118 Improving the GNSS Attitude Ambiguity Success Rate with the Multivariate Constrained LAMBDA Method

G. Giorgi, P.J.G. Teunissen, S. Verhagen, and P.J. Buist

Abstract

GNSS Attitude Determination is a valuable technique for the estimation of platform orientation. To achieve high accuracies on the angular estimations, the GNSS carrier phase data has to be used. These data are known to be affected by integer ambiguities, which must be correctly resolved in order to exploit the higher precision of the phase observables with respect to the GNSS code data. For a set of GNSS antennae rigidly mounted on a platform, a number of nonlinear geometrical constraints can be exploited for the purpose of strengthening the underlying observation model and subsequently improving the capacity of fixing the correct set of integer ambiguities. A multivariate constrained version of the LAMBDA method is presented and tested here.

118.1 Introduction

Attitude determination is an important issue in remote sensing applications: the knowledge of the orientation of the platform which carries the sensors (radars, lasers, etc.) is required for the pointing procedures. Although the accuracy of a stand-alone GNSS attitude system might not be comparable with the one obtainable with other modern attitude sensors, a GNSS-based system presents several advantages. The main assets are that it is driftless and it requires less maintenance. Many works investigated the feasibility and performance of GNSS Attitude Determination, see e.g. [1–6]. The key for a precise attitude estimation is the ambiguity resolution process, since only when the integers inherent to the GNSS carrier phase observations are correctly fixed one is able to exploit the carrier phase data, which are of two orders of magnitude more accurate than the GNSS code observations. In this contribution we focus on the problem of fixing the correct integer ambiguities for data collected on a frame of antennae firmly mounted on a rigid platform: the relative positions between the antennae are assumed to be known and constant. In such configurations, the baselines lengths and the angles between them are known, resulting in a set of nonlinear constraints posed on the baseline vectors which can be exploited to strengthen the underlying observation model. The set of GNSS phase and code observations is cast into a linearized system solvable

G. Giorgi • S. Verhagen (✉) • P.J. Buist
Delft Institute of Earth Observation and Space Systems (DEOS), Delft University of Technology, PO Box 5058, 2600 Delft, The Netherlands
e-mail: a.a.verhagen@tudelft.nl

P.J.G. Teunissen
Delft Institute of Earth Observation and Space Systems (DEOS), Delft University of Technology, PO Box 5058, 2600 Delft, The Netherlands

Department of Spatial Sciences, Curtin University of Technology, GPO Box U1987, Perth, WA 6845, Australia

S. Kenyon et al. (eds.), *Geodesy for Planet Earth*, International Association of Geodesy Symposia 136,
DOI 10.1007/978-3-642-20338-1_118, © Springer-Verlag Berlin Heidelberg 2012

in a least-squares sense, where both the integerness of the ambiguities and the constraints on the baselines must be fulfilled. The method is based on an extension of the Integer Least Squares (ILS, [7]) principle, employed to solve in a least-squares sense a linear system of equations where some of the unknowns are integer-valued. A well-known mechanized implementation of the ILS principle is the Least squares AMBiguity Decorrelation Adjustment (LAMBDA) method [8], widely used for its high computational efficiency. This method has been recently extended to accommodate those baseline applications where the distance between the antennae is known and constant: the baseline Constrained LAMBDA method was introduced in [9, 10]. The inclusion of the baseline length constraint results in a large improvement in the success rate, as shown in [11–15].

We here present and test the performance of the multivariate generalization of the Constrained LAMBDA method [16], which solves the model where the full set of nonlinear geometrical constraints is taken into account, i.e. the different baseline lengths and their relative positions.

118.2 Modeling of the Multi-antennae GNSS Observations

We consider a set of $m + 1$ antennae (m independent baselines) simultaneously tracking the same $n + 1$ GNSS satellites on a single frequency. The set of linearized Double Difference (DD) GNSS phase and code observations obtained on the m baselines can be cast into a Gauss–Markov model as follows:

$$\begin{aligned} &E(Y) = AZ + GB \; Z \in \mathbb{Z}^{n\times m}; B \in \mathbb{R}^{3\times m} \\ &D(vec(Y)) = Q_Y \end{aligned} \qquad (118.1)$$

where

$E(\cdot)$ is the expectation operator

Y is the $2n$ by m matrix whose columns are the code and phase DD observations derived at each baseline:

$$Y = [\, y_1 \quad y_2 \quad \ldots \quad y_m \,]$$

Z is the n by m matrix whose columns are the integer-valued ambiguities for each baseline:

$$Z = [\, z_1 \quad z_2 \quad \ldots \quad z_m \,]$$

B is the 3 by m matrix whose columns are the real-valued baseline coordinates:

$$B = [\, b_1 \quad b_2 \quad \ldots \quad b_m \,]$$

A is the $2n$ by n design matrix which contains the carrier wavelength

G is the $2n$ by 3 design matrix of line-of-sight vectors

$D(\cdot)$ is the dispersion operator

Q_Y is the $2nm$ by $2nm$ variance–covariance matrix of the vector of observations $vec\,(Y)$

We make use of the vec operator, which stacks the columns of a matrix below each other, to define the variance–covariance matrix of the vector of observations $vec\,(Y)$. We assume that the antennae are separated by short baselines, for which the atmospheric effects can be neglected, and the only real-valued unknowns to be estimated are the $3m$ coordinates of the baseline vectors. Also, the short baseline hypothesis allows us to make use of a unique matrix of line-of-sight vectors G for all the baselines. A Gaussian-distributed error is assumed on the observables Y.

Aiming to estimate a platform's orientation solely via GNSS measurements, two or more antennae are assumed to be firmly mounted on the platform, which is here considered as a rigid body. We introduce a system of body axes (local) $u_1u_2u_3$ taken as to have the first axis u_1 aligned with the first baseline, the second axis u_2 perpendicular to u_1 and lying in the plane formed by the first two baselines, and the third axis u_3 oriented that $u_1u_2u_3$ form an orthogonal frame (see Fig. 118.1, where f_i is the ith baseline, and f_{ij} indicates the jth coordinate of the baseline i). The baseline coordinates expressed in the local frame are collected in matrix F, and the relationship between these and the coordinates B expressed in the global frame $x_1x_2x_3$ is:

$$B = R \cdot F \qquad (118.2)$$

where R is the orthogonal matrix which describes the relative orientation between the local and global frames, i.e. the attitude of the platform. For notational convenience, the rotation matrix and the local baseline coordinates F are defined as [16]

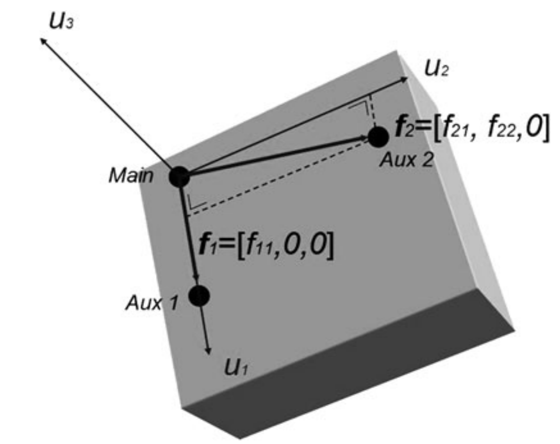

Fig. 118.1 The first baseline f_1 (*Main* antenna – Aux_1 antenna) defines the first body axis u_1, while the second body axis u_2, perpendicular to u_1, lies in the plane formed by f_1 and f_2 (*Main* antenna – Aux_2 antenna). u_3 is taken as to form an orthogonal frame

$$\begin{cases} m \geq 3 \\ q = 3 \end{cases} : \; RF = [r_1, r_2, r_3] \begin{bmatrix} f_{11} & f_{21} & f_{31} & \cdots & f_{m1} \\ 0 & f_{22} & f_{32} & \cdots & f_{m2} \\ 0 & 0 & f_{33} & \cdots & f_{m3} \end{bmatrix}$$
$$\begin{cases} m = 2 \\ q = 2 \end{cases} : \; RF = [r_1, r_2] \begin{bmatrix} f_{11} & f_{21} \\ 0 & f_{22} \end{bmatrix}$$
$$\begin{cases} m = 1 \\ q = 1 \end{cases} : \; RF = [r_1][f_{11}] \tag{118.3}$$

where q is a parameter introduced to cope with the case of $m < 3$ baselines. The relation (118.2) is a linear transformation, which changes the unknowns of the problem: the estimation of the baseline coordinates B turns into the estimation of the components of the matrix R, of which only three are independent. Hence, in addition to the integer constraint on the matrix Z, also the orthogonality of the matrix R has to be considered. This allows to rewrite the set of baseline observations (118.1) as

$$E(Y) = AZ + GRF \;\; Z \in \mathbb{Z}^{n \times m}; R \in \mathbb{O}^{3 \times q}$$
$$D(vec(Y)) = Q_Y \tag{118.4}$$

This is the model that we aim to solve in a least-squares sense. The standard LAMBDA method can be employed to solve the system when the constraint on the matrix R is disregarded, being the system solely subject to the integer constraint on Z. Via a modification of the LAMBDA method, the model (118.4) is solvable in a rigorous least-squares sense taking into account all the different constraints, which are the integer nature of the entries of Z and the orthogonality of the rotation matrix R. This is demonstrated in the following section.

118.3 Integer Least Squares

We aim to solve for the model (118.4) in a rigorous least-squares sense, minimizing the weighted squared norm of the residuals. The solution of the model (118.4) is derived with a three-steps procedure: first obtain a float solution, then search for the integer ambiguities and finally extract the orthogonal matrix R. In this section we describe each of these steps.

118.3.1 The Float Solution

The float solution of (118.4) is the least-squares solution obtained disregarding both the integerness of the matrix Z and the orthogonality of R:

$$N \cdot \begin{pmatrix} vec(\hat{Z}) \\ vec(\hat{R}) \end{pmatrix} = \begin{bmatrix} I_m \otimes A^T \\ F \otimes G^T \end{bmatrix} Q_Y^{-1} vec(Y)$$
$$N = \begin{bmatrix} I_m \otimes A^T \\ F \otimes G^T \end{bmatrix} Q_Y^{-1} \begin{bmatrix} I_m \otimes A & F^T \otimes G \end{bmatrix} \tag{118.5}$$

where $\otimes$ is the *Kronecker* product and we made use of the property

$$vec(X_1 X_2 X_3) = \left(X_3^T \otimes X_1\right) vec(X_2)$$

$vec(\hat{Z})$ and $vec(\hat{R})$ are the float estimators of Z and R; their v–c matrices are obtained via the inversion of the normal matrix N:

$$\begin{bmatrix} Q_{\hat{Z}} & Q_{\hat{Z}\hat{R}} \\ Q_{\hat{R}\hat{Z}} & Q_{\hat{R}} \end{bmatrix} = N^{-1} \tag{118.6}$$

If one assumes that the matrix Z is known, the conditional solution of R (conditioned on the knowledge of the matrix Z) is obtained as

$$vec(\hat{R}(Z)) = vec(\hat{R}) - Q_{\hat{R}\hat{Z}} Q_{\hat{Z}}^{-1} vec(\hat{Z} - Z) \tag{118.7}$$

The precision of the conditional solution $\hat{R}(Z)$ is described by the v–c matrix

$$Q_{\hat{R}(Z)} = Q_{\hat{R}} - Q_{\hat{R}\hat{Z}} Q_{\hat{Z}}^{-1} Q_{\hat{Z}\hat{R}} \tag{118.8}$$

118.3.2 The Search for the Integer Minimizer

Given the float estimator $\hat{Z}$, the conditional solution $\hat{R}(Z)$ and their v–c matrices, we can write the sum-of-squares decomposition of the weighted squared norm of the residuals of (118.4) as

$$\begin{aligned} &\| vec(Y) - (I_m \otimes A)vec(Z) - (F^T \otimes G)vec(R) \|^2_{Q_Y} \\ &\quad = \| vec(\hat{E}) \|^2_{Q_Y} + \| vec(Z - \hat{Z}) \|^2_{Q_{\hat{Z}}} \\ &\quad + \| vec(\hat{R}(Z) - R) \|^2_{Q_{\hat{R}(Z)}} \end{aligned} \tag{118.9}$$

where $\hat{E}$ is the matrix of least-squares residuals. The decomposition shows that the last term can always be made zero for any value assumed by Z, by taking $R = \hat{R}(Z)$, if the orthogonality of R is disregarded. The minimization of the least-squares residuals then reduces to the well known case of finding the integer minimizer of the second term, and the standard LAMBDA method can be directly applied. When the orthogonality constraint on the matrix R is taken, the last term generally differs from zero, since $\hat{R}(Z)$ is usually non orthogonal: this leads to a modification of the search algorithm to be adopted, resulting in a multivariate constrained version of the LAMBDA method.

118.3.2.1 The LAMBDA Method

Disregarding the orthogonality of R, the integer-valued minimizer of (118.9) equals

$$\check{Z}^U = \arg \min_{Z \in \mathbb{Z}^{n\times m}} \| vec(Z - \hat{Z}) \|^2_{Q_{\hat{Z}}} \tag{118.10}$$

The matrix $\check{Z}^U$ has the minimum distance from the float solution $\hat{Z}$ in the metric defined by $Q_{\hat{Z}}$: since no closed-form solution of (118.10) is known, the estimation of the matrix $\check{Z}^U$ involves a direct search inside a set of suitable integer candidates:

$$\Omega^U(\chi^2) = \left\{ Z \in \mathbb{Z}^{n\times m} \,|\, \| vec(Z - \hat{Z}) \|^2_{Q_{\hat{Z}}} \leq \chi^2 \right\} \tag{118.11}$$

The set Ω^U, which geometrically draws an hyper-ellipsoid centered in $vec(\hat{Z})$ and size/shape driven by the entries of $Q_{\hat{Z}}$, is evaluated and the integer matrix Z that minimizes the squared norm (118.10) is extracted. The LAMBDA method is applied to perform the search in an efficient and fast way; it works by decorrelating the ambiguities performing an admissible (i.e. which preserves the integerness of the variables) transformation: the effect of the decorrelation is to have a reduced set of integer candidates, among which the matrix $\check{Z}^U$ is quickly extracted.

118.3.2.2 The Multivariate Constrained LAMBDA Method

The full-constrained least-squares minimization of (118.9) is obtained by taking the minimization with respect both the matrix of ambiguities Z and the orthogonal matrix R:

$$\begin{aligned} &\check{Z}^C = \arg \min_{Z \in \mathbf{Z}^{n\times m}} C(Z) \\ &C(Z) = \| vec(Z - \hat{Z}) \|^2_{Q_{\hat{Z}}} + \| vec(\hat{R}(Z) - \check{R}(Z)) \|^2_{Q_{\hat{R}(Z)}} \end{aligned} \tag{118.12}$$

with

$$vec(\check{R}(Z)) = \arg \min_{R \in \mathbb{O}^{3\times q}} \| vec(\hat{R}(Z) - R) \|^2_{Q_{\hat{R}(Z)}} \tag{118.13}$$

where $\mathbb{O}^{3\times q}$ is the class of $3 \times q$ orthogonal matrices, i.e. $R^T R = I_q$. The integer minimizer $\check{Z}^C$ weighs the sum of two terms: the first is the distance with respect to the float solution $\hat{Z}$ weighted by $Q_{\hat{Z}}$, and the second is the distance between $\hat{R}(Z)$ and the solution of the nonlinear constrained least-squares problem (118.13). The latter gives the orientation of the platform $\check{R}(Z)$ by minimizing in a least-squares sense the distance from the matrix $\hat{R}(Z)$, subject to the orthogonal constraint. Note that for the single-baseline case ($m = 1$) the problem reduces to the one addressed in [9, 10, 17]: the method discussed here is a multivariate generalization. The minimizer of (118.12) is searched in the set defined as

$$\Omega(\chi^2) = \{Z \in \mathbb{Z}^{n\times m} | C(Z) \leq \chi^2\} \tag{118.14}$$

Minimizing the cost function (118.12) in the set $\Omega(\chi^2)$ is a non-trivial task: the evaluation of $C(Z)$ involves the computation of a nonlinear constrained least-squares problem, and if the set contains a large number of candidates the search is very time-consuming. Hence, the choice for the scalar χ is an important issue, since it strongly affects the time dedicated to the minimization process.

To make the search more time-efficient, and to cope with both the problems of setting the value of χ and computing (118.13) a large number of times, the two algorithms coined as the Expansion and the Search and Shrink approaches were developed [12–15]: by using two functions that provide a lower and an upper bound for the cost function $C(Z)$ and that are easier to evaluate (i.e. the computation of (118.13) is avoided), the search for the integer minimizer $\check{Z}^C$ is computed efficiently and in a much faster way.

118.3.3 The Attitude Solution

The two above mentioned search methods provide the ILS minimizer $\check{Z}$ of the expression (118.9), respectively with (Constrained LAMBDA, $\check{Z} = \check{Z}^C$) or without (LAMBDA, $\check{Z} = \check{Z}^U$) considering the orthogonality of R. Note that in general $\check{Z}^U \neq \check{Z}^C$. Given the integer minimizer resolved, the conditional attitude solution is obtained as in (118.7): the solution $\hat{R}(Z)$ is characterized by a better accuracy (118.8), but it is in general non-orthogonal. In order to obtain the sought orthogonal attitude matrix, the following nonlinear constrained least-squares problem has to be solved:

$$vec(\check{R}(\check{Z})) = \arg \min_{R\in\mathbb{O}^{3\times q}} \| vec(\hat{R}(\check{Z}) - R) \|^2_{Q_{\hat{R}(Z)}} \tag{118.15}$$

where $\check{Z} = \check{Z}^U$ for the LAMBDA method and $\check{Z} = \check{Z}^C$ for the Constrained LAMBDA method. The nonlinearity of the problem comes from the orthogonality of R: via a suitable parameterization of the rotation matrix, e.g. Euler angles or Quaternions [18], the orthogonality is implicitly fulfilled, and the least-squares solution of (118.15) can be solved for example with the Newton method.

118.4 Testing the Method

The method presented was tested with simulated data and with data collected during a kinematic experiment. All the data sets were processed with both the LAMBDA method and the Constrained LAMBDA method, in order to compare the different performance obtained in terms of single-epoch, single-frequency success rate.

118.4.1 Simulation Results

Different sets of data were generated via a Monte Carlo simulation, reproducing the set of baseline observations according to the model (118.4). Table 118.1 summarizes the set-up of the simulations: from the actual GPS constellation on 22 January 2008 (as seen from Delft, The Netherlands), we selected five to eight satellites, with corresponding PDOP values between 4.2 and 1.8. Two baselines were simulated, of 1 and 2 m length, forming an angle of 100°. For each of the 24 scenarios, 10^5 samples were generated, aiming to extract an accurate estimation of the success rate, defined as the ratio of samples where the correct integer ambiguity matrix has been fixed and the total number of samples. The data sets were processed applying the LAMBDA and the Constrained LAMBDA methods as described in Sect. 118.3. Table 118.2 shows the single-frequency, single-epoch

Table 118.1 Simulation set up

Frequency	L1
Number of satellite (PRNs) 5/6/7/8	Corresponding PDOP 4.19/2.14/1.92/1.81
Undifferenced code noise σ_p (cm)	30-15-5
Undifferenced phase noise σ_ϕ (mm)	3-1
Baselines f_i (x_1, x_2, x_3)	$\vec{f}_1 = [1, 0, 0]$m $\vec{f}_2 = [-0.35, 1.97, 0]$m
Samples simulated	10^5

Table 118.2 Simulation results: single-frequency, single-epoch success rates for the unconstrained and constrained LAMBDA methods. Success rates higher than 99.9% are stressed

σ_ϕ (mm)	3			1		
σ_p (cm)	30	15	5	30	15	5
N	Two-baselines success rate, LAMBDA Two-baselines success rate, constrained LAMBDA					
5	0.17	5.69	83.87	0.55	10.18	95.48
	99.60	**99.94**	**100**	**100**	**100**	**100**
6	9.81	56.89	96.83	30.12	81.35	**100**
	99.99	**100**	**100**	**100**	**100**	**100**
7	31.97	73.07	99.74	61.16	91.33	**100**
	99.99	**100**	**100**	**100**	**100**	**100**
8	81.72	93.12	**99.99**	**99.99**	**100**	**100**
	100	**100**	**100**	**100**	**100**	**100**

Table 118.3 Vessel experiment results: single-epoch, single frequency success rates for the LAMBDA and Constrained LAMBDA methods

	Baselines	LAMBDA	Constrained LAMBDA
Single-epoch, single frequency success rate (%)	1–2 + 1–3	54.08 $\left(\frac{4867}{9000}\right)$	99.96 $\left(\frac{8996}{9000}\right)$

success rates for the methods: the improvement is large especially for the weaker scenarios (lower number of satellite/higher levels of noise), where the difference between the methods is significant. For example, the weakest simulated data set, with five available satellites and high noise values, shows an improvement from a low 0.17–99.60%. The number of correctly fixed samples for the Constrained LAMBDA method is always higher than 99.6%: as expected, the strengthening of the underlying model due to the embedded geometrical constraints substantially affects the capacity of fixing the correct integer ambiguity vector. The two-baseline case shows success rates higher than 99% on all the data sets processed, obtaining a 100% success rate on 20 out of 24 data sets simulated.

118.4.2 Experimental Results

We tested the new method on a set of data collected onboard a vessel during a kinematic experiment held in Delft, The Netherlands. The vessel was equipped with three couples of antennae-receivers (1) a choke-ring antenna connected to an Ashtech receiver; (2) an antenna connected to a Leica SR530 receiver; (3) an antenna connected to a Novatel OEM3 receiver. The baseline lengths between the antennae are 2 m (antennae 1–2) and 1.5 m (antennae 1–3), and the coordinates F in the local plane u_1u_2 are

$$F = \begin{bmatrix} f_{11} & f_{21} \\ 0 & f_{22} \end{bmatrix} = \begin{bmatrix} 1.5 & 1.89 \\ 0 & 0.74 \end{bmatrix} (m)$$

The vessel sailed for about 2.5 h, collecting a total of 9,000 epochs of GPS observations. The number of tracked GPS satellite varied between seven and eight, except for the first thousands epoch, were only data from six GPS satellite were stored. The PDOP values were between 2.1 and 4. We processed both the baselines (1–2 + 1–3) embedded in the model (118.4). The LAMBDA method gave a 54.08% single-epoch, single-frequency success rate, thus providing the correct attitude solution for about half the epochs processed. As expected, the number of correctly fixed epochs largely increased when the Constrained LAMBDA method was employed: only four epochs were incorrectly fixed, thus achieving 99.96% success rate. This is due to the stronger observation model obtained by including the known geometrical constraints on the different baselines. Each of the (either correctly or incorrectly) fixed ambiguity matrices were used to compute the full attitude of the vessel in the East-North-Up (ENU) frame according to (118.15). Figure 118.2 shows the Heading, Elevation and Bank angles computed epoch by epoch according to the output of the LAMBDA algorithm: the plot is rather scattered due to the low success rate (54.08%), which resulted in many wrong attitude solutions $\check{R}(\check{Z})$. The Constrained LAMBDA method, providing for 99.96% of the epochs the correct integer matrix, produces the plots of Fig. 118.3: as shown, the single-epoch full attitude solution was available for the entire (only four epoch missed) duration of the experiment.

Conclusions

In this contribution it is analyzed how to resolve the integer ambiguities in a rigorous ILS sense for GNSS Attitude Determination applications. It is firstly described how to model the set of GNSS phase and code observations collected on a frame of antennae mounted on the same platform. Then the

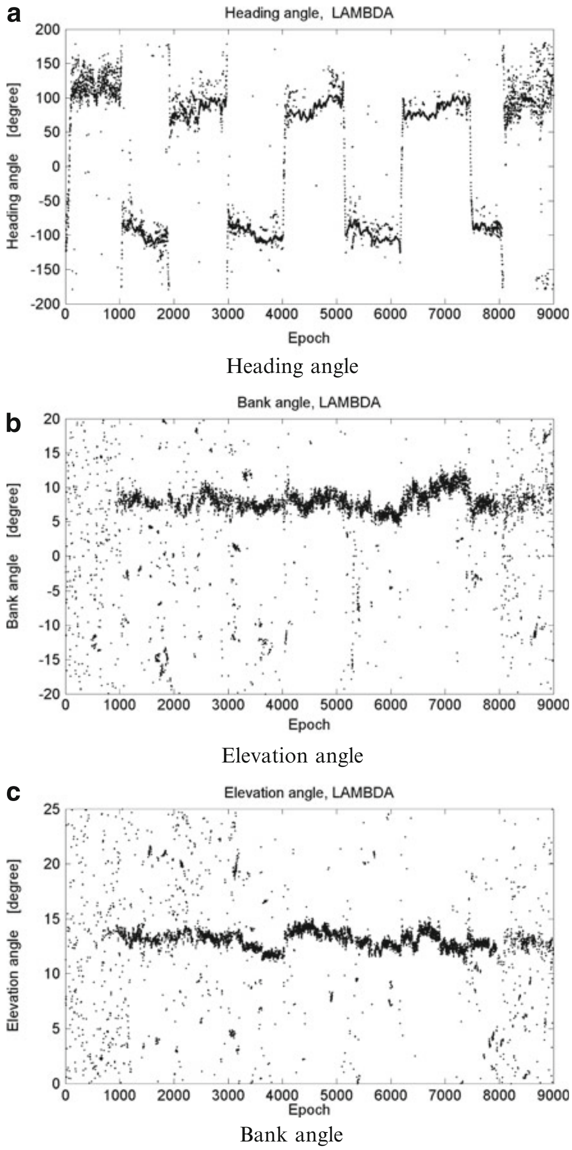

Fig. 118.2 Single-epoch/single-frequency full attitude solution, LAMBDA method

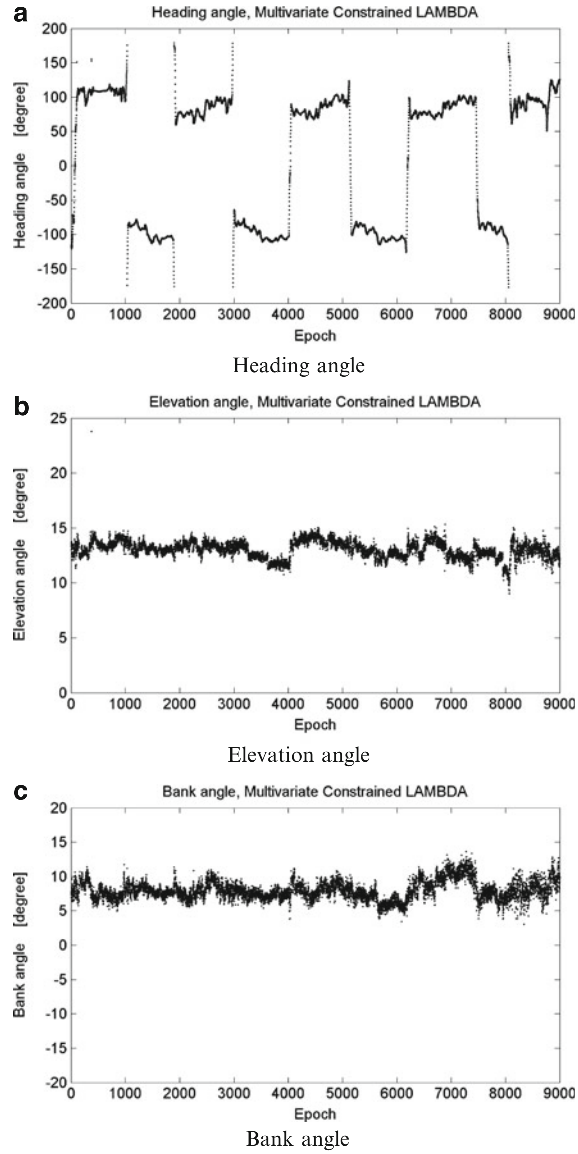

Fig. 118.3 Single-epoch/single-frequency full attitude solution, Multivariate Constrained LAMBDA method

Constrained LAMBDA method is introduced, which solves the problem of minimizing the norm of the least-squares residuals taking into account both the integerness of the ambiguities and the orthogonality of the rotation matrix. The latter constraint is derived assuming the relative positions of the antennae as known and constant. The strengthening of the observation model, due to the inclusion of the geometrical constraints, improves the capacity of fixing the correct set of integer ambiguities, as shown via simulations and as tested on data collected during a kinematic experiment. The high fixing rates obtained from the tests suggest that for GNSS Attitude Determination applications the single-epoch, single-frequency ambiguity resolution is feasible when either the quality of the observation is high or the number of constrained baselines on the platform increases. During the kinematic test was proven that already with a three antennae/two baselines configuration the Constrained LAMBDA method is capable of providing the correct full attitude solution almost at every epoch.

Acknowledgements P.J.G. Teunissen is the recipient of an Australian Research Council Federation Fellowship (project number FF0883188): this support is gratefully acknowledged. The research of S. Verhagen is supported by the Dutch Technology Foundation STW, applied science division of NWO and the Technology Program of the Ministry of Economic Affairs.

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