How Multipath Maps Are Generated

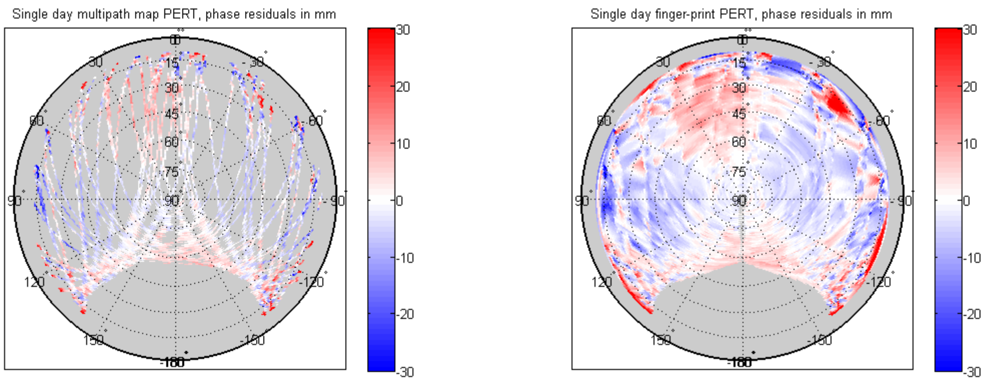
The undifferenced phase residuals depend on the satellite to receiver geometry and thus depend on azimuth and elevation of the observables from a satellite. Since the GPS constellation repeats itself every 23h56m the same residuals are expected to be observed every 23h56m. Multipath maps represent the residuals in an azimuth and elevation dependent graph and they are generated from undifferenced phase residuals that are obtained for all observations. To project the residuals in a multipath map the residuals are first gridded as a function of azimuth α and elevation ε in a sinusoidal equal area projection. A grid size s is chosen in the sinusoidal equal area projection so that each grid cell describes an equal area on the sky (Formula 1). Except for the azimuth α_0 the azimuth in the sinusoidal projection is a curved line, the largest curve is in azimuth α_0+180^o. For multipath maps in the Southern hemisphere $\alpha_0=180^o$ is chosen in this paper, this results in the largest distortion in the south direction where no data is available due to the design of the GPS constellation. The multipath value *m* of a grid point in multipath map *a* is now calculated from the mean of all the undifferenced residuals *r* that are inside the area *S* that surrounds that grid point (Formula 2). Each undifferenced residuals can only be in one area *S*.

$$S_{\alpha,\varepsilon} = \left\{ (x,y) \middle| \begin{array}{c} x \in [(\alpha-\alpha_0)\cos(\varepsilon) - s/2, (\alpha-\alpha_0)\cos(\varepsilon) + s/2) \\ y \in [\varepsilon - s/2, \varepsilon + s/2) \end{array} \right. \tag{1}$$

$$m_a(\alpha,\varepsilon) = \left\{ \frac{1}{N}\sum_{i=1}^{N} r_i \middle| r_i \in S_{\alpha,\varepsilon} \right. \tag{2}$$

The method described above to combine all the observations inside the area *S* is referred to as stacking by Alber *et al.* (2000). The equal area projection is the same as has been used by Van der Marel & Gundlich (2006), but modified for the Southern hemisphere. The GPS constellation will not cover the full field of view at a station, so not all areas S will contain one or more observations. Therefore not all grid points (α,ε) will be assigned a value in the multipath map.

Figure 3a shows a multipath map, the lines in the multipath map show the trajectories of the observed satellites in the time-span that the multipath was created. A cut off angle of 10 degrees has been used to create the maps. Grey areas indicate the parts of the sky where no observations are available. Due to the cut off angle, lines of tracked satellites will start and end at 10 degrees when 23h56m of data is used.



(a) (b)

Figure 3: Single day multipath map (a) and finger-print (b) for PERT. The multipath map shows the stacked residuals, the finger-print is an interpolation of the multipath map.

The single day multipath map shows that between 40-50 degrees azimuth signals below 20 degrees are blocked. PERT IGS site is operated by the European Space Operation Centre (ESOC). On the website (ESOC, 2009) ESOC has provided pictures of the site in different directions with the antenna as origin. Figure 4 shows the view in the north and northeast direction. A tree can be seen close to the antenna which might be causing the blockage of the signals. A possible explanation for the large residuals at 20 degrees where the line stops is that signals are not only blocked in that azimuth but are also influenced by multipath as they

Figure 4; North and northeast view at PERT (ESOC, 2009)

get through the tree. Figure 3b gives a smooth overview of the receiver multipath by interpolating the values in the multipath map to grid points that were not assigned a value. The interpolation method used here is a linear interpolation performed in the sinusoidal equal area projection. The interpolated multipath maps can be interpreted as a finger-print for a CORS site under the assumption that multipath characteristics are unique for a site. From the finger-print the (an)isotropy of a multipath map is easier to interpret and elevation or azimuth dependent residuals stand out more clearly. For example in the above finger-print, Figure 3b, between 40-50 degrees azimuth and 15-30 degrees elevation large residuals show up.

2.3 Repeatability of Multipath Maps

Ideally the multipath map only contains multipath and the multipath is repeatable. Therefore the difference between two multipath maps is expected to be zero at all the grid points. Since multipath maps generated from one or a few days also contain un-modelled atmospheric effects, the difference between multipath maps will not consist of zeros. The residual e at a grid point (α,ε) between the multipath map of a single day i and the multipath maps of j days is:

$$e_i(\alpha,\varepsilon) = m_i^1(\alpha,\varepsilon) - m^j(\alpha,\varepsilon) \tag{3}$$

After stacking the residuals of a certain numbers of days the un-modelled atmospheric delays will have averaged out and only remaining systematic errors (eg. multipath and un-modelled phase centre variations) remain. The empirical standard deviation $\hat{\sigma}_{e^j}(\alpha,\varepsilon)$ (Formula 4) for a grid point (α,ε) will not change significantly from that day as the systematic errors are expected to be constant with a certain standard deviation.

$$\hat{\sigma}_{e^j}(\alpha,\varepsilon) = \sqrt{\sum_{i=1}^{j} e_i^T e_i \,/\, \left(\left(\sum_{i=1}^{j} N_i\right) - 1\right)} \tag{4}$$

Figure 5 shows the empirical standard deviations for stations PERT and KALG. All lines in the graph represent one elevation, it is clear that the lines converge to a constant value. There is an elevation dependency visible in the graphs; the lines with the largest empirical standard deviations belong to the lowest elevations. Figure 5 also shows the difference of the standard deviations between consecutive days. The minimum and maximum number of days to create a reliable multipath map is not clear from these results. The choice has been made to use 30

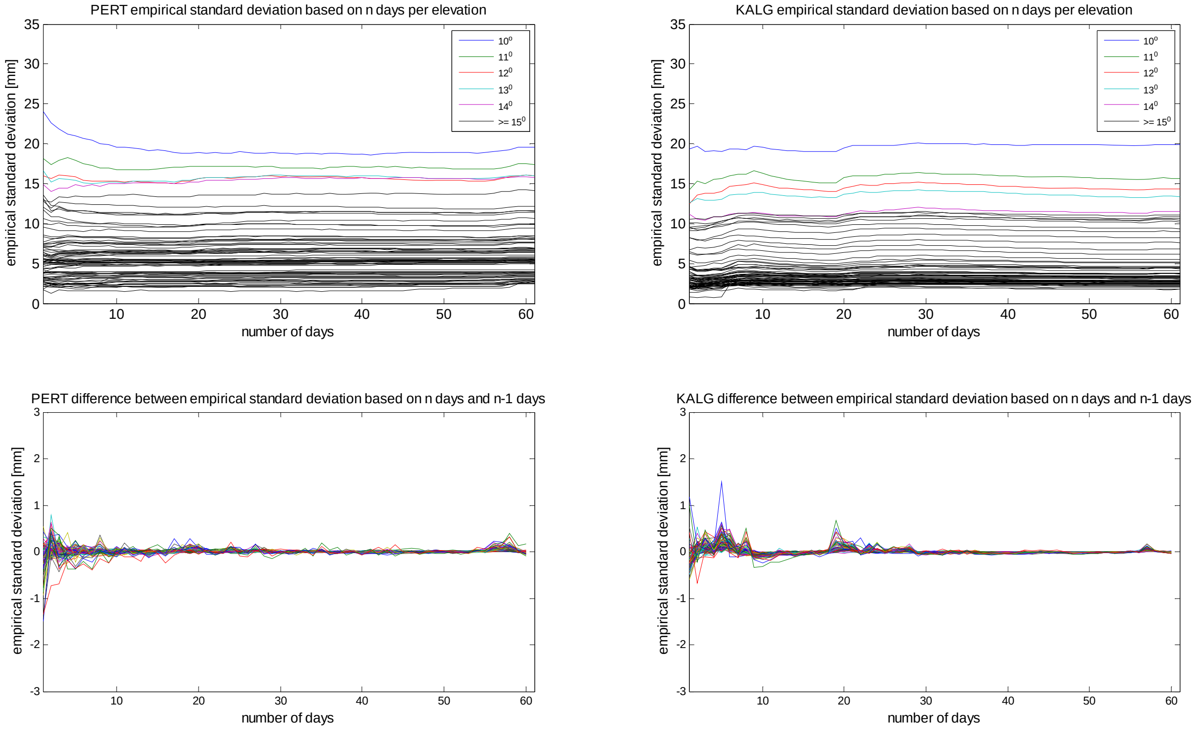


Figure 5: Empirical standard deviations versus number of days (top) and differences between consecutive days empirical standard deviations (bottom) for PERT (left) and KALG (right)

Table 2: Station information for selected stations

Station	Location	Receiver	Antenna	Radome	Network	Established
ALBY	Albany	LEICA GRX1200GGPRO	LEIAT504GG	NONE	AuScope	2008
KALG	Kalgoorly	LEICA GRX1200GGPRO	LEIAT504GG	SCIS	AuScope	2008
NNOR	New Norcia	ASHTECH Z-XII3	ASH701945C_M	NONE	IGS	2002
PERT	Perth	ASHTECH UZ-12	ASH701945C_M	NONE	IGS	1993

days of data to create multipath maps for four stations in the network, ALBY, KALG, NNOR and PERT. Station information can be found in Table 2. The choice for 30 days is quite conservative. The minimal number of days needed to create a multipath map might be less than 30 days, however 30 days is still a manageable period. The number of days used by other researchers to create multipath maps to represent systematic errors varies from 3 days (Braun *et al*, 2003) to 51 days (Shoji *et al.*, 2004). Some applications and analysis of these multipath maps will be give in the next section.

3. RESIDUALS AT THE STATIONS

30 day multipath maps and finger-prints have been created for the selected stations in Western Australia, the maps are shown in Figure 6. For all stations again the elevation dependency of the residuals is visible in the multipath map. At the recently installed stations ALBY and KALG the stacked residuals are clearly smaller than at the stations PERT and NNOR. The difference of ALBY and KALG can be explained by the site selection. ALBY and KALG have been installed at locations with a clear view to the sky, while at NNOR and PERT signals seem to be blocked. Large residuals pop up in the finger-prints at azimuth and elevation locations where the satellite signal is blocked for these long period multipath maps, as they did for the single day multipath map of PERT shown in section 2.2.

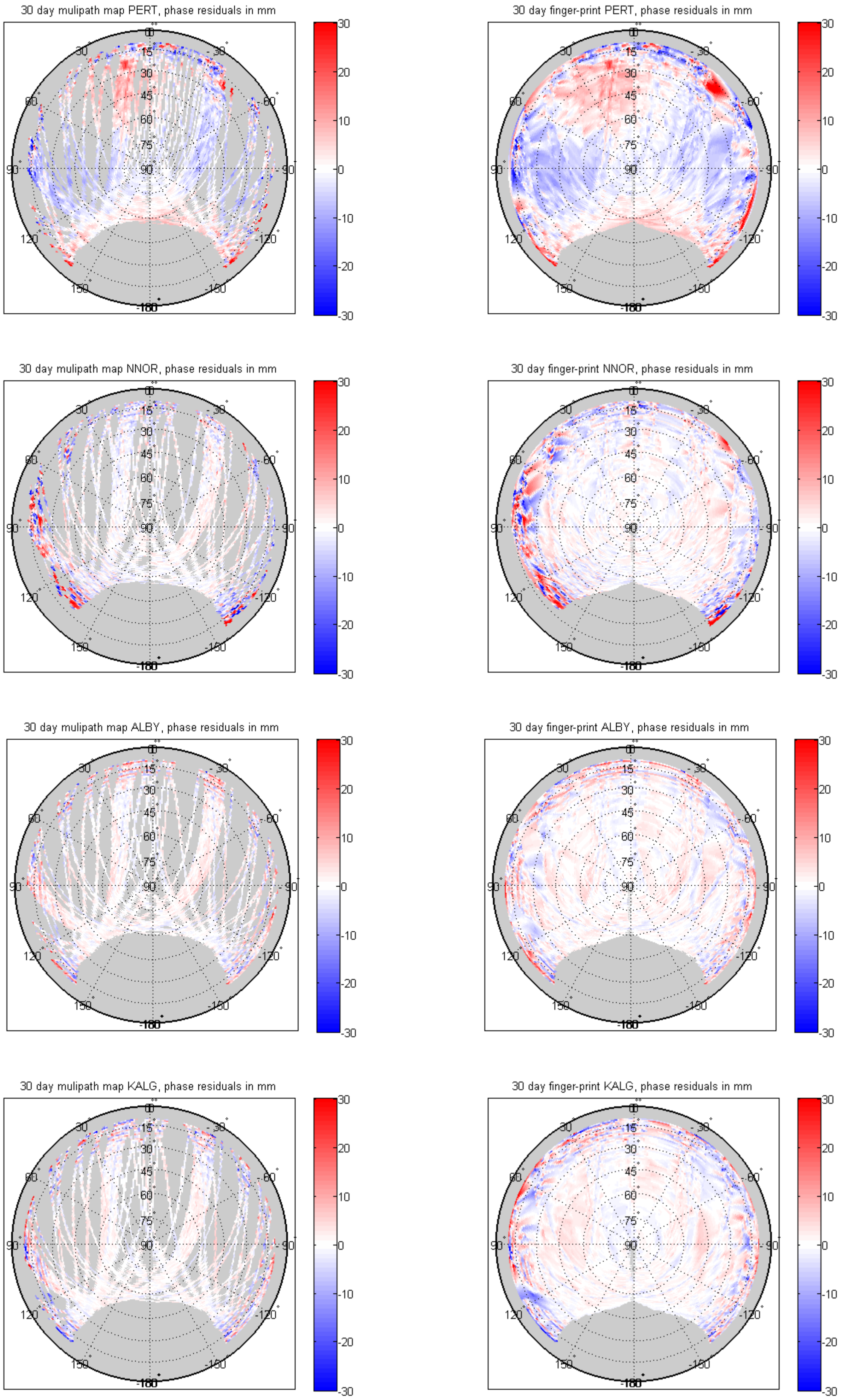


Figure 6: 30 day multipath maps (left) and finger-prints for PERT, NNOR, ALBY and KALG

3.1 NNOR Site Evaluation

The multipath map and finger-print of NNOR show that the signals at lower elevation between 60 and 150 degrees azimuth are blocked and have large residuals at the elevations where the signals are only just received. NNOR IGS site is operated by the European Space Operation Centre (ESOC). The GPS antenna is installed on a pillar close to a 35 diameter deep space antenna. On the website (ESOC, 2009) ESOC has provided pictures of the site in different direction with the antenna as origin. From the IGS-sitelog of NNOR and pictures on the website an explanation is given for the blockage and larger residuals. Figure 7 shows the west and south-west view of the antenna. The signals are blocked due to the vicinity of a hill at the antenna location.

Figure 7: West (left) and Southwest (right) view at NNOR (ESOC, 2009)

3.2 Elevation dependent residuals

The finger-prints show the azimuth and elevation dependent residuals. Instead of mapping to azimuth and elevation, the residuals can also be mapped to elevation only. Azimuth dependent effects then average out and elevation dependent effects will stand out more. In Figure 8 the undifferenced phase residuals have been mapped by their elevation for 60 days, there are some clear systematic effects, especially at lower elevations. These systematic effects can only be explained by elevation dependent, azimuth independent, mis-modelling such as remaining antenna phase centre variations. For example for NNOR there is no systematic effect visible at the lowest elevation, again probably due to the hill. The large systematic effects at station ALBY are remarkable, especially since this stations does not appear to have receiver multipath on the phase observations.

3.3 Applying Multipath Maps to Correct Residuals

From the multipath maps, finger-prints and empirical standard deviations, the elevation dependency of the residuals has already been shown in section 2.3. When visualizing the empirical standard deviation versus the elevation angle the elevation dependency of the residuals again can clearly be seen in Figure 9a.

The formal standard deviations are computed from the co-variance matrix of the least-squares residuals. The variance of the observables in the network processing is based on the elevation dependent weighting function $\sigma^2 = (C / \cos(90 - \varepsilon))^2$ where C is a constant and ε is the elevation angle of the slant observation. As a result the formal standard deviations of the residuals are also elevation angle dependent. The empirical standard deviations are based on the actual residuals. The empirical standard deviation of station ALBY is close to the formal standard deviations at all elevation angles. The other stations follow the shape of the elevation

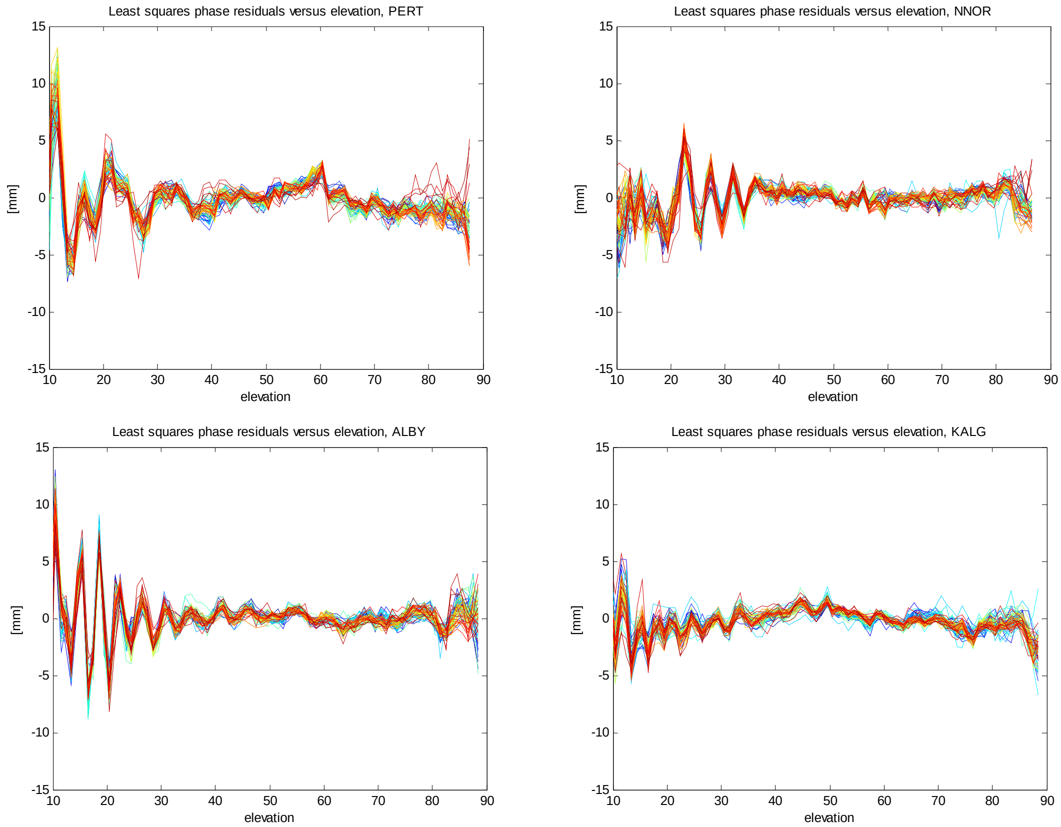


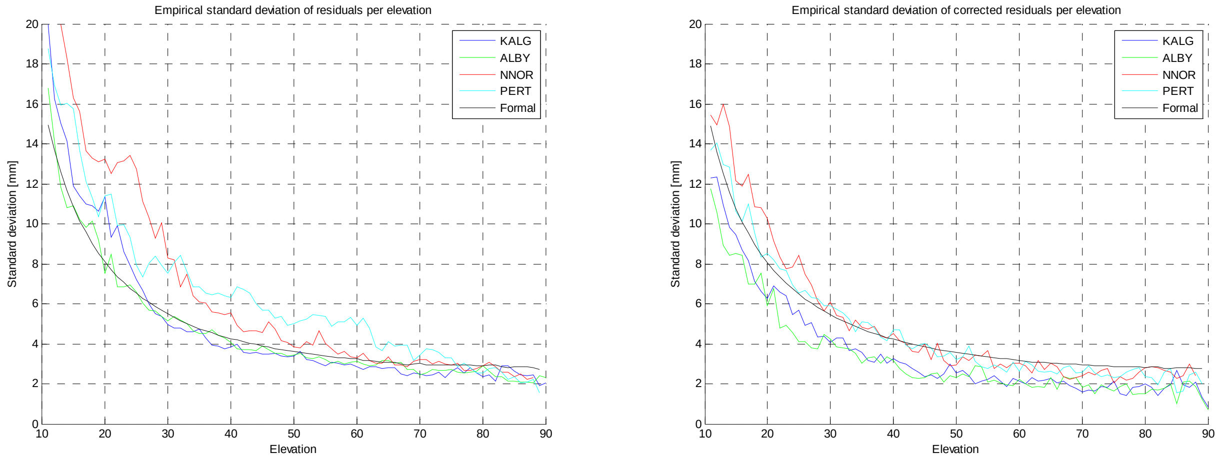
Figure 8: Undifferenced least squares phase residuals per elevation for selected stations for 61 days, each day has a different colour.

dependence residuals but especially at lower elevations the differences become larger. This is due to the anisotropic variation of the residuals that can be seen in the finger-prints. As explained for station NNOR this is caused by the hill at the southwest side of the antenna.

The multipath maps can be used to correct the residuals of a single day for systematic effects. This is done by subtracting the multipath value m that is valid for the grid point (α,ε) in which the undifferenced residual r is located. Figure 9b shows the effect on the empirical standard deviation of the least squares residuals. The residuals plotted in this figure are corrected using the multipath map of the previous 30 days. Except for lower elevations at station NNOR all stations now have an empirical standard deviation that is close to the formal standard deviation. This shows that correcting residuals for systematic effects does help and the resulting empirical standard deviations verify that the choice of the zenith dependent weighting function for phase-observations is valid.

4. CONCLUSIONS

This contribution has described how local site finger-prints can be created for CORS stations. Local site finger-prints are interpolated multipath maps and visualize systematic effects in the undifferenced least-squares GNSS ionosphere-free phase observation residuals. The determined systematic effect in the residuals can be used to analyze the performance of a CORS station, but also to monitor the station and reduce the effects of systematic effects.



(a) (b)

Figure 9: Empirical standard deviations versus elevation for the residuals (a) and the empirical standard deviations of the corrected residuals (b). The black line gives the formal standard deviation.

The recently installed CORS stations ALBY and KALG show less azimuth and elevation dependence systematic errors than older stations. Large systematic effect at intermediate elevations can be assigned to blockage of signals in some cases. The uncorrected residuals of the recently installed stations have a smaller standard deviation than the older stations; this indicates a better overall observation precision. After correcting residuals for systematic effects the performance of all stations is similar.

ACKNOWLEDGMENTS

The authors wish to thank the International GNSS Service, Geoscience Australia for making the necessary data available for this contribution. The work was supported by iVEC and AuScope through the use of advanced computing and data resources provided by the Petabyte Storage Facility located at the Australian Resources Research Centre.

Professor P.J.G. Teunissen is the recipient of an Australian Research Council Federation Fellowship (project number FF0883188): this support is greatly acknowledged.

REFERENCES

Alber, C., Ware, R., Rocken, C. and Braun, J. (2000) Inverting GPS double differences to obtain single path phase delays. *Geophysical Research Letters* 27: 2661–2664

AuScope (2009), http://www.auscope.org.au/category.php?id=15

Braun, J., Rocken, C. and Ware, R. (1999) Operating a dense L1 GPS network for atmospheric sensing, EOS, *Transactions - American Geophysical Union* 80

Braun, J., Rocken, C. and Ware, R. (2001), Validation of line-of sight water vapor measurements with GPS, *RadioScience* 36 (3): 459-472

Braun, J., Rocken, C., Liljegren, J. (2003), Comparisons of Line-of-Sight Water Vapour Observations Using the Global Positioning System and a Pointing Microwave Radiometer, *Journal of Atmospheric and Oceanic Technology* 20: 606-612

Dach, R., Hugentobler, U., Fridez, P., Meindl, M. (2007) *Bernese GPS Software Version 5.0.*, Astronomical Institute, University of Bern, Bern, 612pp

ESOC (2009), http://nng.esoc.esa.de/gps/

Iwabuchi, T., Shoji, Y., Shimada, S., Nakamura, H., Seko, H. and Aonashi, K. (2004) Tsukuba GPS dense net campaign observation: Comparison and evaluation of slant wet delays estimated in three types of software, *Journal of the Meteorological Society of Japan* 82: 315-330

De Haan, S., Van der Marel, H. and Barlag, S. (2002) Comparison of GPS slant delay measurements to a numerical model: case study of a cold front passage, *Physics and Chemistry of the Earth* 27: 317-322

Leick, A. (2004) *GPS Satellite Surveying (third edition)*, John Wiley & Sons, Inc., New Jersey, 435pp

Macias-Valadez, D., Cocard, M., Santerre, R. (2007) 3D Modelling of the Tropospheric Refractivity using a Permanent GPS Network, *Geomatica* 61(4): 445-454

Rocken, C., Mervart, L., Lukes, Z., Johnson, J. (2004) Testing a New Network RTK Software System, *ION GNSS 17th International Technical Meeting of the Satellite Division*

Shoji, Y., Nakamura, H., Iwabuchi, T., Aonashi, K., Seko, H., Mishima, K., Itagaki, A., Ichikawa, R. Ohtani, R. (2004) Tsukuba GPS dense net campaign observation: Improvement of GPS analysis of slant path delay by stacking oneway post fit phase residuals, *Journal of the Meteorological Society of Japan* 82: 301-314

Teunissen, P.J.G. (2007) Best prediction in linear models with mixed integer/real unknowns: theory and application, *Journal of Geodesy* 81 (12), 759-780

Teunissen P.J.G., Kleusberg A (eds) (1998) *GPS for Geodesy (second edition)*, Springer, Berlin Heidelberg New York, 650pp

Van der Marel, H. and Gündlich, B. (2006) Development of Models for Use of Slant Delays, Slant Delay Retrieval and Multipath Mapping Software, *TOUGH Deliverable D33*, Danish Meteorological Institute