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Single-Receiver Single-Channel Real-Time Validation of GNSS Data

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Abstract
Real-time quality control of GNSS measurements can be performed using a single-receiver single-channel approach. The local DIA procedure can be utilised for this purpose using only observations of the epoch where data are collected. A description of the proposed method is given in this paper. A geometry-free observation equation model is proposed. Due to rank deficiency during initialisation of the model, a reparameterisation of the model unknowns was performed. In addition, the ionosphere error is assumed changing relatively smooth as a function of time and its variation from its mean is modelled as a first order autoregressive process. For a short time window, the instrumental code delay and the phase bias including the sum of the initial phase, the phase ambiguity as well as instrumental phase bias are treated as constants. The dynamic and stochastic modeling of the method is presented, and initialisation of the filter is discussed. The quality control procedure is given and the internal reliability quantified by the Minimal Detectable Bias is discussed.

Key words
Validation, Quality Control, Multi-GNSS, RAIM, Minimal Detectable Bias (MDB)

1. Introduction

Successful GNSS software should include a pre-processing step for screening of the data. During this pre-processing step the most severe irregularities in the data can be detected and if necessary repaired [1]. Error detection can be implemented at receiver level as part of a Receiver Autonomous Integrity Monitoring (RAIM) algorithm. RAIM techniques are generally based on statistical tests that use the redundancy of the model to detect errors in the observation.

Real-time quality control of GNSS can be performed using a single-receiver single-satellite approach utilizing the Detection- Identification- Adaptation (DIA) Method [2]. This approach can be applied for single or multi-frequency observations. A geometry-free observation equation model can be used. The advantages of this approach are: no satellite positions need to be known beforehand and thus no complete navigation messages need to be read and used, and due to its flexibility it can be applied to any receiver type and make and under static or kinematic modes. In addition, when the above technique is applied for single-channel (satellite) data screening, one by one, it allows one to present the necessary numerical and graphical statistical diagnostics and collect the very necessary long term statistics of the GNSS PRN specific data quality. This paper discusses a proposed method for error detection using a geometry-free single-receiver single-channel approach during pre-processing of GNSS data.

2. Single-Channel, Geometry-free Modelling

2.1 Measurement Model and Re-parameterisation

The carrier phase and pseudo range observation equations of a single receiver that tracks a single satellite on frequency f_i (for $i = 1, \ldots, n$) at time instant k can be written as:

$$\varphi_{i_k} = \rho_k^* - \mu_i I_k + b_{\varphi_{i_k}} + \varepsilon_{\varphi_{i_k}} \quad (1)$$

$$p_{i_k} = \rho_k^* + \mu_i I_k + b_{p_{i_k}} + \varepsilon_{p_{i_k}} \quad (2)$$

where:

$$\rho_k^* = \rho_k + c(dt_r - dt^s) + T_k \quad (3)$$

where φ_{i_k} and p_{i_k} denote the observed carrier phase and pseudo range code measurements; respectively, with corresponding zero-mean noise terms $\varepsilon_{\varphi_{i_k}}$ and $\varepsilon_{p_{i_k}}$. ρ_k is the receiver-satellite range, c denotes the speed of light, dt_r and dt^s are the receiver and satellite clock errors; respectively, and T_k is the tropospheric delay. The parameter I denotes the ionospheric error expressed in units of range with respect to the first frequency, such that for frequency I, the ionospheric coefficient $\mu_i = \frac{\lambda_i^2}{\lambda_1^2}$ is applied. The parameters $b_{\varphi_{i_k}}$ and $b_{P_{i_k}}$ are the phase bias and the instrumental code delay, respectively. The phase bias is the sum of the initial phase, the phase ambiguity and the instrumental phase delay.

Assuming equal number of phase and code measurements, the model given in Eq. (1 & 2) shows that the problem at hand is underdetermined. The parameters b_{φ_i} and b_{p_i} are

assumed constant for a short time window. If measurements from a second epoch are added, then there is no redundancy for a single-frequency receiver, but a redundancy of two for a dual-frequency receiver. The rank-defect is caused by the fact that the information content of the observables together with the time-constancy of the bias vectors is such that only time-differences of the parameters can be determined. One way to take that into consideration is to re-parameterize the unknowns in the observation equations as follows:

$$\rho_k^{**} = \rho_k^{*} - \rho_{k_o}^{*} \quad (4)$$

$$I_k^{*} = I_k - I_{k_o} \quad (5)$$

$$b_{\varphi_{i_k}}^{*} = b_{\varphi_{i_k}} + [\rho_{k_o}^{*} - \mu_i I_{k_o}] \quad (6)$$

$$b_{p_{i_k}}^{*} = b_{p_{i_k}} + [\rho_{k_o}^{*} + \mu_i I_{k_o}] \quad (7)$$

Where k_o refers to the initial epoch of data processing.

2.2 Dynamic Modelling

The dynamic and stochastic properties of the re-parameterised range ρ_k^{**} in Eq. (4) can be modelled as a random walk. It is assumed that no information is known about the dynamic behaviour of the object, and thus allowing the range to change freely. Accordingly, the variance of its stochastic noise may be set to infinity. The dynamic model reads:

$$\rho_k^{**} = \rho_{k-1}^{**} + d_{\rho_k} \quad (8)$$

where d_ρ is the process noise of ρ^{**}. Other approaches can include elimination of this parameter in the predicted states of the model and consequently from the dynamic model.

The ionosphere error can be further parameterised to describe its dynamic variability with time. The ionosphere can be assumed changing relatively smooth as a function of time and for a short time window, we can assume that its mean is constant. Thus, we can decompose the ionosphere error into two components; its mean value ($\bar{I}$) and the deviation from its mean (δI), such that at epoch k we have:

$$I_k = \bar{I} + \delta I_k \quad (9)$$

and:

$$I_k^{*} = \delta I_k + \bar{I} - I_{k_o} \quad (10)$$

The temporal correlation of δI, denoted here as (β) can be modelled exponentially decaying with time by using a first-order autoregressive stochastic process (e.g. 1st order Gauss-Markov process) as follows:

$$\beta = e^{-\alpha|\Delta t|} \quad (11)$$

where α is the inverse of the correlation time length (time constant), and Δt is the time interval between processing epochs. Thus, with small values of α (i.e. large correlation time), the temporal correlation function will start large and reduces slowly with time, whereas with a large value for α (i.e. short correlation time), the correlation starts small and quickly damped. For the ionosphere, a correlation time in the range of 600 seconds to 2400 seconds can be assumed, depending on ionospheric activity, time of day and year, and location (latitude). The dynamic model of (δI) can thus be taken as:

$$\delta I_k = \beta\, \delta I_{k-1} + d_{\delta I_k} \quad (12)$$

Where $d_{\delta I_k}$ is the process noise for δI_k. For a short window of time, the bias terms $b_{\varphi_{i_k}}^{*}$ and $b_{p_{i_k}}^{*}$ can be modelled as constants. The ionosphere term $\{\bar{I} - I_{k_o}\}$ can thus be lumped together with these terms, such that the bias terms become:

$$b_{\varphi_{i_k}}^{**} = b_{\varphi_{i_k}} + [\rho_{k_o}^{*} - \mu_i \bar{I}] \quad (13)$$

$$b_{p_{i_k}}^{**} = b_{p_{i_k}} + [\rho_{k_o}^{*} + \mu_i \bar{I}] \quad (14)$$

The transition matrix for the unknowns $[\rho_k^{**}, \delta I_k, b_{\varphi_{i_k}}^{**}, b_{p_{i_k}}^{**}]$, denoted by $\Phi_{k/k-1}$, between the times k and k-1, can thus be given as:

$$\Phi_{k/k-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix} \quad (15)$$

2.2 Stochastic Modelling

In the single-channel single-receiver case, no auto-correlation or cross-correlation is assumed between code and phase measurements in the stochastic model, and thus, the covariance matrix of the undifferenced measurements is a diagonal matrix. The zenith-referenced values of standard deviations of phase and code can be taken as follows [1]:

Table 1: Standard deviation of undifferenced GNSS measurements

	GPS			Galileo				
	L1	L2	L5	E1	E5a	E5b	E5	E6
code (cm)	15	15	3.9	6.1	3.9	3.7	0.9	4.4
phase (mm)	1.0	1.3	1.3	1.0	1.3	1.3	1.3	1.2

The variance of phase and code measurements are weighted according to the elevation angle, for instance using a weight factor of $\frac{1}{(1+10\times\exp(-\frac{\theta}{10}))}$, with θ is the elevation angle in degrees.

The covariance matrix of the process noise of the unknowns (Q_{dd}) can be given as:

$$Q_{dd} = \begin{bmatrix} \infty & 0 & 0 & 0 \\ 0 & \sigma_{\delta I}^2(1-\beta^2) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (16)$$

Where $\sigma_{\delta I}^2$ is the variance of the ionospheric error from its mean, which is assumed as $1cm^2$.

3. Kalman Filtering

In a general form, the GNSS multi-frequency geometry-free single-receiver single-channel undifferenced observations (y) can be formulated in terms of the unknown state vector (x) at epoch (k) as:

$$y_k = A_k x_k + e_k \quad (17)$$

with (A) denotes the design matrix. In real time, a recursive LS filter (e.g. Kalman Filter) can be applied. The dynamic model can be given as:

$$x_k = \Phi_{k/k-1}\, x_{k-1} + d_{x_k} \quad (18)$$

where d_{x_k} is the process noise of the unknowns at epoch k. For the functional models, it is assumed that for the epochs k and p we have:

$D(x_o) = Q_{x_o x_o}$, $E(e_k)=0$,

$C(e_k, x_o)=0$, $C(e_k,e_p) = Q_{ee}\, \delta_{kp}$

Where D( ), E( ), and C( ) denote the dispersion, expectation, and covariance operators; respectively, $\delta_{kp} = 1$ for k=p, 0 for $k \neq p$, and Q_{ee} is the covariance matrix of the residuals.

and for the dynamic model, it is assumed that:

$E(d_x)=0$, $C(d_{x_k}, d_{x_p})= Q_{dd}\ \delta_{kp}$

$C(d_{x_k}, x_o)=0$, $C(d_{x_k}, e_k)=0$.

The time update of the recursive filter can be formulated as:

$$\hat{x}_{k/k-1} = \Phi_{k/k-1}\, \hat{x}_{k-1/k-1} \quad (19)$$

$$P_{k/k-1} = \Phi_{k/k-1}\, P_{k-1/k-1}\, \Phi_{k/k-1}^T + Q_{dd} \quad (20)$$

where $P_{k/k-1}$ and $P_{k-1/k-1}$ are the covariance matrices of the predicted and estimated unknowns, respectively. For m measurements, the measurement update can be applied as follows:

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k\,(y_k - A_k\, \hat{x}_{k/k-1}) \quad (21)$$

$$P_{k/k} = (I - K_k\, A_k)\, P_{k/k-1} \quad (22)$$

$$K_k = P_{k/k-1}\, A_k^T\, (Q_{yy} + A_k\, P_{k/k-1}\, A_k^T)^{-1} \quad (23)$$

From the Eq. (4-7, 10, 13-14) one can conclude that initialisation of the filter for the case at hand, where $k = k_o$, can be performed such that for all available phase and codes on different frequencies ($1 < i \leq n$):

$$x_{o/o} = \begin{bmatrix} \rho^{**} \\ \delta I \\ b^{**}_{\varphi_{i=1..n}} \\ b^{**}_{P_{i=1..n}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \varphi_{i=1..n} \\ P_{i=1..n} \end{bmatrix} \quad (24)$$

where ρ^{**} is zero at $k = k_o$ since ρ_k^* equals $\rho_{k_o}^*$, and its variance is thus taken zero. δI_{k_o} is assumed equals zero at k_o. Finally, the initialization covariance matrix is then given as:

$$P_{x_{o/o}} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \sigma_{\delta I}^2 & 0 & 0 \\ 0 & 0 & \sigma_{\varphi_{i=1..n}}^2 & 0 \\ 0 & 0 & 0 & \sigma_{P_{i=1..n}}^2 \end{bmatrix} \quad (25)$$

where $\sigma_{\varphi_{i=1..n}}^2$ and $\sigma_{P_{i=1..n}}^2$ denote the phase and code variances; respectively, for all frequencies between 1 to n.

4. Quality Control and Model Validation

For simplicity, since local testing is considered in this context, time indices will be ignored in the remaining equations. For the measurement model presented in Eq. (17), the vector of least squares (LS) observation residuals ($\hat{e}$) and their covariance matrix ($Q_{\hat{e}\hat{e}}$) can be used for validation of the observations. In statistical testing, one can take the null hypothesis H_o to represent an error-free model and the mean of the LS residuals will equal zero. The null hypothesis can then be defined by:

$$H_o: y \sim N(A\, x, Q_{yy}) \quad (26)$$

With

$$\hat{e}_o \sim N(0, Q_{\hat{e}_o \hat{e}_o}) \quad (27)$$

where Q_{yy} is the covariance matrix of the observations, and the (o) subscript denotes the null hypothesis. On the other hand, the alternative hypothesis H_a is assumed to represent the presence of errors in the model. For instance, in the presence of a measurement bias in observation j, with mean ∇_j, the observation equation corresponding to the alternative hypothesis representing this bias reads:

$$y = A\, x + c_j \nabla_j + e \quad (28)$$

where c_j is the vector describing the presence of the error in the observation j, with zero entries for all its elements except 1 for the row element corresponding to the observation j. If more than one error is present, the vector c_j becomes a matrix C_j, with ∇_j becomes a vector comprising the errors, where j here is a vector referring to the observations that include these errors. Thus, C_j represents the general case, where all possible errors that are under consideration for testing can be presented. As a result, the mean of the LS residuals will be biased and in this general case the alternative hypothesis can be defined as:

$$H_a: y \sim N(A\,x + C_j \nabla_j, Q_{yy}) \tag{29}$$

where the subscript (a) denotes the alternative hypothesis.

4.1 Local testing and error detection

In real-time applications, one may consider examining the model at the present epoch using observations only from the current epoch. This is referred to as Local Testing [3]. For the general case of (m) measurements, (u) number of unknowns, and testing (q) number of errors, where q < m, the best estimator of the error vector corresponding to measurement(s) j that contain the errors can be computed as follows:

$$\hat{\nabla}_j = (C_j^T\, Q_{yy}^{-1}\, Q_{\hat{e}_o \hat{e}_o}\, Q_{yy}^{-1} C_j)^{-1}\, C_j^T\, Q_{yy}^{-1}\, \hat{e}_o \tag{30}$$

where $\hat{e}_o$ is the values of (e) estimated from Eq. (17) under the null hypothesis, and the covariance matrix of the best estimator of the error vector reads:

$$Q_{\hat{\nabla}_j \hat{\nabla}_j} = (C_j^T\, Q_{yy}^{-1}\, Q_{\hat{e}_o \hat{e}_o}\, Q_{yy}^{-1} C_j)^{-1} \tag{31}$$

Detection of the presence of model errors in local testing can be performed by using the Local Over-all Model (LOM) test statistic T_{LOM}, which can be formulated as:

$$T_{LOM} = \hat{e}_o\, Q_{yy}^{-1}\, \hat{e}_o \tag{32}$$

where one may reject H_o in favour of H_a when:

$$T_{LOM} \geq \chi^2_\alpha(df, 0) \tag{33}$$

where χ^2_α is the chi-square for a preset significance level (α), and (df) is the degrees of freedom, which equal (m-u).

If the detection test passes, then testing stops at the current epoch and the same procedure is applied for the next epoch. However, if the test fails, identification of the possible error(s) should be performed.

4.2 Local identification and Adaptation

Once the presence of model errors is detected, one needs to identify the erroneous measurement that causes such model error. The characterizing matrix C_j is set to test the alternative hypothesis corresponding to each type of possible model errors. For local testing, two cases are of particular interest:

- The case of a single outlier in one code or phase measurement, i.e. q = 1 and the C_j matrix reduces to a vector c_j. For instance, in testing the possibility of having an error in a single observation (j), c_j reads:

$$c_j = [\,0, 0, 1_j, 0, \ldots 0_n]^T$$

- The case of multiple outliers, or complete loss of lock either in phase or in code, where q > 1 and C_j remains as a matrix. For instance, for n frequencies and organizing all phase observations to precede code observations in our measurement model, and for the case of testing the hypothesis that all phase measurements may have errors, the C_j matrix reads:

$$C_j = \begin{bmatrix} I_{n\times n} \\ 0_{n\times n} \end{bmatrix}$$

For outlier identification in testing single observations where q = 1, $\hat{\nabla}_j$ becomes a scalar, and the test static (w_j) can be given as [4]:

$$w_j = \frac{\hat{\nabla}_j}{\sigma_{\hat{\nabla}_j}} \quad \text{or} \quad w_j = \frac{c_j^T\, Q_{yy}^{-1}\, \hat{e}_o}{\sqrt{(c_j^T\, Q_{yy}^{-1}\, Q_{\hat{e}_o \hat{e}_o}\, Q_{yy}^{-1}\, c_j)}} \tag{34}$$

and the null hypothesis can be rejected in favour of the alternative hypothesis when:

$$|w_j| \geq N_{\frac{\alpha}{2}}(0,1) \tag{35}$$

In case of testing the possibilities of more than one single observation error, i.e. when $1 < q \leq m - u$, C_j remains a matrix with dimensions $m \times q$ and the identification test can be formulated as follows:

$$T = \hat{\nabla}_j^T\, Q_{\hat{\nabla}_j \hat{\nabla}_j}^{-1}\, \hat{\nabla}_j \tag{36}$$

and H_o can be rejected in favour of H_a when:

$$T \geq \chi^2_\alpha(q, 0) \tag{37}$$

For the two cases mentioned above, where different alternative hypotheses are examined, we have mixed size cases (i.e. q = 1 when considering outliers in the observations and q >1 for the case of multiple outliers). Thus, a unified criterion needs to be set to compare the statistical testing outcomes of different alternative hypotheses. This can be achieved by comparing the P-

value under the χ^2 distribution. Thus, both w_j and T_j (where $T_j = w_j^2$) are computed for the case of q=1, but the probability of the latter can be used (as it has χ^2 distribution) in the sought comparison between different alternative hypotheses. Next, all alternative hypotheses are ranked according to their P-value in a descending order, where hypothesis of smallest P-value put on top of the list as the most suspected error, and the one associated with the largest P-value in the bottom of the list.

Once possible error, or errors, is/are identified; the estimated values of the unknowns, which were originally determined as $\hat{x}$ assuming an error-free model (of the null hypothesis) can be adjusted to adapt for the presence of the errors as follows:

$$\hat{x}_a = \hat{x}_o - Q_k \, C_j \, \hat{\nabla}_j \tag{38}$$

where $\hat{x}_a$ is the adapted vector of the unknowns, and its adapted covariance matrix $Q_{\hat{x}_a\hat{x}_a}$ can be determined from its model-free equivalent $Q_{\hat{x}_o\hat{x}_o}$, such that:

$$Q_{\hat{x}_a\hat{x}_a} = Q_{\hat{x}_o\hat{x}_o} + Q_k C_j \, Q_{\hat{\nabla}_j\hat{\nabla}_j} \, C_j^T \, Q_k^T \tag{39}$$

where Q_k matrix is determined from:

$$Q_k = (A^T Q_{yy}^{-1} A)^{-1} A^T Q_{yy}^{-1} \tag{40}$$

After identification and before finalizing the adapted values of the unknowns, another cycle of detection and identification has to be performed [6]. In this case, the detection test given in Eq. (32) has to be performed, but with reduced df by the number of the found error(s), and replacing ($\hat{e}_o$) by their updated values of the LS residuals ($\hat{e}_a$), where:

$$\hat{e}_a = y - \hat{y}_a \tag{41}$$

and

$$\hat{y}_a = A \, \hat{x}_a + C_j \, \hat{\nabla}_j \tag{42}$$

4.3 The Minimal Detectable Bias

In quality control practice, internal reliability, quantified by the Minimal Detectable Bias (MDB) can be computed even before actual measurements have been carried out using only a functional model and the expected stochastic properties of the data. The MDB is a measure for the size of the errors that can be detected with the model with a certain power and probability of false alarm [1, 5]. The MDB can be applied to a single receiver geometry-free model. To compute the MDB, the noncentrally parameter (λ_o) needs first to be determined, which can be computed based on pre-set values of the probability of false alarm (α), power of the test (γ), and the number of possible errors q (number of alternative hypotheses), such that:

$$\lambda_o = f(\alpha, \gamma, q) \tag{43}$$

When q=1, the MDB can be computed from:

$$\text{MDB} = |\hat{\nabla}_j| = \sqrt{\frac{\lambda_o}{(c_j^T \, Q_{yy}^{-1} \, Q_{\hat{e}_o\hat{e}_o} \, Q_{yy}^{-1} \, c_j)}} \tag{44}$$

5. Summary

The paper presents the procedure of quality control of GNSS measurements using a single-channel single-receiver approach that can be applied in the pre-processing stage for any GNSS system measurements. The proposed functional, dynamic and stochastic models are described and initialisation of the filter is given. The process of detection of a single or multiple measurement errors that may take place at one epoch is illustrated. Identification is performed by ranking the test statistic according to their P values. The adaptation process is discussed, which is performed to adjust the unknown parameters if an error is detected.

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